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Robust tests for heteroskedasticity in the one-way error components model☆

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ABSTRACT

This paper constructs tests for heteroskedasticity in one-way error components models, in line with Baltagi et al. [Baltagi, B.H., Bresson, G., Pirotte, A., 2006. Joint LM test for homoskedasticity in a one-way error component model. Journal of Econometrics 134, 401–417]. Our tests have two additional robustness properties. First, standard tests for heteroskedasticity in the individual component are shown to be negatively affected by heteroskedasticity in the remainder component. We derive modified tests that are insensitive to heteroskedasticity in the component not being checked, and hence help identify the source of heteroskedasticity. Second, Gaussian-based LM tests are shown to reject too often in the presence of heavy-tailed (e.g. t-Student) distributions. By using a conditional moment framework, we derive distribution-free tests that are robust to non-normalities. Our tests are computationally convenient since they are based on simple artificial regressions after pooled OLS estimation.

1. Introduction

Typical panels in econometrics are largely asymmetric, in the sense that their cross-sectional dimension is much larger than its temporal one. Consequently, most of the concerns that affect cross-sectional models harm panel data models similarly. This is surely the case of heteroskedasticity, a subject that has played a substantial role in the history of econometric research and practice, and still occupies a relevant place in its pedagogical side: all basic texts include a chapter on the subject. As is well known, heteroskedasticity invalidates standard inferential procedures, and usually calls for alternative strategies that either accommodate heterogeneous conditional variances, or are insensitive to them. The one-way error components model is the most basic extension of simple linear models to handle panel data, and it is widely used in the applied literature. In this model, heteroskedasticity may now be present in either the 'individual' error component, in the observation-specific 'remainder' error component, or in both simultaneously.

Consider the case of testing for heteroskedasticity. In the cross-sectional domain, the landmark paper by Breusch and Pagan (1979) derives a widely used, asymptotically valid test in the Lagrange multiplier (LM) maximum-likelihood (ML) framework under normality. Further work by Koenker (1981) proposed a simple 'studentization' that avoids the restrictive Gaussian assumption. This is an important result since non-normalities severely affect the performance of the standard LM based test, as clearly documented by Evans (1992) in a comprehensive Monte Carlo study. Wooldridge (1990, 1991) and Dastoor (1997) consider a more general framework allowing for heterokurtosis.

The literature on panel data has only recently produced results analogous to those available for the cross-sectional case.[1] For the one-way error component, Holly and Gardiol (2000) study the case where heteroskedasticity is only present in the individual-specific component, and derive a test statistic that is a direct analogy of the classic Breusch–Pagan test in an LM framework under normality.[2] Baltagi et al. (2006) allow for heteroskedasticity in both components and derive a test for the joint null of homoskedasticity, again, in the Gaussian LM framework. They also derive 'marginal' tests for homoskedasticity in either component, that is, tests that assume that heteroskedasticity is absent in the component not

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1 An early contribution on this topic is the seminal paper by Mazodier and Trognon (1978).

2 Recently, Baltagi et al. (2010) extended this test to incorporate serial correlation as well.

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being checked, of which, naturally, the test by Holly and Gardiol (2000) is a particular case. Both articles propose LM-type tests and, consequently, are based on estimating a null homoskedastic model, which makes them computationally attractive.[3] Closer to our work is Lejeune (2006), who proposes a pseudo-maximum-likelihood framework for estimation and inference of a full heteroskedastic model.

This paper derives new tests for homoskedasticity in the error components model that possess two robustness properties. Though the term robust has a long tradition in statistics (Huber, 1981), in this paper it is used to mean being resistant to (1) misspecification of the conditional variance of the remainder term, and (2) departures away from the strict Gaussian framework used in the ML-LM context.

The first robustness property is related to resistance to misspecification of the *a priori* admissible hypotheses, that is, to 'type-III errors' in the terminology of Kimball (1957) (see Welsh, 1996, pp. 119–120, for a discussion of these concepts). The negative effects of this type of misspecification on the performance of LM tests have been studied by Davidson and MacKinnon (1987), Saikkonen (1989) and Bera and Yoon (1993), and are found to occur when the score of the parameter of interest is correlated with that of the nuisance parameter. This type of misspecification affects the Holly and Gardiol (2000) test in the case where the temporal dimension of the panel is fixed, which assumes that heteroskedasticity is absent in the remainder term, and therefore, rejects its null spuriously not due to heteroskedasticity being present in the individual component being tested, but in the other one. This problem can be observed directly in the corresponding non-zero element of the Fisher information matrix presented in Baltagi et al. (2006). As discussed in Section 4, Lejeune's (2006) tests are similarly affected. In such cases, it is difficult to identify the presence of heteroskedasticity in the individual component since it is 'masked' by the other source. We propose a modified test for heteroskedasticity in the individual component that is immune to the presence of heteroskedasticity in the remainder term, and hence can identify the source of heteroskedasticity.

The second robustness property is related to the idea of *robustness of validity* of Box (1953), that is, tests that achieve an intended asymptotic level for a rather large family of distributions (see Welsh, 1996, ch. 5, for a discussion). In this paper, through an extensive Monte Carlo experiment, non-normalities are shown to severely affect the performance of the tests by Holly and Gardiol (2000) and Baltagi et al. (2006), consistent with the results of Evans (1992) for the cross-sectional case. We derive new tests using a conditional moment framework, and thus, they are distribution free by construction, subject to mild regularity assumptions. In this context, the LM-type tests proposed by Lejeune (2006) are also resistant to non-normalities. We also consider the case of possible heterokurtosis as a simple extension of our framework, along the line of the work by Wooldridge (1990; 1991) and Dastoor (1997).

An additional advantage of all our proposed statistics is that of simplicity, since they are based on simple transformations of pooled OLS residuals of a fully homoskedastic model, unlike the case of the tests by Holly and Gardiol (2000) and Baltagi et al. (2006) that require ML estimation. Furthermore, all tests proposed in this paper can be computed based on the R^2 coefficients from simple artificial regressions.

The paper is organized as follows. Section 2 presents the heteroskedastic error components model and the set of moment conditions used to derive test statistics in Section 3. Section 4 presents the results of a detailed Monte Carlo experiment that compares all our statistics and those obtained by Holly and Gardiol (2000), Baltagi et al. (2006) and Lejeune (2006). Section 5 considers an extension of the proposed statistics to handle heterokurtosis. Section 6 concludes and presents suggestions for practitioners and future research.

2. Moment conditions for the one-way heteroskedastic error components model

Baltagi et al. (2006) use a parametric error components model under normality and a ML estimator. In order to highlight differences and similarities, our search for distribution-free tests for heteroskedasticity will be based on a set of appropriate moment conditions. Consider the following regression model with general heteroskedasticity in a one-way error components model:

$$y_{it} = x_{it}'\beta + u_{it}, \qquad u_{it} = \mu_i + \nu_{it},$$
$$i = 1, \dots, N;\ t = 1, \dots, T, \tag{1}$$

where y_{it}, u_{it}, μ_i and ν_{it} are scalars, x_{it}' is a k_β-vector of regressors, and β is a k_β-vector of parameters. As usual, the subscript i refers to individual, and t to temporal observations. We follow the conditional moment framework introduced by Newey (1985), Tauchen (1985) and White (1987), and consider a set of conditioning variables w_{it}, containing the not necessarily disjoint elements x_{it}, $z_{\mu i}$ and $z_{\nu it}$. Here $z_{\mu i}$ and $z_{\nu it}$ are vectors of regressors of dimensions k_{θ_μ} and k_{θ_ν}, respectively. For notational convenience we also define $w_i = \{w_{i1}, \dots, w_{it}, \dots, w_{iT}\}$ and $x_i = \{x_{i1}, \dots, x_{it}, \dots, x_{iT}\}$. Throughout the paper we assume that the conditional mean of model (1) is well specified, that is, $E[u_{it}|w_i] = E[u_{it}|x_i] = 0$. In the context of the general framework specified by Wooldridge (1990, p. 18) this implies that the validity of the derived tests actually imposes more than just the hypothesis of interest, by ruling out misspecification in the conditional mean.[4]

Further, we assume that the conditional processes $\mu_i|w_i$ and $\nu_{it}|w_i$ are conditionally uncorrelated, independent across i, with $\nu_{it}|w_i$ also uncorrelated across t, and with zero conditional mean, conditional variances given by

$$\sigma^2_{\mu i} \equiv V[\mu_i|w_i] = \sigma^2_\mu h_\mu(z_{\mu i}'\theta_\mu) > 0, \quad i = 1, \dots, N, \tag{2}$$

$$\sigma^2_{\nu it} \equiv V[\nu_{it}|w_i] = V[\nu_{it}|w_{it}] = \sigma^2_\nu h_\nu(z_{\nu it}'\theta_\nu) > 0,$$
$$i = 1, \dots, N;\ t = 1, \dots, T, \tag{3}$$

and finite fourth moments. $h_\mu(.)$ and $h_\nu(.)$ are twice continuously differentiable functions satisfying $h_\mu(.) > 0$, $h_\nu(.) > 0$, $h_\mu(0) = 1$, $h_\nu(0) = 1$, $h_\mu^{(1)}(0) \neq 0$ and $h_\nu^{(1)}(0) \neq 0$, where $h^{(j)}$ denotes their jth derivatives.

