

NBER WORKING PAPER SERIES

EFFICIENT UNEMPLOYMENT INSURANCE

Daron Acemoglu  
Robert Shimer

Working Paper 6686  
<http://www.nber.org/papers/w6686>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
August 1998

We thank Olivier Blanchard, Peter Diamond, Hugo Hopenhayn, John Leahy, Juan Pablo Nicolini, Robert Topel, an anonymous referee, and seminar participants at Berkeley, Cleveland Fed, Econometric Society, Georgetown, Harvard/MIT, LSE, Maryland, NBER, NYU, Princeton and Universidad Di Tella (Argentina) for useful comments. Financial support from the National Science Foundation Grants SBR-9602116 and SBR-9709881 are gratefully acknowledged. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

© 1998 by Daron Acemoglu and Robert Shimer. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Efficient Unemployment Insurance  
Daron Acemoglu and Robert Shimer  
NBER Working Paper No. 6686  
August 1998  
JEL No. D83, J64, J65

**ABSTRACT**

This paper constructs a tractable general equilibrium model of search with risk-aversion. An increase in risk-aversion reduces wages, unemployment, and investment. Unemployment insurance (UI) has the reverse effect due to market generated moral hazard: insured workers seek high wage jobs with high unemployment risk. An economy with risk-neutral workers achieves maximal output without any UI. In contrast, in an economy with risk-averse workers, a positive level of UI maximizes output. Therefore, moderate UI not only improves risk-sharing, but also increases output.

Daron Acemoglu  
Department of Economics  
MIT, E52-371  
50 Memorial Drive  
Cambridge, MA 02142-1347  
and NBER  
daron@mit.edu

Robert Shimer  
Department of Economics  
Princeton University  
001 Fisher Hall  
Princeton, NJ 08544-1021  
shimer@princeton.edu

# 1 Introduction

We develop a general equilibrium model of search and matching with risk-averse agents and incomplete insurance. Firms make irreversible investments and post wages. Workers optimally search among posted wages. There is an unavoidable risk for workers in this frictional economy as they may suffer unemployment. When workers are more risk-averse, wages and unemployment decrease, and firms invest less. The reason is that risk-averse workers wish to avoid the risk of unemployment, and in response, the labor market offers its own version of insurance, an equilibrium with higher employment but lower wages. In a frictional market, when the unemployment risk of workers is reduced, the vacancy risk of firms increases, implying lower utilization of their ex ante investment. Anticipating this, firms reduce their capital-labor ratio.

UI encourages workers to apply to high wage jobs with high unemployment risk. The impact of UI on worker and firm behavior is driven by a form of moral hazard. Because the insurers cannot directly control workers' actions, the increased utility of unemployment induces them to search for higher wage jobs. Firms respond by creating high wage jobs, with greater unemployment risk and greater capital-labor ratios, enabling workers to seek riskier opportunities.

Next, we investigate how risk-aversion and UI affect the level of output. We find that the decentralized equilibrium fails to maximize output as long as workers are risk-averse. This is because when opening low wage jobs with lower unemployment risk, firms are creating high vacancy risk for themselves, and so prefer to reduce their ex ante investments. This moves the economy away from “productive efficiency” and reduces net output. We show that there always exists a level of incomplete UI which ensures productive efficiency and maximizes output. Hence the title of our paper: “Efficient” Unemployment Insurance. Although an allocation that maximizes output does not maximize ex ante utility, i.e. is not *optimal*, our results imply that moderate UI not only creates risk-sharing benefits, but also increases the level of output. As a result, an economy with optimal UI may have *higher* output than an economy with no UI. This contrasts with existing results on optimal insurance which emphasize risk-sharing/output (equity/efficiency) tradeoffs. In our basic model, this contrast is sharp because conventional moral hazard is absent — UI does not reduce search effort. We show, however, that even when search effort is endogenous, UI continues to increase the level of output for plausible parameter values.

A feature of our model is that despite the dynamic nature of the decisions, the analysis can be carried out in a static model. We prove this by fully analyzing a dynamic model of search with precautionary savings, and establishing that all the qualitative

results are the same as in a static model. Since the equilibrium of the static model is equivalent to the solution of a constrained maximization problem and can be analyzed diagrammatically, it can easily be used for other applications.

Our model is closely related to the general equilibrium search literature (e.g. Diamond, 1982, Mortensen, 1982, Pissarides, 1990) and to the wage posting models of Peters (1991) and Montgomery (1991). We extend these papers by including risk-aversion and capital investments. In the process, we also generalize the efficiency results of Hosios (1990), Moen (1997), and Shimer (1996).

Perhaps the most surprising result of our analysis is that an appropriate level of UI restores the economy to the output maximizing allocation. We are aware of no other paper that has obtained this result, although others have also demonstrated that unemployment benefits may improve the allocation of resources in search models (Diamond, 1981; Acemoglu, 1997; Marimon and Zilibotti, 1997). However, because these papers assume risk-neutrality, UI is simply a subsidy to search. In contrast, we work with a model in which UI reduces output when workers are risk-neutral, demonstrating that the role of UI is not to subsidize search, but to undo the distortions introduced by uninsured risks. Many of our results, including the comparative static results with respect to attitudes towards risk and the finding that UI is the right instrument to achieve productive efficiency, rely explicitly on our modeling of risk-aversion. Our model also implies that the impact of UI on search behavior crucially depends on the wealth level of individuals, an empirical prediction that distinguishes our mechanism from models in which unemployment benefits are simply a subsidy to search.

The result that UI is welfare improving is also related to the implicit contract literature developed by Azariadis (1975), Bailey (1974), and Gordon (1974). In this literature, firms provide insurance to risk-averse workers by increasing employment above the first-best level. Introducing UI would make workers more willing to take the job loss risk and improve welfare. There are, however, important differences between this story and our's. First, endogenous job composition (capital-labor ratios) and free entry play key a role in our model, and imply that UI not only increases ex ante welfare but also raises *the level of output* and improves the composition of jobs. In the standard implicit contract model, UI reduces output, and there are no implications about job composition. Second, our result is derived in a general equilibrium search model, which has two advantages: (i) the source of the inefficiencies are fully specified, and we show that when frictions disappear, the economy is efficient and there is no room for UI; (ii) firms cannot improve upon the decentralized equilibrium, which contrasts with the implicit contract setting where firms can increase their profits by introducing severance payments.

Analyses of optimal UI in the presence of asymmetric information, e.g. Shavell and Weiss (1979), Hansen and Imrohorglu (1992), Atkeson and Lucas (1995) and Hoppen-

hayn and Nicolini (1997), and partial equilibrium analyses of UI such as Mortensen (1977) also relate to our work. But because they treat the distribution of jobs and wage offers as exogenous, they do not share our result that UI may increase output.

We start in the next section with a static model that illustrates most of our main points. To highlight our main innovations, we also abstract from conventional moral hazard. In Section 3, we analyze the output implications of risk-aversion and UI. In Section 4, we characterize the optimal level of UI. Section 5 extends our analysis to a dynamic setup and introduces conventional moral hazard. Section 6 discusses extensions and empirical implications of our analysis. Section 7 concludes. Appendix A contains the important proofs, while the remaining proofs are available upon request.

## 2 A Model of Job Search by Risk-Averse Agents

### 2.1 Preferences and Technology

There is a continuum 1 of identical workers, each with the von Neumann-Morgenstern utility function  $u(c)$  over final consumption.  $u$  is twice continuously differentiable, strictly increasing, and weakly concave. All workers are endowed with initial wealth  $A_0$ , which they may either store or invest in a mutual fund. The absence of aggregate risk ensures that the mutual fund's gross rate of return is equal to the return from storage,  $R = 1$ . Worker  $i$ 's consumption is therefore equal to his assets  $A_0$ , minus lump-sum taxes  $\tau$ , plus net income from wages or unemployment benefits  $y_i$ , so his utility is

$$u(A + y_i),$$

where  $A \equiv A_0 - \tau$  is after-tax assets. In most of the paper, we will think of UI as provided by the government and financed by taxes, but it could also be provided by a private insurance firm or the worker's family.

There is a larger continuum of potential firms, each with access to a production technology  $f : (0, \infty) \rightarrow (0, \infty)$  that requires one worker and capital  $k > 0$  to produce  $f(k)$  units of the consumption good.  $f$  is continuously differentiable, strictly increasing, and strictly concave, and satisfies standard conditions to ensure an interior solution:  $\lim_{k \rightarrow 0} f(k) = 0$ ,  $\lim_{k \rightarrow 0} f'(k) > 1$ , and there exists  $\bar{k}$  such that  $f'(\bar{k}) \equiv 1$ . The price of capital is normalized to  $R = 1$ , and since workers own a diversified portfolio, firms maximize expected profit. The large number of potential firms ensures free-entry, so aggregate profits are zero in equilibrium.

Workers and firms come together via search. At the beginning of the period, each firm  $j$  decides whether to buy capital  $k_j > 0$ . If it does, the firm is active, and posts a

wage  $w_j$ .<sup>1</sup> In the next stage, each worker observes all the wage offers and decides where to apply. That is, worker  $i$  seeks jobs with a wage  $w_i \in \mathcal{W}$  where  $\mathcal{W} = \{w_j, \text{ for all } j \text{ active}\}$  is the set of wage offers. If he is hired, he earns  $y_i = w_i$ . Otherwise he is unemployed and obtains UI  $y_i = z$ . A firm that hires a worker produces  $f(k)$ , while an unfilled vacancy produces nothing and its capital remains idle.

Depending on workers' application decisions, there may be more competition for some jobs than others. To capture this, we let  $q_j$  be the ratio of workers who apply for jobs at firms offering wage  $w_j$  to the number firms posting that wage. We refer to this as the job's *expected queue length*, an endogenous measure of the extent of competition for jobs offering  $w_j$ . We assume that a worker applying to wage  $w_j$  is hired with probability  $\mu(q_j)$ , where  $\mu : \mathbb{R}_+ \cup \infty \rightarrow [0, 1]$  is decreasing and continuously differentiable; if many workers apply for one type of job, each has a low employment probability. Symmetrically, the probability that firm  $j$  hires a worker is  $\eta(q_j)$ , where  $\eta : \mathbb{R}_+ \cup \infty \rightarrow [0, 1]$  is increasing and continuously differentiable.<sup>2</sup> This implies that holding constant the number of workers applying for a given wage, if more firms post that wage, each has a lower hiring probability. We impose the boundary conditions  $\eta(0) = \mu(\infty) = 0$  and  $\eta(\infty) = \mu(0) = 1$ .