In this setup, θ_μ and θ_ν will be the parameters of interest. A test for heteroskedasticity in the individual-specific component is based on the null hypothesis $H_0^{\sigma^2_\mu}: \theta_\mu = 0$; and a test for heteroskedasticity in the remainder error term is based on $H_0^{\sigma^2_\nu}: \theta_\nu = 0$. Testing for the validity of the full homoskedastic model implies a joint test with null hypothesis $H_0^{\sigma^2_\mu, \sigma^2_\nu}: \theta_\nu = \theta_\mu = 0$. Because, in general, the nature of the heteroskedasticity is unknown, z_μ and z_ν may be similar, when not identical, hence we cannot rely on them to distinguish among different types of heteroskedasticity.

Let $\bar{u}_i \equiv T^{-1}\sum_{t=1}^{T} u_{it}$ be the *between* residuals and $\tilde{u}_{it} \equiv u_{it} - \bar{u}_i$ the *within* residuals. Different moment conditions on these errors provide alternative ways of testing for both sources of heteroskedasticity.

[3] Other related contributions include Roy (2002) and Phillips (2003).

[4] Before testing for heteroskedasticity, it would be necessary first to check that the conditional mean is correctly specified. Lejeune (2006) provides robust tests for that purpose.

The squared *between* residual provides moment conditions for testing $H_0^{\sigma_\mu^2}$:

$$E[\bar{u}_i^2|w_i] = \sigma_\mu^2 h_\mu(z'_{\mu i}\theta_\mu) + T^{-2}\sigma_\nu^2 \sum_{t=1}^{T} h_\nu(z'_{\nu it}\theta_\nu). \tag{4}$$

If $H_0^{\sigma_\nu^2}$ is true, that is, if there is no heteroskedasticity in the remainder component, it simplifies to

$$E[\bar{u}_i^2|w_i] = \sigma_\mu^2 h_\mu(z'_{\mu i}\theta_\mu) + T^{-1}\sigma_\nu^2. \tag{5}$$

Moreover, if $H_0^{\sigma_\nu^2}$ does not hold, but $N \to \infty$ and $T \to \infty$, the presence of heteroskedasticity in the remainder component has no effect on a test for homoskedasticity in the individual component based on (5). In this case a test for $H_0^{\sigma_\mu^2}$ is said to be *robust* to the validity of $H_0^{\sigma_\nu^2}$. Second, if $N \to \infty$ and T is fixed, but $H_0^{\sigma_\nu^2}$ is true, the moment condition in (5) holds. A test for these cases can be based on N times the centered R^2 of an auxiliary regression of $\bar{u}^2$ on z_μ and a constant, as shown in the next section.

However, if $N \to \infty$, T is fixed and $H_0^{\sigma_\nu^2}$ does not hold, tests based on (4) may led to spurious rejections because of the presence of heteroskedasticity in the remainder component. For this case, define

$$\begin{aligned}\tilde{\bar{u}}_i^2 &= \bar{u}_i^2 - T^{-2}\sum_{t=1}^{T}\tilde{u}_{it}^2 - T^{-3}\sum_{t=1}^{T}\tilde{u}_{it}^2 - T^{-4}\sum_{t=1}^{T}\tilde{u}_{it}^2 \ldots \\ &= \bar{u}_i^2 - \frac{T^{-2}}{1-T^{-1}}\sum_{t=1}^{T}\tilde{u}_{it}^2,\end{aligned}$$

and note that

$$E[\tilde{\bar{u}}_i^2|w_i] = E\left[\bar{u}_i^2 - \frac{T^{-2}}{1-T^{-1}}\sum_{t=1}^{T}\tilde{u}_{it}^2|w_i\right] = \sigma_\mu^2 h_\mu(z'_{\mu i}\theta_\nu). \tag{6}$$

Unlike (4), this moment condition does not involve parameters related to heteroskedasticity in the remainder component, and, hence, it will be used in Section 3.2 to construct tests for heteroskedasticity in the individual component in short panels that are robust to the presence of heteroskedasticity in the remainder component.

Consider now the moment condition based on the squared *within* residual:

$$E[\tilde{u}_{it}^2|w_i] = \sigma_\nu^2\left[(1-2T^{-1}+T^{-2})h_\nu(z'_{\nu it}\theta_\nu) + T^{-2}\sum_{j\neq t}^{T} h_\nu(z'_{\nu ij}\theta_\nu)\right]. \tag{7}$$

This condition can be used to construct tests for $H_0^{\sigma_\nu^2}$. Note that σ_μ^2 and θ_μ do not appear anywhere in (7), which means that a test based on this moment condition will be robust to the presence of heteroskedasticity in the individual error component, i.e. when $\theta_\mu \neq 0$. A test for heteroskedasticity in the remainder component will be based on $NT \times R^2$, where R^2 is the centered coefficient of determination of an auxiliary regression of $\tilde{u}^2$ on z_ν and a constant (see Section 3.3). Note, there may be differences between short and long panels because $E[\tilde{u}_{it}^2|w_i] = \sigma_\nu^2(h_\nu(z'_{\nu it}\theta_\nu) + O(T^{-1}))$. This is explored in Section 3.4.

3. Robust tests for heteroskedasticity

Our tests will be based on the moment conditions considered in the previous section, following Koenker's (1981) studentization procedure. We use the asymptotic framework of Dastoor (1997) adapted to the one-way error components model structure described above.

Assumption 1. For each $i = 1, \ldots, N$ and $t = 1, \ldots, T$, $E[w_{j,it} w'_{j,it}]$ is a finite positive definite matrix, where $w_{j,it}$ is a column vector containing the distinct elements of w and 1. Moreover, $E[|w_{j,it}|^{2+\epsilon}]$, $E[|w_{j,it}\mu_i^2|^{2+\epsilon}]$ and $E[|w_{j,it}\nu_{it}^2|^{2+\epsilon}]$ are uniformly bounded for some $\epsilon > 0$.

Dastoor's framework includes Wooldridge's (1990; 1991) setup for heterokurtosis, that is, the case where the error term is allowed to have different conditional fourth moments. In our case, this would involve allowing that both $E[(\mu_i^2 - \sigma_\mu^2\, h_\mu(z'_{\mu i}\theta_\mu))^2|w_i]$ and $E[(\nu_{it}^2 - \sigma_\nu^2\, h_\nu(z'_{\nu it}\theta_\nu))^2|w_i]$ are not constants. In this section we derive tests assuming homokurtosis, since it provides an intuitive framework to motivate the statistics. The heterokurtic case and a related Monte Carlo exploration are treated as an extension in Section 5.

Assumption 2. For each $i = 1, \ldots, N$, and $t = 1, \ldots, T$, $E[(\mu_i^2 - \sigma_\mu^2\, h_\mu(z'_{\mu i}\theta_\mu))^2|w_i] = G_\mu < \infty$ and $E[(\nu_{it}^2 - \sigma_\nu^2\, h_\nu(z'_{\nu it}\theta_\nu))^2|w_i] = G_\nu < \infty$.

The test statistics will be based on transformations of the OLS residuals $\hat{u}_{it} \equiv y_{it} - x'_{it}\hat{\beta}$, where $\hat{\beta}$ is the OLS estimator of regression model (1).

3.1. Test for $H_0^{\sigma_\mu^2}$. Cases $N, T \to \infty$ and $N \to \infty$, T finite and $\theta_\nu = 0$

For these two cases, a test for $H_0^{\sigma_\mu^2}$ will be based on $\hat{\bar{\eta}}_i = \hat{\bar{u}}_i^2$, where $\hat{\bar{u}}_i \equiv T^{-1}\sum_{t=1}^{T}\hat{u}_{it}$. Define $\hat{\bar{\eta}}$, a N-vector containing the sample squared between residuals, $\mathbf{Z}_\mu$, a $N \times k_{\theta_\mu}$ matrix with the sample matrix of covariates for testing this hypothesis, and $M_N \equiv I_N - \bar{J}_N$, where $\bar{J}_N = \iota_N \iota'_N/N$ and ι_N is a $(N \times 1)$ vector of ones. Consider a sequence of alternatives *à la* Pitman such that $\theta_\mu = \delta_\mu/\sqrt{N}$ and $0 \leq \|\delta_\mu\| < \infty$, where $\|.\|$ is the Euclidean norm. The following Theorem derives a valid test statistic for $H_0^{\sigma_\mu^2}$ for the two cases being considered.