This formulation of the matching technology encompasses many reasonable possibilities. One can think of firms opening jobs in different geographic regions or industries, and workers directing their search towards one of these labor markets (see Acemoglu, 1997). In labor market  $j$ , all firms offer a common wage  $w_j$ , and the ratio of workers to firms,  $q_j$ , determines the matching probabilities. Standard matching frictions ensure that within an individual labor market, unemployment and vacancies coexist. Moen (1997) offers a model where labor markets with different wages are created by a competitive "market making" sector. Montgomery (1991), Peters (1991) and Burdett, Shi, and Wright (1997) offer another story. Workers use identical mixed strategies in making their applications. A firm may receive multiple applications, in which case it hires one applicant and the others remain unemployed; or it may receive no applications, in which case its capital remains idle. One can prove that if in expectation  $q$  workers apply to each firm posting a wage of  $w$ , then  $\eta(q) = 1 - \exp(-q)$  and  $\mu(q) = \eta(q)/q$ . In this case, search frictions are due to a lack of coordination among workers and firms.

Another possibility is the "frictionless" matching process, where the shorter side of the market is fully employed,  $\eta^F(q) = \min\langle 1, q \rangle$  and  $\mu^F(q) = \min\langle 1/q, 1 \rangle$ . This matching process does not satisfy the differentiability assumption at  $q = 1$ , but serves as a useful

---

<sup>1</sup>Observe that firms choose their capital *irreversibly* before they hire a worker as in Acemoglu (1996) and Acemoglu and Shimer (1997). This implies that when buying capital, they take into account how easy it will be to find a worker. Partially irreversible capital decisions are empirically plausible; most equipment is purchased in advance, and many specific costs are incurred before workers are recruited.

<sup>2</sup>By having matching probabilities only as a function of ratio of workers to firms, we have implicitly imposed constant returns to scale in matching (see Pissarides, 1990). This immediately implies that  $\eta(q) \equiv q\mu(q)$ , which we use in one of the proofs.

limiting case. Although our results obtain under any smooth approximation to this matching process, they *do not hold* in this frictionless limit, demonstrating that search frictions are crucial to our analysis.

## 2.2 Definition of Equilibrium

An allocation is a tuple  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  where  $\mathcal{K} \subset \mathbb{R}_+$  is a set of capital investment levels,<sup>3</sup>  $\mathcal{W} : \mathbb{R}_+ \rightrightarrows \mathbb{R}_+$  is a set of wages offered by firms making particular capital investments,  $Q : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \cup \infty$  is the queue length associated with each wage, and  $U \in \mathbb{R}_+$  is workers' utility level. We also define the set of wages offered by some firm,  $\bar{\mathcal{W}} = \{w | w \in \mathcal{W}(k) \text{ for some } k \in \mathcal{K}\}$ . Note that if  $w \notin \bar{\mathcal{W}}$ ,  $Q(w)$  is not actually observed. Instead, these “off-the-equilibrium path actions” represent conjectures that help determine equilibrium behavior.

**Definition 1** *An equilibrium is an allocation  $\{\mathcal{K}^*, \mathcal{W}^*, Q^*, U^*\}$  such that:*

1. [Profit Maximization]  $\forall w, k,$

$$\eta(Q^*(w)) (f(k) - w) - k \leq 0$$

*with equality if  $k \in \mathcal{K}^*$  and  $w \in \mathcal{W}^*(k)$*

2. [Optimal Application]  $\forall w,$

$$U^* \geq \mu(Q^*(w))u(A + w) + (1 - \mu(Q^*(w)))u(A + z) \quad \text{and} \quad Q^*(w) \geq 0,$$

*with complementary slackness, where*

$$U^* = \sup_{w' \in \bar{\mathcal{W}}^*} \mu(Q^*(w'))u(A + w') + (1 - \mu(Q^*(w'))u(A + z)$$

*or  $U^* = u(A + z)$  if  $\bar{\mathcal{W}}^*$  is empty.*

Profit Maximization ensures that given the queue length associated with each wage, firms choose wages and capital investments to maximize profits. Also, free entry drives the maximized value of profits to zero. Optimal Application ensures that workers make their application decisions to maximize utility, and imposes a form of subgame perfection: queue lengths adjust to make workers earn the maximal level of utility  $U^*$  at any wage, including wages not offered along the equilibrium path. This feature rules out situations

---

<sup>3</sup>For notational simplicity, we omit the fraction of firms making different investments from the definition of an allocation. This only matters if  $\mathcal{K}$  is not a singleton, which is only a non-generic possibility (see below), and even then the actual distribution of capital is unimportant for computing the equilibrium. A similar statement holds for  $\mathcal{W}$ .

in which firms may not deviate to a potentially profitable wage, incorrectly conjecturing that very few workers would apply. Finally, complementary slackness ensures that if a wage  $w$  does not deliver utility  $U^*$  to a worker who is hired with probability 1, then no one applies for that wage, that is  $Q(w) = 0$ .

Observe that the queue length function  $Q^*$  contains two other pieces of useful information: first, if  $w^*$  is the unique equilibrium wage, the number of active firms is  $1/Q^*(w^*)$ ; and second, the rate of unemployment of workers applying to a wage  $w'$  is  $u(w') = 1 - \mu(Q^*(w'))$ . Clearly  $u(w')$  is increasing in  $Q^*(w')$ , since workers who apply to jobs with longer queues suffer a higher probability of unemployment.

Finally, note that at this point we do not impose a balanced budget condition. Instead, UI  $z$  and taxes  $\tau$  are treated as parameters. We add a balanced budget condition for the analysis of efficient and optimal UI in Sections 3 and 4.

### 2.3 Existence and Characterization

**Proposition 1** *There always exists an equilibrium. If  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  is an equilibrium, then any  $k^* \in \mathcal{K}$ ,  $w^* \in \mathcal{W}(k^*)$ , and  $q^* = Q(w^*)$  solve*

$$U = \max_{k, w, q} \mu(q)u(A + w) + (1 - \mu(q))u(A + z) \quad (1)$$

$$\text{subject to } \eta(q)(f(k) - w) - k = 0 \quad (2)$$

$$\text{and } w \geq z. \quad (3)$$

*Conversely, if some  $\{k^*, w^*, q^*\}$  solves the above program, then there exists an equilibrium  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  such that  $k^* \in \mathcal{K}$ ,  $w^* \in \mathcal{W}(k^*)$ , and  $q^* = Q(w^*)$ .*

An equilibrium allocation maximizes workers' utility subject to firms earning zero profits, and an allocation which does so is part of an equilibrium. In view of this result, we will refer to a triple  $\{k^*, w^*, q^*\}$  that satisfies Proposition 1 as an "equilibrium".

Let us also define  $f'(\bar{k}) \equiv 1$  and  $\bar{z} \equiv f(\bar{k}) - \bar{k}$ . Then, whenever  $z > \bar{z}$ , the constraint set, given by (2) and (3), is empty, and there are no active firms in equilibrium (i.e.  $\mathcal{K} = \bar{\mathcal{W}} = \emptyset$ ). In contrast, if  $z < \bar{z}$ , then  $\mathcal{K}, \bar{\mathcal{W}} \neq \emptyset$  (see Step 3 of the proof of Proposition 1 in the Appendix), and any equilibrium wage is strictly greater than  $z$ , so (3) is slack. In the rest of the paper, we restrict attention to the (interesting) case of  $z < \bar{z}$ , and ignore constraint (3).

Using Proposition 1, we can characterize firms' investment decisions. Since  $k$  only affects (2), an equilibrium  $k^*$  must satisfy the optimal capital choice condition  $\eta(q^*)f'(k^*) = 1$ . Combining this with zero profits gives:

$$w^* = f(k^*) - k^* f'(k^*). \quad (4)$$



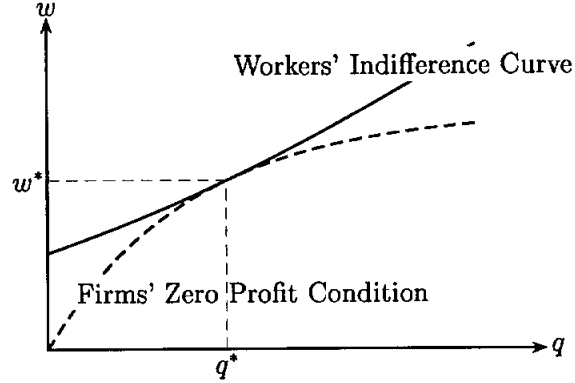


Figure 1: An equilibrium maximizes workers' utility subject to firms earning zero profits. Higher wages and shorter queues raise utility and lower profits.

Despite the search frictions, capital earns its marginal product and labor keeps the residual output. Since  $f$  is concave, (4) defines  $w^*$  as an increasing function of  $k^*$ .

Proposition 1 and equation (4) allow us to depict an equilibrium graphically. First, the optimal capital choice and (4) imply that the constraint, firms' zero profit condition, is an upward sloping curve in  $\{q, w\}$  space, as shown in Figure 1. Similarly, workers' indifference curves are upward sloping in  $\{q, w\}$  space. An equilibrium then is a point of tangency between these two indifference curves. Either curve, however, could be non-convex, so uniqueness of the equilibrium is not guaranteed.

## 2.4 Comparative Statics

### Proposition 2

1. Let  $\{k_i, w_i, q_i\}$  be an equilibrium when the utility function is  $u_i$ . If  $u_1$  is a strictly concave transformation of  $u_2$ , then  $k_1 < k_2$ ,  $w_1 < w_2$ , and  $q_1 < q_2$ .
2. Let  $\{k_i, w_i, q_i\}$  be an equilibrium when the initial asset level is  $A_i$ . If  $A_1 < A_2$  and the utility function has decreasing absolute risk aversion, then  $k_1 < k_2$ ,  $w_1 < w_2$ , and  $q_1 < q_2$ . With increasing absolute risk aversion, the inequalities are reversed. With constant absolute risk aversion, asset levels do not affect the set of equilibria.
3. Let  $\{k_i, w_i, q_i\}$  be an equilibrium when the unemployment benefit is  $z_i$ . If  $z_1 < z_2$ , then  $k_1 < k_2$ ,  $w_1 < w_2$ , and  $q_1 < q_2$ .

The first part states that with more risk-averse workers, wages are lower, job queues are shorter and firms invest less. The second part shows that with decreasing absolute risk

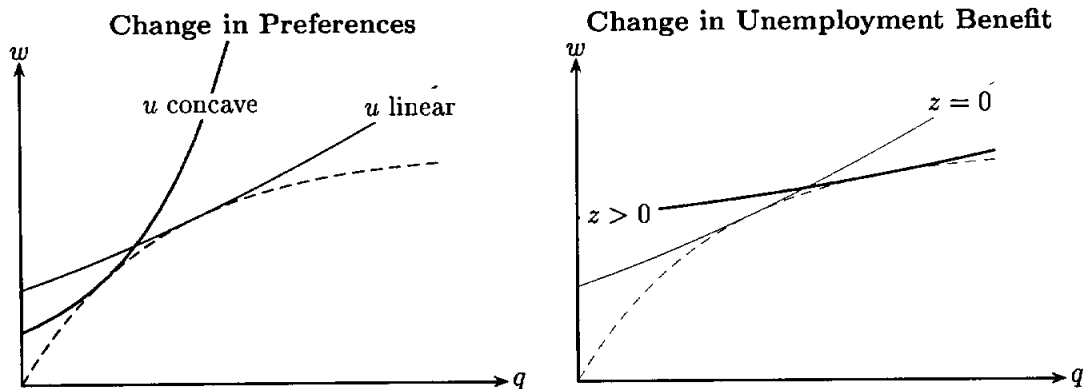


Figure 2: The left panel shows that more risk-aversion makes indifference curves steeper, and lowers the equilibrium wage and queue length. The right panel shows that higher UI makes indifference curves flatter, having the opposite effect.

aversion (DARA), poorer workers are effectively more risk-averse. The third part shows that higher UI increases wages, unemployment and capital-labor ratios.