Theorem 1. *Let* $\phi_\mu = Var[\bar{u}_i^2|w_i]$, $\mathbf{D}_\mu = \lim_{N\to\infty} E[\frac{1}{N}\mathbf{Z}_\mu M'_N \mathbf{Z}_\mu]$ *and* $\lambda_\mu = \frac{\sigma_\mu^4 h_\mu^{(1)}(0)^2}{\phi_\mu}\delta'_\mu \mathbf{D}_\mu \delta_\mu$. *Then, under Assumptions 1 and 2, as* $N, T \to \infty$ *or* $N \to \infty$, T *fixed and* $H_0^{\sigma_\nu^2}$, *and under* $H_A^{\sigma_\mu^2}: \theta_\mu = \delta_\mu/\sqrt{N}$,

$$\begin{aligned} m_\mu &\equiv N \times (\hat{\bar{\eta}}' M_N \hat{\bar{\eta}})^{-1} \hat{\bar{\eta}}' M_N \mathbf{Z}_\mu (\mathbf{Z}'_\mu M_N \mathbf{Z}_\mu)^{-1} \mathbf{Z}'_\mu \\ &\quad \times M_N \hat{\bar{\eta}} \xrightarrow{d} \chi^2_{k_{\theta_\mu}}(\lambda_\mu). \end{aligned} \tag{8}$$

Proof. Note that the sequence of random variables $\{\bar{u}_i^2\}$ is independent. Moreover, by taking a Taylor series expansion of the function $h_\mu(.)$ and Assumption 1, $\frac{1}{\sqrt{N}}\mathbf{Z}'_\mu M_N \bar{\eta} = \sigma_\mu^2 h_\mu^{(1)}(0)$ $\delta'_\mu D_\mu + o_p(1)$ and $\lim_{N\to\infty} Var[\frac{1}{\sqrt{N}}\mathbf{Z}'_\mu M_N \bar{\eta}] = \phi_\mu D_\mu$, where $\bar{\eta} = \{\bar{u}_1^2, \ldots, \bar{u}_N^2\}$. Also note that $\phi_\mu = \frac{1}{N}\bar{\eta}' M_N \bar{\eta} + o_p(1)$. Now we apply Theorem 1 in Dastoor (1997) for our sequence of squared OLS between residuals on $i = 1, \ldots, N$, which under Assumption 2 (homokurtosis) gives the desired result. □

Note that if μ is Gaussian, $\phi_\mu = 2 \times (\sigma_\mu^2 + T^{-1}\sigma_\nu^2)^2$, and then the Koenker-type test reduces to the Holly and Gardiol (2000) marginal test, which is similar to the Breusch and Pagan (1979) test where the between OLS residuals are used instead of the untransformed OLS residuals.

Consider now the auxiliary regression model (see Davidson and MacKinnon, 1990, on the use of artificial regressions)

$$\bar{\tilde{u}}_i^2 = \alpha + z'_{\mu i}\gamma + \text{residual}. \tag{9}$$

Note that m_μ is $N \times R_\mu^2$ where R_μ^2 is the centered coefficient of determination of this regression model, i.e. an auxiliary regression of $\bar{\tilde{\eta}}$ on z_μ and a constant (see Koenker, 1981, p. 111).

3.2. Test for $H_0^{\sigma_\mu^2}$. Case $N \to \infty$, T finite and $\theta_\nu \neq 0$

A test for the individual component in short panels with potential heteroskedasticity in the remainder component requires the use of condition (6). A test for $H_0^{\sigma_\mu^2}$ will be based on $\tilde{\bar{\tilde{\eta}}}_i = \tilde{\bar{\tilde{u}}}_i^2$, where $\tilde{\bar{\tilde{u}}}_i^2 = \bar{\tilde{u}}_i^2 - \frac{T^{-2}}{1-T^{-1}} \sum_{t=1}^T \tilde{\hat{u}}_{it}^2$ and $\tilde{\hat{u}}_{it} \equiv \hat{u}_{it} - \bar{\hat{u}}_i$. Define $\tilde{\bar{\tilde{\eta}}}$, a N-vector containing the transformed sample residuals.

Theorem 2. *Let $\phi_\mu^* = \lim_{N\to\infty} Var[\tilde{\bar{\tilde{u}}}_i^2|w_i]$ and $\lambda_\mu^* = \frac{\sigma_\mu^4 h_\mu^{(1)}(0)^2}{\phi_\mu^*}\delta'_\mu \mathbf{D}_\mu \delta_\mu$. Then, under Assumptions 1 and 2, as $N \to \infty$ and under $H_A^{\sigma_\mu^2}$: $\theta_\mu = \delta_\mu/\sqrt{N}$,*

$$m_\mu^* \equiv N \times (\tilde{\bar{\tilde{\eta}}}' M_N \tilde{\bar{\tilde{\eta}}})^{-1} \tilde{\bar{\tilde{\eta}}}' M_N \mathbf{Z}_\mu (\mathbf{Z}'_\mu M_N \mathbf{Z}_\mu)^{-1} \mathbf{Z}'_\mu M_N \tilde{\bar{\tilde{\eta}}} \xrightarrow{d} \chi^2_{k_{\theta_\mu}}(\lambda_\mu^*). \tag{10}$$

Proof. Similar to that in Theorem 1. □

Consider the auxiliary regression model

$$\tilde{\bar{\tilde{u}}}_i^2 = \alpha + z'_{\mu i}\gamma + \text{residual}. \tag{11}$$

Using a similar argument as before, $m_\mu^* = N \times R_\mu^{2*}$ where R_μ^{2*} is the centered coefficient of determination of the regression model. Note that the auxiliary regression model (11) covers that in model (9), and therefore, the case analyzed here is a generalization of the former.

3.3. Test for $H_0^{\sigma_\nu^2}$. $N, T \to \infty$

Consider a test for homoskedasticity in the remainder component in long panels with $N, T \to \infty$. Define $\tilde{\tilde{\eta}}_{it} = \tilde{\tilde{u}}_{it}^2$, where $\tilde{\tilde{u}}_{it} \equiv \hat{u}_{it} - \bar{\hat{u}}_i$, $\tilde{\tilde{\eta}}$, a NT-vector containing the sample *within* residuals squared, $\mathbf{Z}_\nu$, a $NT \times k_{\theta_\nu}$ matrix with the sample matrix of covariates for testing this hypothesis, and $M_{NT} = I_{NT} - (\bar{J}_N \otimes \bar{J}_T)$, where $\bar{J}_T = \iota_T\iota'_T/T$, $\otimes$ is the Kronecker product, and ι_T is a $(T\times 1)$ vector of ones. Consider a sequence of local alternatives (Pitman drift) such that $\theta_\nu = \delta_\nu/\sqrt{NT}$ and $0 \leq \|\delta_\nu\| < \infty$. The following Theorem derives an asymptotically valid test for this hypothesis.

Theorem 3. *Let $\phi_\nu = \lim_{N,T\to\infty} Var[\tilde{u}_{it}^2|w_i] = G_\nu$, $\mathbf{D}_\nu = \lim_{N,T\to\infty} E[\frac{1}{NT}\mathbf{Z}_\nu M'_{NT}\mathbf{Z}_\nu]$ and $\lambda_\nu = \frac{\sigma_\nu^4 h_\nu^{(1)}(0)^2}{\phi_\nu}\delta'_\nu \mathbf{D}_\nu\delta_\nu$. Then, under Assumptions 1 and 2, as $N, T \to \infty$ and under $H_A^{\sigma_\nu^2}$: $\theta_\nu = \delta_\nu/\sqrt{NT}$,*

$$m_\nu \equiv NT \times (\tilde{\tilde{\eta}}' M_{NT}\tilde{\tilde{\eta}})^{-1}\tilde{\tilde{\eta}}' M_{NT}\mathbf{Z}_\nu(\mathbf{Z}'_\nu M_{NT}\mathbf{Z}_\nu)^{-1}\mathbf{Z}'_\nu M_{NT}\tilde{\tilde{\eta}} \xrightarrow{d} \chi^2_{k_{\theta_\nu}}(\lambda_\nu). \tag{12}$$

Proof. Note that the sequence of random variables $\{\tilde{u}_{it}^2\}$ is asymptotically independent as $T \to \infty$, because $Cov[\tilde{u}_{it}^2, \tilde{u}_{kh}^2|w_i, w_k] = 0$, $i \neq k$ and $Cov[\tilde{u}_{it}^2, \tilde{u}_{ih}^2|w_i] = O(T^{-2})$, $t \neq h$. Then follow the proof of Theorem 1 for our sequence on $i = 1, \ldots, N$ and $t = 1, \ldots, T$, which under Assumption 2 (homokurtosis) gives the desired result. □

Note that if ν_{it} is Gaussian, $\phi_\nu = 2 \times \sigma_\nu^4$, so this Koenker-type test is the same as the Breusch–Pagan style test where the *within* OLS residual is used instead of the untransformed OLS residual.

Consider now the auxiliary regression model

$$\tilde{\tilde{u}}_{it}^2 = \alpha + z'_{\nu it}\gamma + \text{residual}. \tag{13}$$

Again, $m_\nu = NT \times R_\nu^2$, where R_ν^2 is the centered coefficient of determination of the regression model.[5]

3.4. Test for $H_0^{\sigma_\nu^2}$. $N \to \infty$ and T finite

Consider now the case where $N \to \infty$ and T is finite. For this case, consider a Taylor expansion of Eq. (7) where θ_ν is expanded about 0,

$$\begin{aligned} E[\tilde{u}_{it}^2|w_i] &= \sigma_\nu^2 + \sigma_\nu^2\left((1-2T^{-1})h_\nu^{(1)}(0)\, z'_{\nu it}\theta_\nu + T^{-2}\sum_{j=1}^T h_\nu^{(1)}(0)\, z'_{\nu ij}\theta_\nu\right) + o(\|\theta_\nu^*\|) \\ &= \sigma_\nu^2 + \sigma_\nu^2((1-2T^{-1})h_\nu^{(1)}(0)\, z'_{\nu it}\theta_\nu + T^{-1}h_\nu^{(1)}(0)\bar{z}'_{\nu i}\theta_\nu) + o(\|\theta_\nu^*\|) \end{aligned}$$

where $\bar{z}_{\nu i} = T^{-1}\sum_{t=1}^T z_{\nu it}$, $i = 1, \ldots, N$ and θ_ν^* is between θ_ν and 0. Moreover, note that $Cov[\tilde{u}_{it}^2, \tilde{u}_{ih}^2|w_i] = c = O(T^{-2})$, then, for T finite, additional covariance terms need to be taken into consideration. Define $\lim_{N\to\infty} Var[\frac{1}{\sqrt{NT}}\tilde{Z}'_\nu M_{NT}\tilde{\eta}] = \Omega_\nu$, where $\tilde{Z}_\nu$ is a $NT \times k_{\theta_\nu}$ matrix with the sample matrix of covariates with typical element $\{(1-2T^{-1})z_{\nu it} + T^{-1}\bar{z}_{\nu i}\}$, $\tilde{\eta}$ is vector of within residuals $\{\tilde{u}_{it}\}$, and let $\hat{\Phi}_\nu$ be a consistent estimate of that variance–covariance matrix of $\tilde{\eta}$.

Theorem 4. *Let $\lambda_\nu = \sigma_\nu^4 h_\nu^{(1)}(0)^2\delta'_\nu\tilde{D}_\nu\Omega_\nu^{-1}\tilde{D}_\nu\delta_\nu$ where $\tilde{D}_\nu = \lim_{N\to\infty} E[\frac{1}{NT}\tilde{Z}_\nu M'_{NT}\tilde{Z}_\nu]$. Then, under Assumptions 1 and 2, as $N \to \infty$, T fixed and under $H_A^{\sigma_\nu^2}$: $\theta_\nu = \delta_\nu/\sqrt{NT}$,*

$$m_\nu^* \equiv NT \times \tilde{\tilde{\eta}}' M_{NT}\tilde{Z}_\nu(\tilde{Z}'_\nu M_{NT}\tilde{Z}_\nu)(\tilde{Z}'_\nu M_{NT}\hat{\Phi}_\nu M_{NT}\tilde{Z}_\nu)^{-1} \times (\tilde{Z}'_\nu M_{NT}\tilde{Z}_\nu)\tilde{Z}'_\nu M_{NT}\tilde{\tilde{\eta}} \xrightarrow{d} \chi^2_{k_{\theta_\nu}}(\lambda_\nu).$$

Proof. The proof follows from Theorem 3 and Dastoor's (1997) Theorem 1. □

A convenient way to implement this test is based on the auxiliary regression model

$$\tilde{\tilde{u}}_{it}^2 = \alpha + \tilde{z}'_{\nu it}\gamma + \text{residual}, \tag{14}$$

and note that $NT \times R_\nu^{2*} = m_\nu^* + o(T^{-(2+\epsilon)})$ for any $\epsilon > 0$, where R_ν^{*2} is the centered coefficient of determination of this regression model.[6]

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[5] As noted by an anonymous referee a significant limitation of this test is that $\nu_{it}|w_i$ is not serially correlated and it should not be very difficult to construct a modified test that do not rely on this assumption (see for instance the next subsection, where additional covariance terms are considered).

[6] The Monte Carlo experiments of the next section are carried out with $T \geq 5$, and we find no significant discrepancies between the results obtained from model (14) and those carried out based on the statistic in Theorem 4, where the within individuals covariance terms c in $\hat{\Phi}_\nu$ are estimated as $\frac{1}{NT(T-1)}\sum_{i=1}^N\sum_{t=1}^T\sum_{h\neq t}^T \tilde{\tilde{u}}_{it}^2\tilde{\tilde{u}}_{ih}^2$.

3.5. Test for $H_0^{\sigma_\nu^2,\sigma_\mu^2}$

Following Baltagi et al. (2006) we construct a joint test based on the sum of the individual tests,

$$m_{\mu,\nu} = m_\mu + m_\nu. \tag{15}$$

With N and T tending to infinity, the joint test is trivially derived by exploiting the two orthogonal moment conditions (5) and (7) and hence a valid test is based on the sum of the marginal tests for each source of heteroskedasticity, which involve the sum of independent chi-squared random variables, and therefore, we have that $m_{\mu,\nu} \xrightarrow{d} \chi^2_{k_{\theta_\mu}+k_{\theta_\nu}}$. Note that the joint test by Baltagi et al. (2006) also reduces to the sum of two marginal tests when $T \to \infty$. A preliminary analysis of the Monte Carlo experiments showed that with T small, $m_{\mu,\nu}$ behave similarly to the large T case, and therefore, we find that it is not necessary to make a small panel correction.

4. Monte Carlo experiments

In order to explore the robustness properties of the proposed tests in small samples, the design of our Monte Carlo experiment will initially follow very closely that of Baltagi et al. (2006), to which we refer for further details on the experimental design, and will be modified accordingly to highlight some specific features of our tests. The baseline model is:

$$y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + \nu_{it}, \quad i = 1, \ldots, N; t = 1, \ldots, T, \tag{16}$$

where $x_{it} = w_{i,t} + 0.5 w_{i,t-1}$ and $w_{i,t} \sim iid\, U(0, 2)$. The parameters β_0 and β_1 are assigned values 5 and 0.5, respectively. For each x_i, we generate $T + 10$ observations and drop the first 10 observations in order to reduce the dependency on initial values.

The experiment considers three cases, corresponding to different sources of heteroskedasticity. In all of them, the total variance is set to $\bar{\sigma}_\mu^2 + \bar{\sigma}_\nu^2 = 8$, where $\bar{\sigma}_\mu^2 = E(\sigma_{\mu_i}^2)$ and $\bar{\sigma}_\nu^2 = E(\sigma_{\nu_{it}}^2)$. For all DGPs, ν_{it} has zero mean and variance $\sigma_{\nu_{it}}^2$, while μ_i has zero mean and variance $\sigma_{\mu_i}^2$. For each case we consider exponential heteroskedasticity, $h(z'\theta) = \exp(z'\theta)$.[7] The following heteroskedastic models are considered:

Heteroskedasticity in the remainder component (case a): $\sigma_{\nu_{it}}^2 = \sigma_\nu^2 h_\nu(\theta_\nu x_{it}), \sigma_{\mu_i}^2 = \sigma_\mu^2, \theta_\nu \in \{0, 1, 2, 3\}$, and $\theta_\mu = 0$.

Heteroskedasticity in the remainder component (case b): $\sigma_{\nu_{it}}^2 = \sigma_\nu^2 h_\nu(\theta_\nu \bar{x}_i), \bar{x}_i = T^{-1}\sum_{t=1}^T x_{it}, \sigma_{\mu_i}^2 = \sigma_\mu^2, \theta_\nu \in \{0, 1, 2, 3\}$, and $\theta_\mu = 0$.

Heteroskedasticity in the individual component: $\sigma_{\mu_i}^2 = \sigma_\mu^2 h_\mu(\theta_\mu \bar{x}_i)$, $\bar{x}_i = T^{-1}\sum_{t=1}^T x_{it}, \sigma_{\nu_{it}}^2 = \sigma_\nu^2, \theta_\mu \in \{0, 1, 2, 3\}$, and $\theta_\nu = 0$.

For each replication we have computed the test statistics proposed in this paper, those based on Lejeune's (2006) framework (based on pooled OLS residuals), and those of Baltagi et al. (2006) and Holly and Gardiol (2000), using residuals after ML estimation. In particular, the statistics considered and their corresponding null hypotheses are:

- m_μ. $H_0^{\sigma_\mu^2}$: $\theta_\mu = 0$. The statistic is N-times the R^2 from the pooled OLS regression of $\bar{\hat{u}}_i^2$ on $\bar{x}_i$ and a constant (see Section 3.1, Eq. (8)).
- m_μ^*. $H_0^{\sigma_\mu^2}$: $\theta_\mu = 0$. This test statistic is robust to the validity of $H_0^{\sigma_\nu^2}$ in short panels, and is N-times the R^2 from the pooled OLS regression of $\tilde{\hat{u}}_{it}^2$ on x_{it} and a constant (see Section 3.2, Eq. (10)).
- HG_μ. $H_0^{\sigma_\mu^2}$: $\theta_\mu = 0$. Holly and Gardiol's (2000) 'marginal' test for no heteroskedasticity in the individual component.
- L_μ. H_0 : $\theta_\mu = \theta_\nu = 0$. Lejeune's (2006) 'marginal' test for no heteroskedasticity in the individual component.
- m_ν. $H_0^{\sigma_\nu^2}$: $\theta_\nu = 0$. The statistic is NT-times the R^2 from the pooled OLS regression of $\tilde{\hat{u}}_{it}^2$ on x_{it} and a constant (see Section 3.3, Eq. (12)).
- m_ν^*. $H_0^{\sigma_\nu^2}$: $\theta_\nu = 0$. This is a finite T corrected version of the previous statistic, and is NT-times the R^2 from the pooled OLS regression of $\tilde{\hat{u}}_{it}^2$ on $\tilde{x}_{it}$ and a constant, with $\tilde{x}_{it}^* = (1 - 2T^{-1})x_{it} + T^{-1}\bar{x}_i$. (see Section 3.4, Eq. (14)).
- BBP_ν. $H_0^{\sigma_\nu^2}$: $\theta_\nu = 0$. This is the marginal tests for the null of no heteroskedasticity in the remainder component in Baltagi et al. (2006), for the case where heteroskedasticity varies with i and t; see their Section 3.2, Eq. (10).
- BBP'_ν. $H_0^{\sigma_\nu^2}$: $\theta_\nu = 0$. In this case, it is assumed that the variance of ν_{it} varies only with $i = 1, \ldots, N$. See Baltagi et al. (2006), Section 3.2, Eq. (11).
- L_ν. H_0 : $\theta_\mu = \theta_\nu = 0$. Lejeune's (2006) 'marginal' test for no heteroskedasticity in the remainder component.
- $m_{\mu,\nu}$. H_0 : $\theta_\mu = \theta_\nu = 0$. This is the proposed statistic for the joint null of homoskedasticity in both components, and is the sum of m_μ and m_ν (see Section 3.5, Eq. (15)).
- $BBP_{\mu,\nu}$. H_0 : $\theta_\mu = \theta_\nu = 0$. This is Baltagi et al.'s (2006) test for the joint null; see their Section 3.2, Eq. (13).
- $L_{\mu,\nu}$. H_0 : $\theta_\mu = \theta_\nu = 0$. This is Lejeune's (2006) test for the joint null.