The Appendix contains a formal proof of Proposition 2, but the idea can be seen graphically in Figure 2. Firms' zero profit condition (2) is unaffected by preferences, assets, and UI. The comparative statics are therefore due exclusively to changes preferences. More risk-averse workers have indifference curves that are everywhere steeper. In contrast, when UI increases, the new set of indifference curves is everywhere flatter. For example, as indifference curves become flatter, the point of tangency must shift to the right, to a point of higher wages and longer job queues. The comparative static results are always unambiguous because the sets of indifference curves cross only once.

The intuition of Proposition 2 is a good way of illustrating the key innovations of our analysis. Frictional matching introduces an inherent risk as workers may remain unemployed and suffer low consumption. In the absence of UI, the labor market offers *its own brand of insurance* as firms see the profit opportunities in creating jobs with lower unemployment risk, and charge an "insurance premium" to risk-averse workers by offering lower wages. This market insurance also affects the form of production in the economy, because opening a large number of jobs creates high "vacancy risk" for firms, reducing investment and capital intensity. The reason why there is no separation between the consumption and production sides of this economy is incomplete insurance. In the presence of complete insurance, workers would maximize their expected income and capital-labor ratios would not respond to changes in risk-aversion.

When UI increases, workers wish to apply to higher wages which are associated with higher unemployment risk. Firms once more cater to these preferences, and wages, unemployment and capital intensity increase. Underlying this labor market adjustment

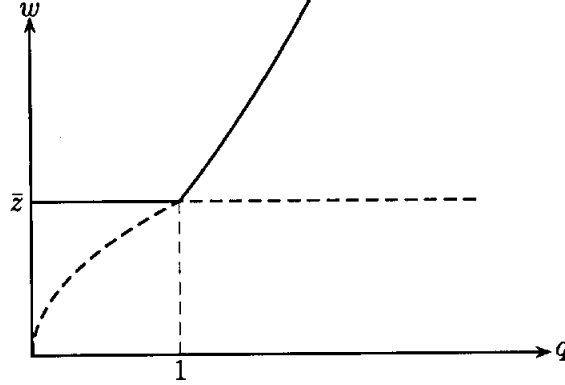


Figure 3: This depicts the equilibrium under the frictionless matching technology  $\eta^F, \mu^F$ . It is unaffected by unemployment benefits and preferences.

is *market generated moral hazard* (distinct from conventional moral hazard discussed in Section 5). As is common, the insurer would like to prevent workers from taking on more risk. The inability to control agents' actions directly is the essence of the moral hazard problem (e.g. Holmstrom, 1982). In our model, the insurer would like the worker not to apply to higher wages after receiving UI. But, since we assume that the worker's application decision is private information, insurance cannot be conditioned on it. We call this market generated moral hazard, because the response of the labor market is crucial: if firms did not change their wage offers, workers would be unable to apply to higher wages.

A corollary of Proposition 2 is that the equilibrium is generically unique. For example, holding all the other parameters fixed, the equilibrium is unique for almost every level of UI. To see why, consider the correspondence  $K(z)$ , defined as the set of all  $k$  that solve the constrained optimization problem in Proposition 1. The third part of Proposition 2 implies  $K$  is strictly monotonic, and so there are at most a countable number of  $z$  for which  $K$  is not a singleton. At all other  $z$ , the equilibrium is unique. A similar argument implies generic uniqueness in an appropriate space of utility functions.

We conclude by observing that with the frictionless matching technology described above, Proposition 2 does not hold. If  $\eta^F(q) = \min(1, q)$  and  $\mu^F(q) = \min(1/q, 1)$ , the unique equilibrium is  $\{k, w, q\} = \{\bar{k}, \bar{z}, 1\}$  for any degree of risk aversion and any UI  $z < \bar{z}$ . Workers gain nothing by applying for a job with queue length less than 1, and firms gain nothing by offering a wage that yields a queue length greater than 1. Therefore, indifference curves and the zero profit condition are kinked at  $q = 1$ , ensuring that this is the point of "tangency" (see Figure 3). Then  $q = \eta^F(q) = 1$  implies  $f'(k) = 1$ , and hence  $k = \bar{k}$ . Finally, (4) yields  $w = f(\bar{k}) - \bar{k} \equiv \bar{z}$ . Nevertheless, with any continuously differentiable approximation to  $\eta^F$  and  $\mu^F$ , the comparative statics results

in Proposition 2 obtain. The fact that our results limit to the competitive equilibrium and fail to hold at this limit point (with  $\eta^F$  and  $\mu^F$ ) is reassuring: frictions, as well as incomplete insurance, are crucial for our conclusions.