We have performed 5000 replications for each case, and the proportion of rejections was obtained based on a 5% nominal level. The main goals of the experiment are to quantify (1) the effects of misspecified heteroskedasticity on new and existing tests, (2) the effects of departures away from Gaussianity, (3) the 'cost of robustification', that is, the potential power losses due to using robust tests when the 'ideal' conditions (normality and correct specification) used to derive the ML-LM based tests hold, and hence a robustification is not necessary. In order to isolate each problem, in the first subsection we will focus on robustness to misspecification, and in the second one on robustness of validity, measuring robustification costs for each case.

4.1. Robustness to misspecified heteroskedasticity

Tables 1–3 present simulation results for the Gaussian DGP, for $(N, T) = (50, 5)$ and $(N, T) = (25, 10)$ panel sizes, with $\mu_i \sim N(0, \sigma_{\mu_i}^2)$, $\nu_{it} \sim N(0, \sigma_{\nu_{it}}^2)$. Each table is split into four horizontal panels, corresponding to different variance values and panel sizes.

It is important to note that all tests are constructed using parameters estimated under the joint null hypothesis of full homoskedasticity. The Holly and Gardiol (2000), Baltagi et al. (2006) and Lejeune (2006) statistics may be affected by the presence of heteroskedasticity in the other component not being tested and which is ignored. For instance, as discussed in Section 3, misspecified heteroskedasticity is expected to affect the performance of the Holly and Gardiol (2000) statistic, that is, a test for heteroskedasticity in the individual component assuming no heteroskedasticity in the remainder component. Similarly, it

[7] Simulations were also run for quadratic heteroskedasticity, $h(z'\theta) = (1 + z'\theta)^2$, and the results are similar for size and power to those of exponential heteroskedasticity. Following the referees' suggestions we omit these results but they are available from the authors upon request.

Table 1
Empirical rejection probabilities. DGP: Normal. Heteroskedasticity in the remainder component (case a).

θ_μ	θ_ν	Exponential heteroskedasticity											
		m_μ	m_μ^*	HG_μ	L_μ	m_ν	m_ν^*	BBP_ν	BBP_ν'	L_ν	$m_{\mu,\nu}$	$BBP_{\mu,\nu}$	$L_{\mu,\nu}$
$\sigma_\mu^2 = 6, \bar{\sigma}_\nu^2 = 2$ $N = 25, T = 10$													
0	0	0.047	0.047	0.045	0.032	0.052	0.093	0.045	0.045	0.050	0.041	0.044	0.025
0	1	0.053	0.049	0.048	0.054	1.000	0.463	0.999	0.900	0.998	0.364	0.998	0.361
0	2	0.055	0.054	0.056	0.080	1.000	0.808	1.000	0.998	1.000	0.634	1.000	0.654
0	3	0.063	0.061	0.061	0.097	1.000	0.889	1.000	0.999	1.000	0.698	1.000	0.661
$N = 50, T = 5$													
0	0	0.053	0.054	0.040	0.041	0.062	0.092	0.057	0.043	0.047	0.057	0.045	0.034
0	1	0.056	0.055	0.046	0.083	1.000	0.428	1.000	0.695	0.324	1.000	1.000	0.343
0	2	0.054	0.054	0.042	0.164	1.000	0.788	1.000	0.949	0.619	1.000	1.000	0.658
0	3	0.053	0.054	0.045	0.209	1.000	0.878	1.000	0.975	0.695	1.000	1.000	0.693
$\sigma_\mu^2 = 2, \bar{\sigma}_\nu^2 = 6$ $N = 25, T = 10$													
0	0	0.055	0.054	0.049	0.039	0.056	0.052	0.053	0.047	0.047	0.050	0.049	0.035
0	1	0.099	0.069	0.092	0.288	0.999	0.996	1.000	0.903	0.999	0.985	1.000	0.944
0	2	0.181	0.088	0.183	0.485	1.000	1.000	1.000	0.998	1.000	0.996	1.000	0.981
0	3	0.276	0.119	0.300	0.512	1.000	1.000	1.000	0.999	1.000	0.980	1.000	0.937
$N = 50, T = 5$													
0	0	0.049	0.049	0.042	0.045	0.050	0.053	0.049	0.048	0.047	0.050	0.044	0.041
0	1	0.053	0.053	0.046	0.610	1.000	0.997	1.000	0.698	0.990	1.000	1.000	0.968
0	2	0.066	0.055	0.052	0.877	1.000	1.000	1.000	0.956	0.998	1.000	1.000	0.993
0	3	0.076	0.069	0.069	0.865	1.000	1.000	1.000	0.970	0.987	1.000	1.000	0.966

Notes: Monte Carlo simulations based on 5000 replications. Theoretical size 5%. *Heteroskedasticity in the remainder component, case a*: $\sigma_{\nu_{it}}^2 = \sigma_\nu^2 h_\nu(\theta_\nu x_{it})$, $\sigma_{\mu_i}^2 = \sigma_\mu^2$, $\theta_\nu \in \{0, 1, 2, 3\}$, and $\theta_\mu = 0$.

Table 2
Empirical rejection probabilities. DGP: Normal. Heteroskedasticity in the remainder component (case b).

θ_μ	θ_ν	Exponential heteroskedasticity											
		m_μ	m_μ^*	HG_μ	L_μ	m_ν	m_ν^*	BBP_ν	BBP_ν'	L_ν	$m_{\mu,\nu}$	$BBP_{\mu,\nu}$	$L_{\mu,\nu}$
$\sigma_\mu^2 = 6, \bar{\sigma}_\nu^2 = 2$ $N = 25, T = 10$													
0	0	0.050	0.050	0.047	0.036	0.048	0.049	0.047	0.050	0.039	0.045	0.040	0.023
0	1	0.053	0.053	0.046	0.053	0.205	0.238	0.194	0.745	0.054	0.165	0.146	0.034
0	2	0.053	0.053	0.045	0.090	0.493	0.567	0.554	0.995	0.073	0.396	0.479	0.053
0	3	0.054	0.053	0.041	0.143	0.680	0.755	0.787	1.000	0.102	0.582	0.730	0.074
$N = 50, T = 5$													
0	0	0.044	0.045	0.043	0.044	0.057	0.063	0.052	0.051	0.047	0.056	0.047	0.037
0	1	0.052	0.048	0.046	0.088	0.547	0.694	0.531	0.924	0.074	0.454	0.446	0.062
0	2	0.058	0.056	0.056	0.197	0.917	0.978	0.943	1.000	0.153	0.874	0.912	0.121
0	3	0.070	0.064	0.065	0.281	0.979	0.998	0.993	1.000	0.214	0.956	0.990	0.170
$\sigma_\mu^2 = 2, \bar{\sigma}_\nu^2 = 6$ $N = 25, T = 10$													
0	0	0.046	0.048	0.038	0.040	0.048	0.051	0.045	0.051	0.047	0.047	0.044	0.032
0	1	0.052	0.053	0.043	0.336	0.212	0.245	0.191	0.735	0.117	0.167	0.447	0.199
0	2	0.072	0.071	0.064	0.697	0.509	0.586	0.553	0.996	0.261	0.436	0.917	0.481
0	3	0.096	0.093	0.093	0.764	0.687	0.757	0.777	1.000	0.374	0.618	0.990	0.554
$N = 50, T = 5$													
0	0	0.046	0.048	0.043	0.050	0.053	0.055	0.051	0.040	0.051	0.055	0.044	0.043
0	1	0.103	0.065	0.095	0.666	0.556	0.696	0.514	0.934	0.337	0.492	0.447	0.498
0	2	0.232	0.117	0.242	0.932	0.922	0.979	0.934	1.000	0.679	0.906	0.917	0.823
0	3	0.377	0.182	0.396	0.897	0.979	0.997	0.992	1.000	0.774	0.970	0.990	0.775

Notes: Monte Carlo simulations based on 5000 replications. Theoretical size 5%. *Heteroskedasticity in the remainder component, case b*: $\sigma_{\nu_{it}}^2 = \sigma_\nu^2 h_\nu(\theta_\nu \bar{x}_i)$, $\sigma_{\mu_i}^2 = \sigma_\mu^2$, $\theta_\nu \in \{0, 1, 2, 3\}$, and $\theta_\mu = 0$.

should affect the performance of m_μ, our test robustified to non-normalities only. We expect our fully robust test m_μ^* to be more resistant to this type of misspecification.

Consider first Tables 1 and 2, that is, when there is heteroskedasticity in the remainder component only, cases *a* and *b*, respectively. As predicted by the results in Section 3, in terms of size distortion, m_μ and HG_μ become negatively affected by the presence of heteroskedasticity in the remainder component, that is, they tend to reject their nulls not due to the presence of heteroskedasticity in the individual component but in the other one. For example, in Table 1, with small T, the rejection rates reach 0.3 for a nominal size of 0.05. The Monte Carlo results show that this problem affects the corresponding test by Lejeune (L_μ) as well. Monte Carlo results on Lejeune's (2006) procedures are new, so it is relevant to observe that the test designed specifically to detect heteroskedasticity in the remainder component, L_ν, has correct

Table 3
Empirical rejection probabilities. DGP: Normal. Heteroskedasticity in the individual component.

θ_μ	θ_ν	Exponential heteroskedasticity											
		m_μ	m^*_μ	HG_μ	L_μ	m_ν	m^*_ν	BBP_ν	BBP'_ν	L_ν	$m_{\mu,\nu}$	$BBP_{\mu,\nu}$	$L_{\mu,\nu}$
$\bar{\sigma}^2_\mu = 6, \sigma^2_\nu = 2$ $N = 25, T = 10$													
0	0	0.048	0.047	0.045	0.035	0.054	0.095	0.049	0.050	0.043	0.049	0.044	0.027
1	0	0.326	0.327	0.344	0.067	0.055	0.172	0.049	0.049	0.072	0.255	0.276	0.042
2	0	0.776	0.773	0.815	0.151	0.054	0.330	0.049	0.051	0.131	0.662	0.737	0.077
3	0	0.952	0.953	0.974	0.232	0.051	0.494	0.046	0.055	0.205	0.881	0.950	0.126
$N = 50, T = 5$													
0	0	0.053	0.053	0.039	0.039	0.050	0.092	0.049	0.044	0.042	0.048	0.043	0.033
1	0	0.122	0.121	0.121	0.235	0.049	0.368	0.047	0.048	0.193	0.095	0.098	0.137
2	0	0.298	0.298	0.315	0.547	0.049	0.743	0.049	0.044	0.459	0.217	0.253	0.334
3	0	0.511	0.511	0.562	0.642	0.050	0.911	0.047	0.050	0.575	0.373	0.467	0.430
$\bar{\sigma}^2_\mu = 2, \sigma^2_\nu = 6$ $N = 25, T = 10$													
0	0	0.047	0.050	0.047	0.040	0.054	0.053	0.049	0.047	0.046	0.053	0.050	0.034
1	0	0.175	0.169	0.175	0.050	0.052	0.067	0.047	0.057	0.055	0.141	0.139	0.040
2	0	0.476	0.462	0.504	0.088	0.053	0.094	0.050	0.071	0.055	0.377	0.413	0.052
3	0	0.721	0.694	0.747	0.119	0.055	0.145	0.053	0.084	0.076	0.598	0.654	0.073
$N = 50, T = 5$													
0	0	0.052	0.054	0.042	0.049	0.056	0.056	0.053	0.040	0.046	0.052	0.051	0.042
1	0	0.093	0.096	0.088	0.095	0.050	0.094	0.050	0.046	0.078	0.076	0.079	0.067
2	0	0.218	0.219	0.220	0.202	0.051	0.193	0.045	0.051	0.119	0.159	0.173	0.123
3	0	0.380	0.378	0.412	0.265	0.051	0.313	0.050	0.051	0.157	0.279	0.333	0.162

Notes: Monte Carlo simulations based on 5000 replications. Theoretical size 5%. *Heteroskedasticity in the individual component*: $\sigma^2_{\mu_i} = \sigma^2_\mu h_\mu(\theta_\mu \bar{x}_i)$, $\bar{x}_i = T^{-1}\sum_{t=1}^{T} x_{it}$, $\sigma^2_{\nu_{it}} = \sigma^2_\nu$, $\theta_\mu \in \{0, 1, 2, 3\}$, and $\theta_\nu = 0$.

size and power increasing with the strength of heteroskedasticity, as can be seen in Table 1. Interestingly, the robustified test m^*_μ presents much lower rejection rates (almost a third of their competitors), hence being more resistant to misspecifications in the alternative hypothesis.

It is important to observe that, as predicted by the results of Section 2, the effects of misspecification are stronger the smaller the T is and the more important is the between variation in the remainder component. The first effect can be appreciated by comparing results for different panel sizes, and the second by comparing the cases $\sigma^2_\mu = 6, \bar{\sigma}^2_\nu = 2$ and $\sigma^2_\mu = 2, \bar{\sigma}^2_\nu = 6$ in Tables 1 and 2.

In order to highlight these points, consider the following experiments, which are a variation of the exponential heteroskedasticity in the remainder component, case a, where $\sigma^2_\mu = 2$ for all i, $\lambda_\nu = 3$, and $\bar{\sigma}^2_\nu = 6$. First, to assess the sensitivity of the proposed statistics to the panel size, we fix $N = 50$ and consider 1000 simulations for each $T \in \{2, 3, \ldots, 30\}$. Simulation results are presented graphically in Fig. 1, and show that the main problem arises because of short panels. Moreover, it shows that the main gain of using m^*_μ is in the small T case, the most likely situation in practice. All tests achieve correct size for large T, but m^*_μ achieves the correct size in shorter panels.

Second, we have also computed rejection rates depending on the size of the cross-sectional dimension of the panel, N, keeping fixed the temporal dimension; see Figs. 2 and 3. In particular, we fix $T = 2, 5$ and consider 1000 simulations for each $N \in \{10, 20, \ldots, 200\}$. Results show that m_μ, HG_μ and L_μ increasingly (and wrongly) reject as N increases. Nevertheless, m^*_μ remains insensitive to changes in N, although rejection rates are above 0.05.

Finally, we explored the effects of the relative importance of between vs. within heteroskedasticity in the remainder component. Consider now the following form of functional heteroskedasticity:

$$\sigma^2_{\nu_{it}} = \sigma^2_\nu * \exp(\lambda_\nu * (\alpha * (x_{it} - \bar{x}_i) + (1 - \alpha) * x_{it})),$$

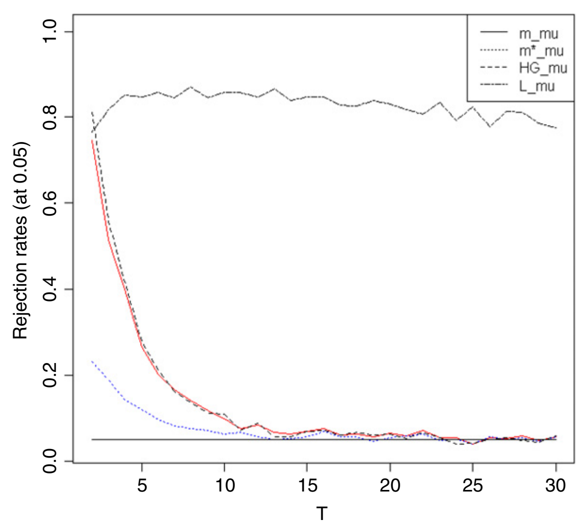

Fig. 1. Heteroskedasticity in the remainder component with T varying.

with $\alpha \in [0, 1]$. If $\alpha = 0$, this corresponds to case a in Table 1. If $\alpha = 1$, by construction, there is only *within* heteroskedasticity, and therefore no differences in the variance across individuals. For different values of α, we have generated 1000 replications for $(N, T) = (50, 5)$, and calculate the empirical size at a theoretical level of 5% of HG_μ, L_μ, m_μ and m^*_μ. Results are shown graphically in Fig. 4. HG_μ, L_μ and m_μ reject too often for small α, while m^*_μ has better size properties. Moreover, for the four statistics, the simulated empirical size approaches the theoretical level as α goes to 1.

Regarding robustification costs, tests specifically designed to detect heteroskedasticity in the remainder (m_ν, BBP_ν, BBP'_ν, L_ν) increase their empirical power with the strength of this type of heteroskedasticity and, as expected under normality, the power of BBP_ν is the largest. Interestingly, our robust test m_ν performs relatively close to the Baltagi et al. (2006) LM statistics, implying

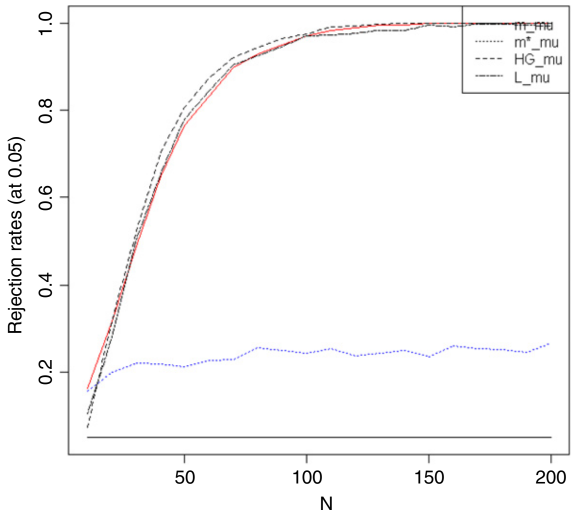

Fig. 2. Heteroskedasticity in the remainder component with N varying, $T = 2$.