## 2.5 Worker Heterogeneity

We have simplified our analysis by assuming that all workers have the same level of assets, the same utility function, and receive the same level unemployment benefits. Our results generalize to an environment in which workers differ with respect to all of these features. In particular, suppose that there are  $s = 1, 2, \dots, S$  types of workers, where type  $s$  has utility function  $u_s$ , after-tax asset level  $A_s$ , and unemployment benefit  $z_s$ . Let  $U$  now be a vector in  $\mathbb{R}^S$ , and assume, for simplicity, that  $z_s < \bar{z}$  for all  $s$ . Then:

**Proposition 3** *There always exists an equilibrium. If  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  is an equilibrium, then any  $k_s^* \in \mathcal{K}, w_s^* \in \mathcal{W}(k_s^*)$ , and  $q_s^* = Q(w_s^*)$ , solve*

$$U_s = \max_{k, w, q} \mu(q) u_s(A_s + w) + (1 - \mu(q)) u(A_s + z_s)$$

*subject to (2) for some  $s = 1, 2, \dots, S$ . If  $\{k_s^*, w_s^*, q_s^*\}$  solves the above program for some  $s$ , then there exists an equilibrium  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  such that  $k_s^* \in \mathcal{K}$ ,  $w_s^* \in \mathcal{W}(k_s^*)$ , and  $q_s^* = Q(w_s^*)$ .*

This proposition states that any triple  $\{k^*, w^*, q^*\}$  that is part of an equilibrium maximizes the utility of one group of workers, subject to firms making zero profits. In other words, the market *endogenously* segments into  $S$  different submarkets, each catering to the preferences of one type of workers, and receiving applications only from that type. The proof of the proposition is analogous to Proposition 1, and is omitted.

We can repeat the analysis of Proposition 2 for each submarket. If workers of type  $s$  are more risk-averse or receive less UI, their submarket will create lower wage jobs with shorter queues and less investment, but the rest of the submarkets will be unaffected. This result crucially depends on the assumption that workers observe all posted wages, and so can choose to ignore the presence of other submarkets.

This analysis predicts that a group of workers with higher UI will apply for higher wage jobs with longer queues and suffer longer unemployment spells. This prediction is verified empirically (Ehrenberg and Oaxaca, 1976; Katz and Meyer, 1990). This analysis also shows that (except in the case of constant absolute risk aversion), workers' asset levels will systematically affect their search behavior, and will alter the impact of UI. This last prediction distinguishes the model from one in which UI is simply a subsidy to search by risk-neutral workers.

### 3 Risk-Aversion, UI and Output

This section demonstrates that if workers are risk-averse, a moderate amount of UI funded by lump-sum taxation will raise output. Since this says nothing about the riskiness of consumption, it is not a normative statement. Nevertheless, it distinguishes our approach from existing theories of UI and implicit contracts more generally.

Suppose workers are homogeneous, and all firms invest  $k$ , offer a common wage  $w$ , and attract an average queue length of  $q$ . Net output in such an economy is given by

$$Y(q, k) \equiv \mu(q)f(k) - k/q.$$

The number of jobs is equal to the measure of workers who find a job,  $\mu(q)$ , times the output of each filled job,  $f(k)$ . From this we subtract investment expenditures, which is the measure of firms,  $1/q$ , times capital expenditure,  $k$ . If firms choose different queue lengths and capital, net output would be a weighted average of each firm's expected output. We suppress this nongeneric possibility to simplify our notation.

**Definition 2**  $\{k^e, w^e, q^e\}$  is efficient if  $Y(q^e, k^e) = \max_{q,k} Y(q, k)$ .  
 UI  $z^e$  and tax  $\tau^e$  are efficient if any associated equilibrium  $\{k_{z^e, \tau^e}, w_{z^e, \tau^e}, q_{z^e, \tau^e}\}$  is efficient and has a balanced government budget,  $\tau^e = (1 - \mu(q_{z^e, \tau^e})) z^e$ .

Note that UI is efficient only if all possible equilibria achieve the highest possible output level that is technologically possible. We begin with an important benchmark:

**Proposition 4** *If agents are risk-neutral, the unique efficient level of UI is  $z^e = \tau^e = 0$ .*

Intuitively, when there is no UI, risk-neutral agents maximize the expected value of their wages, which is identical to net income. This result is a generalization of Moen (1997) and Shimer (1996) to the case in which firms choose their physical capital investments.

**Proposition 5** *If agents are risk-averse and  $z^e = \tau^e = 0$ , output is below its maximum.*

The proof of this proposition is straightforward and omitted: changes in preferences do not affect the efficient allocation, but unambiguously change the equilibrium (Proposition 2). Since  $z^e = \tau^e = 0$  maximizes output with risk-neutral agents, it yields a different, hence lower, level of output when workers are risk-averse. Proposition 5 therefore shows that labor market insurance is “inefficient”, because in the process of creating more jobs, firms reduce capital investments and labor productivity.

A different way of expressing the intuition is to relate it to the efficiency results in search models with bargaining (e.g. Diamond, 1982; Sattinger, 1990; Hosios, 1990). In these models, if bargaining dictates that workers receive too small a fraction of the output

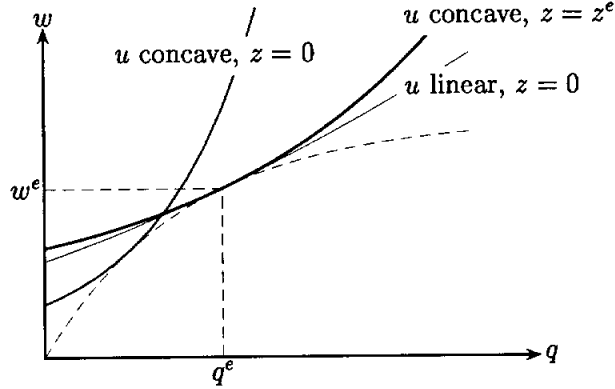


Figure 4: Risk aversion makes indifference curves steeper, while UI makes them flatter. The curve for  $u$  concave and  $z = z^e$  lies everywhere above the  $u$  linear curve.

they produce, unemployment is inefficiently low. In our model, risk-averse workers without UI prefer to receive a small fraction of output in order to reduce unemployment risk, and firms cater to these preferences by offering low wage jobs. This makes the level of frictional unemployment too low compared to the efficient level.<sup>4</sup>

We now show that output can be restored to its maximum through moderate UI.