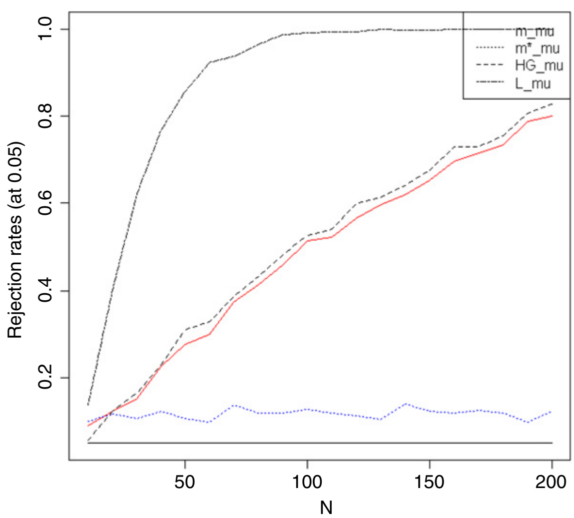

Fig. 3. Heteroskedasticity in the remainder component with N varying, $T = 5$.

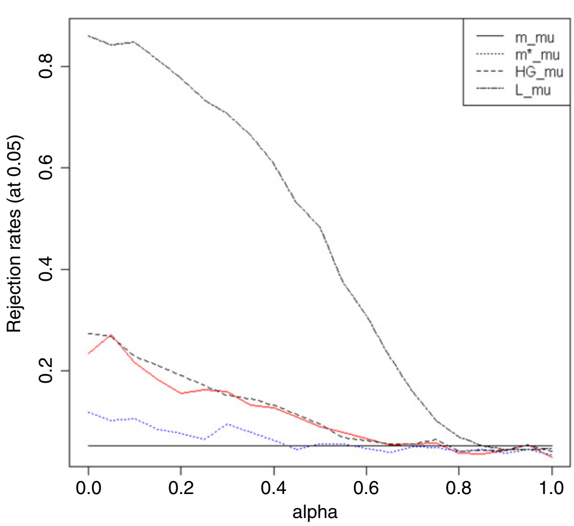

Fig. 4. Within–between heteroskedasticity in the remainder component.

that robustification costs for these particular experiments are low, that is, the loss in power for unnecessarily using a robust test is minor. Finally, note that the performance of m_ν^*, our proposed statistic designed to increase its power in small samples, is not as good as expected. First, it shows over-rejection for the $(\sigma_\mu^2 = 6, \sigma_\nu^2 = 2)$ case. Second, its power outperforms that of m_ν only in Table 2.

Consider now Table 3, where we have heteroskedasticity in the individual component only under Gaussianity. The Holly and Gardiol (2000) test is locally optimal and should have correct asymptotic size, so robustification is not necessary. Our robust statistics m_μ and m_μ^* have very similar rejection rates for all values of θ_μ, suggesting that robustification cost are small in this case too. Interestingly, the test by Lejeune (2006) has increasing power, and for the (50, 5) case it outperforms the optimal test by Holly and Gardiol (2000).

As heteroskedasticity in the individual component increases, $(m_\nu, BBP_\nu, BBP'_\nu)$ present rejection rates similar to their nominal levels, consistent with the fact that tests that check heteroskedasticity in the remainder component are immune to the presence of heteroskedasticity in the individual one. Interestingly L_ν and m_ν^* present unwanted power, that is, they reject their nulls due to heteroskedasticity in the other component, and hence are not robust to this misspecification.

Finally, joint tests present increasing power, though, as expected, they are outperformed by the marginal tests specifically designed to detect departures in a single component. The distribution-free joint statistic $m_{\mu,\nu}$ has less power than $BBP_{\mu,\nu}$ (which assumes Gaussianity) but the power loss is very small, suggesting again that robustification costs are negligible. Results are similar when the relative importance of each component is altered (that is, by comparing the two horizontal panels). Again, for the $N = 50, T = 5$ case and when the individual variance is relatively larger than the individual one (second panel of Table 3), the joint test by Lejeune (2006) presents the highest power.

Although not reported (results are available from the authors upon request), for completeness, we have also considered the case of heteroskedasticity in both components.[8] Our proposed moment-based marginal tests do not diminish their power as we add misspecification of the type not being tested. That is, in general, their power performance increases for greater heteroskedasticity in the other component, and in fact, they have a similar performance to the Baltagi et al. (2006) LM tests.

To summarize, the robustification costs incurred by all our new statistics are small, as measured by the loss in power by unnecessarily using resistant tests in the Gaussian case.

4.2. Robustness of validity

In order to explore the effect of departures away from Gaussianity, we evaluate the performance of all the test statistics under $H_0 : \theta_\mu = \theta_\nu = 0, N = 50$ and $T = 5$, for non-normal DGPs using 5000 replications. First, we generate t-Student DGPs with 3 and 5 degrees of freedom. Second, we consider skewed-normal distributions constructed as in Azzalini and Capitanio (2003).[9] Finally, we have also considered log-normal, exponential, χ_1^2 and uniform distributions. In all cases, the random variables are standardized to have the required variances. Results appear in Table 4.

The effects of departures away from Gaussianity are dramatic. For the t-Student cases, the empirical sizes of the LM Gaussian-based statistics are considerably large. Moreover, the simulations

8 Parameters were set as follows: $\sigma_{\mu_i}^2 = \sigma_\mu^2 h_\mu(\theta_\mu \bar{x}_i)$, $\sigma_{\nu_{it}}^2 = \sigma_\nu^2 h_\mu(\theta_\nu x_{it})$, $\theta_\mu \in \{0, 1, 2, 3\}$, and $\theta_\nu \in \{0, 1, 2, 3\}$.

9 We are grateful to an anonymous referee for pointing out this distribution. We have used the SN package in R and the `rsn` command, with a shape parameter $\alpha = 20$. This random variable has a kurtosis of 1 and considerable skewness.

show that rejection rates decrease as degrees of freedom increase, and thus the DGP becomes closer to normal. Even higher rejection rates are observed for the log-normal, exponential, χ_1^2 and uniform DGPs. For instance, the log-normal has rejection rates above 0.24 for HG_μ, and close to 0.50 for BBP_ν. However, rejection rates are close to the nominal level for the skewed-normal distribution (with considerable skewness but limited kurtosis). These results are in line with Evans' (1992) simulations for the Breusch–Pagan cross-sectional test, which was found to be highly sensitive to excess kurtosis but less so to skewness.

Interestingly our new test statistics and those of Lejeune's (2006) are robust to departures away from Gaussianity, presenting empirical sizes very close to their nominal values. Surprisingly, we also find a good empirical size for the t-Student case with 3 degrees of freedom, which has infinite fourth moment, and therefore, it does not satisfy the assumptions used in the theorems of Section 3. Finally, all tests derived under Lejeune's (2006) framework present good empirical size and are, hence, robust to distributional misspecifications. Although not reported, in all cases, the proposed tests have monotonically increasing empirical power as heteroskedasticity in the tested component augments.

To summarize, the analysis confirms that, although optimal in the Gaussian case, LM tests derived under this assumption are severely affected by non-normalities, and that, on the contrary, our new statistics and those based on Lejeune's (2006) remain unaltered by changes in the underlying distribution of the error terms.

5. An extension: the heterokurtic case

We consider an extension of the tests proposed above to the case of finite but non-identical fourth moments, i.e. heterokurtosis. This is, thus, a generalization of the procedures of Wooldridge (1990, 1991) and Dastoor (1997) in the cross-sectional case, to the error components model in panel data. In this case, Assumption 2 should be dropped and the asymptotic results should be modified to allow for different variances of the conditional squared residuals. We illustrate this procedure by modifying Theorem 1 (for the tests for heteroskedasticity in the individual component), which provides a guidance for straightforward extensions for Theorems 2, 3 and 4.

Recall from Section 3.1 that $\bar{\hat{\eta}}_i = \bar{\hat{u}}_i^2$. Define

$$\hat{\Phi}_\mu = \operatorname{diag}\left\{ \left(\bar{\hat{\eta}}_1 - \frac{1}{N}\sum_{i=1}^{N}\bar{\hat{\eta}}_i\right)^2, \ldots, \left(\bar{\hat{\eta}}_N - \frac{1}{N}\sum_{i=1}^{N}\bar{\hat{\eta}}_i\right)^2 \right\}.$$

Consider the following assumption, that ensures the existence of the fourth moments:

Assumption 2′. Let $\bar{\eta} = \{\bar{u}_1^2, \ldots, \bar{u}_N^2\}$, then $\lim_{N\to\infty} Var[\frac{1}{\sqrt{N}}\mathbf{Z}_\mu' M_N \bar{\eta}] = \Omega_\mu$ is a finite positive define matrix.

The following theorem provides the asymptotic distribution of a Wooldridge (1990) type statistic for testing heteroskedasticity in the individual component with heterokurtosis. The intuition is that, as argued in Wooldridge (1990, p. 23), the White (1980) covariance matrix (in our case based on $\hat{\Phi}_\mu$) can be used to compute heteroskedasticity tests that are not affected by heterokurtosis. A similar procedure can be used to construct tests that are robust to heterokurtosis for all the test statistics considered in this paper.

Theorem 5. *Let $\lambda_\mu^h = \sigma_\mu^4 h_\mu^{(1)}(0)^2 \delta_\mu' D_\mu \Omega_\mu^{-1} D_\mu \delta_\mu$. Then, under Assumptions 1 and 2, as $N, T \to \infty$ or $N \to \infty$, T fixed and $H_0^{\sigma_\nu^2}$, and under $H_A^{\sigma_\mu^2} : \theta_\mu = \delta_\mu/\sqrt{N}$,*

$$m_{\theta_\mu}^h \equiv N \times \bar{\hat{\eta}}' M_N \mathbf{Z}_\mu (\mathbf{Z}_\mu' M_N \mathbf{Z}_\mu)(\mathbf{Z}_\mu' M_N \hat{\Phi}_\mu M_N \mathbf{Z}_\mu)^{-1} \times (\mathbf{Z}_\mu' M_N \mathbf{Z}_\mu)\mathbf{Z}_\mu' M_N \bar{\hat{\eta}} \xrightarrow{d} \chi^2_{k_{\theta_\mu}}(\lambda_\mu^h).$$

Proof. The proof follows from Theorem 1 and Dastoor's (1997) Theorem 1. □

Interestingly, following Wooldridge (1990, Example 3.2, p. 32–34) this test can also be implemented in an artificial regression setup, as $N \times R_\mu^{2h}$ of the regression of a vector of ones on $(\bar{\hat{\eta}} - \frac{1}{N}\sum_{i=1}^{N}\bar{\hat{\eta}}_i)(z_\mu - \frac{1}{N}\sum_{i=1}^{N} z_{\mu i})$, where R_μ^{2h} is the uncentered coefficient of determination of the regression.

Note that this procedure can be extended for a general variance–covariance matrix of the transformed residual η. In this case, we could define a general matrix $\Phi = (M\eta\eta' M) \odot A$, where M is a square matrix with the dimension of η (either M_N or M_{NT}), A is a selector matrix of the same dimension, with 0s and 1s that indicate which elements are non-zero, and '$\odot$' denotes the element-by-element matrix multiplication operator. By imposing adequate restrictions on the type of dependence, test statistics that are robust to heterokurtosis and several types of panel dependences can be constructed.

We conduct a small Monte Carlo experiment to evaluate the effect of heterokurtosis on our proposed statistics, and the corresponding heterokurtic-robust modifications based on the artificial regression setup explained above. We generate 1000 replications under $H_0 : \theta_\mu = \theta_\nu = 0$, $N = 50$ and $T = 5$, for non-normal DGPs with varying kurtosis. We consider 3 different cases. First, we generate half of the observations with a t-Student with 5 degrees of freedom and half with 10 degrees of freedom. Second, half with t-Student ($df = 5$) and half with a log-normal. Finally, one fifth with t-Student ($df = 5$), one fifth with t-Student ($df = 5$), one fifth with t-Student ($df = 5$), one fifth normal and one fifth with log-normal. In all cases we use the adjustment explained in the Monte Carlo section to get the required variances. Results appear in Table 5. The tests based on homokurtosis have good empirical size. In general, the Wooldridge-type statistics show rejection rates below the nominal size of 5%. Overall this suggests that heterokurtosis may not produce great size distortions.

6. Concluding remarks and suggestions for practitioners

As in the cross-sectional case, heteroskedasticity is likely to affect panel models as well. A further complication in the standard error components model is to correctly identify in which of the two components, if not in both, it is present. Available LM based tests are shown to have difficulties solving this problem. First, by relying strictly on distributional assumptions, they are prone to be negatively affected by departures away from the Gaussian framework in which they are derived. This paper shows that this is clearly the case, since alternative distributions (in particular, heavy-tailed ones) lead to spurious rejections of the null of homoskedasticity. Second, joint tests of the null of homoskedasticity in both components, though helpful in serving as a starting diagnostic check, are by construction unable to identify the source of heteroskedasticity. More importantly, the marginal LM test for the individual component rejects its null in the presence of heteroskedasticity in *either* component, and hence, cannot help in identifying which error is causing it.

Our new tests are robust in these two senses, that is, they have correct asymptotic size for a wide family of distributions and they have power only in the direction intended for. An extensive Monte Carlo experiment confirms the severity of these problems and the adequacy of our new tests in small samples. Our new tests are computationally convenient, since they are based on simple algebraic transformations of pooled OLS residuals, unlike the tests by Baltagi et al. (2006) or Holly and Gardiol (2000) that require ML or pseudo-ML estimation. Also, the extension to the case of unbalanced panels is immediate in the case of our tests, due to the use of simple moment conditions, in contrast with

Table 4
Empirical rejection probabilities. Size distortions with different DGPs. $N = 50$, $T = 5$.

	Exponential heteroskedasticity											
	m_μ	m^*_μ	HG_μ	L_μ	m_ν	m^*_ν	BBP_ν	BBP'_ν	L_ν	$m_{\mu,\nu}$	$BBP_{\mu,\nu}$	$L_{\mu,\nu}$
DGP	$\sigma^2_\mu = 6, \bar{\sigma}^2_\nu = 2$											
Gaussian	0.053	0.053	0.039	0.039	0.050	0.092	0.049	0.044	0.042	0.048	0.043	0.033
t_3	0.049	0.049	0.207	0.042	0.055	0.083	0.320	0.324	0.049	0.055	0.384	0.042
t_5	0.055	0.055	0.105	0.050	0.050	0.086	0.176	0.189	0.047	0.051	0.192	0.052
Skewed-N	0.049	0.051	0.065	0.047	0.056	0.074	0.092	0.088	0.055	0.049	0.091	0.044
Log-normal	0.051	0.050	0.314	0.046	0.054	0.065	0.485	0.500	0.051	0.061	0.590	0.041
Exponential	0.048	0.048	0.177	0.032	0.059	0.072	0.238	0.242	0.043	0.055	0.297	0.031
χ^2_1	0.057	0.056	0.275	0.051	0.064	0.080	0.333	0.353	0.048	0.064	0.439	0.039
Uniform	0.055	0.055	0.193	0.049	0.053	0.091	0.013	0.006	0.053	0.051	0.141	0.041
	$\sigma^2_\mu = 2, \sigma^2_\nu = 6$											
Gaussian	0.052	0.054	0.042	0.049	0.056	0.056	0.053	0.040	0.046	0.052	0.051	0.042
t_3	0.048	0.050	0.153	0.043	0.054	0.052	0.341	0.344	0.046	0.054	0.359	0.049
t_5	0.050	0.051	0.077	0.047	0.050	0.052	0.182	0.187	0.046	0.049	0.170	0.045
Skewed-N	0.056	0.057	0.057	0.054	0.054	0.049	0.092	0.087	0.065	0.051	0.088	0.055
Log-normal	0.054	0.054	0.243	0.051	0.054	0.055	0.494	0.496	0.046	0.063	0.543	0.049
Exponential	0.050	0.049	0.115	0.039	0.059	0.052	0.240	0.251	0.049	0.053	0.248	0.033
χ^2_1	0.056	0.055	0.166	0.045	0.056	0.048	0.359	0.364	0.046	0.056	0.386	0.051
Uniform	0.057	0.057	0.202	0.050	0.049	0.056	0.011	0.007	0.046	0.050	0.140	0.045

Notes: Monte Carlo simulations based on 5000 replications. Theoretical size 5%.

Table 5
Empirical rejection probabilities. Heterokurtosis. $N = 50$, $T = 5$.

Test statistic	m_μ	m^*_μ	m^h_μ	m^{*h}_μ	m_ν	m^h_ν	m^*_ν	m^{*h}_ν	$m_{\mu,\nu}$	$m^h_{\mu,\nu}$
DGP	$\sigma^2_\mu = 6, \sigma^2_\nu = 2$									
t_5 & t_{10}	0.045	0.043	0.033	0.031	0.045	0.044	0.055	0.050	0.044	0.036
t_5 & $\log -N$	0.058	0.054	0.026	0.021	0.058	0.038	0.077	0.054	0.056	0.032
t_5 & t_7 & t_{10} & *Normal* & $\log -N$	0.054	0.059	0.038	0.043	0.070	0.055	0.072	0.068	0.064	0.048
DGP	$\sigma^2_\mu = 2, \sigma^2_\nu = 6$									
t_5 & t_{10}	0.049	0.048	0.043	0.042	0.050	0.038	0.057	0.049	0.046	0.044
t_5 & $\log -N$	0.051	0.052	0.022	0.021	0.057	0.038	0.071	0.053	0.062	0.034
t_5 & t_7 & t_{10} & *Normal* & $\log -N$	0.048	0.052	0.030	0.031	0.054	0.045	0.060	0.056	0.051	0.039

Notes: Monte Carlo simulations based on 1000 replications. Theoretical size 5%.

many other error component procedures whose derivation for the unbalanced case requires complicated algebraic manipulations (see Sosa-Escudero and Bera, 2008, for a recent case). Note that Lejeune's (2006) tests allow for unbalanced panels too.

In practice, the use of our new tests will depend on the hypothesis of interest. Obviously, joint tests are a useful starting point as a general diagnostic test, since they have correct size and power to detect departures away from the general null of homoskedasticity. Marginal tests can be used when the interest lies in one particular direction, our tests being particularly helpful in small samples. Additionally, marginal tests can be combined in a Bonferroni approach, to produce a joint test that is compatible with the marginal ones (see Savin, 1984, for further details). That is, compute both marginal tests, and reject the joint null if at least one of them lies in its rejection region, where the significance level for the marginal tests is halved, in order to guarantee that the resulting joint test has the desired asymptotic size. This is the essence of the 'multiple comparison procedure' in Bera and Jarque (1982).

Regarding further research, this paper focuses mostly on preserving consistency and correct asymptotic size, with minimal power losses with respect of existing ML based tests. Power improvements can be expected from using a quantile regression framework, as in Koenker and Bassett (1982), which finds power gains by basing a test for heteroskedasticity on the difference in slopes in a quantile regression framework, for the cross-sectional case. The literature on quantile models for panels is still incipient, though promising (see Koenker, 2004; Canay, 2008; Galvao, 2009), so further developments along the results of this research line seem promising.

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