**Proposition 6** *Let  $\{k^e, w^e, q^e\}$  be an efficient allocation where  $w^e = f(k^e) - k^e f'(k^e)$ . Then  $(z^e, \tau^e)$  is efficient if  $\tau^e \equiv (1 - \mu(q^e)) z^e$  and  $z^e \in (0, w^e)$  satisfies*

$$w^e \equiv \frac{u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)}{u'(A_0 - \tau^e + w^e)} \quad (5)$$

The technical intuition behind this result is given in Figure 4. With risk-neutral workers and no UI, any equilibrium maximizes output (Proposition 4). If instead workers are risk-averse, indifference curves are steeper, leading to a lower equilibrium wage and queue length.<sup>5</sup> Efficient UI undoes the effects of risk aversion, flattens indifference curves and restores tangency between workers' indifference curves and the constraint set at the allocation which maximizes output. However, the result is more subtle and surprising than this intuition suggests. In essence, Proposition 6 establishes that a simple and

<sup>4</sup>To emphasize the relation to existing efficiency results, note that all of our results hold in the limiting case where capital investment  $k$  is fixed at some level  $\underline{k}$ , which can be interpreted as the cost of a vacancy. The reason why an equilibrium with risk-aversion and no UI would fail to maximize output in this case is that, as in traditional search models with excessively low worker bargaining power, firms would open too many vacancies. We concentrate on the case with  $k$  variable because we believe that the distortion in capital intensity is more interesting and relevant.

<sup>5</sup>In proving Propositions 5 and 6, we compare the efficient allocation, which is the equilibrium when workers are risk-neutral, to the equilibrium with risk averse workers. We do not wish to imply that the same allocation is optimal in both economies. Instead, the comparison is simply a method of proof.

linear intervention takes the economy back to productive efficiency, which is generally not possible in non-convex economies like ours. We prove this by establishing that the thick indifference curve in Figure 4 ( $u$  concave and  $z = z^e$ ) lies everywhere above the risk-neutral indifference curve ( $u$  linear and  $z = 0$ ) and is tangent to it at  $(q^e, w^e)$ .

## 4 Optimal Unemployment Insurance

This section analyzes the “optimal” level of UI which maximizes the ex ante expected utility of the representative worker. To begin, suppose a benevolent social planner is constrained by the search technology  $\mu$  and  $\eta$ , but can choose the measure of active firms and the wage and capital level of each firm. The planner would maximize total output and divide it equally among all workers, providing full insurance to each. This is trivially the unconstrained optimum.

A more interesting exercise, however, is to ask what level of UI maximizes workers’ expected utility, with entry, investment, and wage offers determined in equilibrium (rather than being directly chosen by the planner). To this end, we define optimal UI:

**Definition 3** *Let  $\{k_{z,\tau}, w_{z,\tau}, q_{z,\tau}\}$  be an equilibrium with UI and taxes  $(z, \tau)$ . Then, the policy  $(z^o, \tau^o)$  is optimal if it maximizes  $\mu(q_{z,\tau})u(A - \tau + w_{z,\tau}) + (1 - \mu(q_{z,\tau}))u(A - \tau + z)$  subject to  $\tau = (1 - \mu(q_{z,\tau}))z$ .*

**Conjecture 1** *Optimal UI  $(z^o, \tau^o)$  exists. For all  $u$  strictly concave,  $z^o \in (z^e, \bar{z})$ .*

Intuitively, at the point of maximal output, a further increase in the UI leads to a second-order loss of net output, while it increases the income of unemployed workers and decreases the (after-tax) income of employed workers. This suggests workers will prefer the new, high UI “lottery”. Although we are unable to prove Conjecture 1, simulations support it.<sup>6</sup> Also in our simulations it is always the case that an economy functioning at the level of Optimal UI has higher output than the economy with no UI. Therefore, contrary to conventional wisdom, increasing UI to moderate levels not only increases ex ante utility due to better risk-sharing, but also due to higher output.

## 5 A Dynamic Model Without Wealth Effects

Forward looking agents can self-insure by saving even in the absence insurance markets. To explore this issue, we now analyze a dynamic model. We show that all of our results

---

<sup>6</sup>The problem that prevents a simple proof is that the high UI lottery does not second order stochastically dominate the efficient lottery. This is because equilibrium search behavior implies that a worker is more likely to be unemployed when there is more UI. This implies that some risk-averse utility functions prefer the efficient lottery to the high UI lottery. However, since both lotteries are determined by the utility function, this does not disprove the Conjecture.

generalize to this environment. This analysis may have a number of useful applications, as it is much more tractable than existing general equilibrium models of search by risk-aversion agents.<sup>7</sup> At the same time, we demonstrate the flexibility of our environment by introducing a more conventional form of moral hazard so that UI affects search effort.

We assume that workers have constant absolute risk aversion (CARA). Although we believe our results generalize to a dynamic environment with arbitrary preferences, CARA is a useful simplification. With other utility functions, a worker's wealth affects his preferences over wages and queue lengths (see part 2 of Proposition 2). The labor market segments as in Proposition 3, but more importantly a worker recognizes that his consumption decision today influences the probability distribution of jobs in the future. Since this determines the budget constraint, it feeds back and affects his consumption choice today. CARA allows us to avoid these complexities.

## 5.1 Preferences and Technology

Consider an infinite horizon economy in discrete time. Each worker  $i$  makes consumption and job search decisions to maximize:

$$\sum_{t=0}^{\infty} \beta^t \frac{1 - e^{-\theta c_{it}}}{\theta}, \quad (6)$$

where  $c_{it}$  is consumption at time  $t$ ,  $\beta < 1$  is the discount factor, and  $\theta > 0$  is the coefficient of absolute risk aversion. Firms maximize the expected value of profits, discounted at the constant gross interest rate  $R > 1$ .

The search and production technologies are generalizations of the static model. At the start of a period firms may be either inactive, vacant, or have a filled job. Similarly, workers may either be unemployed or employed. Inactive firms can create a vacancy by buying perfectly durable capital at a unit cost conveniently normalized to  $\frac{R}{R-1}$ . After this, newly created and existing vacancies post wages.<sup>8</sup> Next, unemployed workers see the menu of wages, and decide which wage, if any, to seek. The matching technology is the same as above. If the expected number of applicants is  $q$ , a worker is hired with probability  $\mu(q)$  and a firm's probability of hiring is  $\eta(q)$ . Finally, all employed workers and filled jobs produce  $f(k)$  units of output. If the firm cannot find a worker, its capital remains idle for the period. An unemployed worker receives UI equal to  $z$ . Unfilled

---

<sup>7</sup>For other models of search with risk-aversion, see Andalfatto and Gomme (1996), Gomes, Greenwood and Rebelo (1997), and Costain (1996), all of which are solved numerically. Also, the first two papers assume an exogenous distribution of wages, and the third imposes a restrictive structure to ensure that UI and preferences do not affect wages.

<sup>8</sup>We assume that firms post flat-wage contracts, rather than more complex wage profiles (Shimer, 1996; Acemoglu and Shimer, 1997). It can be shown that in this setting, more complex contracts are no better than flat-wage contracts.



vacancies and unemployed workers search again in the following period. We assume that a productive relationship between a worker and a firm never ends. Instead, we maintain a steady state population of unemployed workers by assuming the labor force  $L_t$  grows at rate  $\delta$  and all the  $\delta L_{t-1}$  new workers are unemployed at the start of period  $t$ , with initial assets  $A_{it} = A_0$ .<sup>9</sup>

Workers face the dynamic budget constraint:  $A_{i,t+1} = R(y_{it} + A_{it} - \tau_t - c_{it})$  where  $y_{it}$  is the gross income of worker  $i$  at time  $t$ ,  $\tau_t$  is the lump-sum tax and  $c_{it}$  is consumption at time  $t$ . We also impose the standard transversality condition,  $\lim_{s \rightarrow \infty} R^{-s} A_{i,t+s} = 0$ . We further simplify our analysis by assuming that this is a small open economy with an interest rate equal to the rate of time preference,  $R\beta = 1$ , so workers will equalize the expected marginal utility of consumption across time. This assumption is unimportant.

We introduce *conventional moral hazard* as follows. Each worker  $i$  decides whether to participate in the market economy described above or not to participate, receiving a per-period income  $x_i$  (say from home production). The distribution of  $x$  within each generation is the same and denoted by  $G(x)$ . Whether an individual is searching in the market is not observed publicly, so nonparticipating agents also receive UI. Hence, as in the conventional moral hazard models, UI discourages search. This formulation implies that market opportunities for all agents are identical, yielding a simple recursive structure. We first solve for the utility of a representative participating agent. Since the matching technology has constant returns to scale, the number of agents who participate in the market is unimportant for this analysis. We then compare this market utility to the utility from not participating to determine equilibrium participation.

## 5.2 Optimal Consumption Decisions and Value Functions

We begin by defining Bellman values for employed, unemployed, and nonparticipating workers. We focus on steady state equilibria in which wages, capital stocks, queue lengths, and unemployment rates are constant. Let  $E(A, w)$  be the lifetime utility of a worker who is employed at wage  $w$  with assets  $A$  at the start of the time period. Similarly, let  $J(A, w, q)$  be the expected value of an unemployed worker if he applies for wage  $w$  with queue length  $q$  in this period, and then follows an optimal consumption rule and application strategy thereafter. Let  $U(A)$  denote the Bellman value of an unemployed participant with assets  $A$ . Finally, define  $N(A, x)$  as the utility of a nonparticipating agent with asset level  $A$ , receiving home-produced income  $x$ .

These Bellman values depend on workers' consumption and savings decisions. To

---

<sup>9</sup>This formulation, rather than one with separations, is adopted for simplicity. With random separations, it is not optimal for firms to offer workers constant wages. Instead, since firms are risk-neutral, they would optimally bear the risk of future random separations. This could be done, for example, by paying workers large signing bonuses, and then holding them to their reservation wage.

solve unemployed workers' consumption problem, we assume that they apply for the same wage-queue combination in every period, regardless of their asset level. We later verify that this policy is optimal.

**Lemma 1** 1. *An employed worker who starts a period with assets  $A_t$ , pays taxes  $\tau$ , and earns wage  $w$  in every period, has optimal consumption  $c_t^e = w + (1 - \beta)A_t - \tau$ , so  $A_{t+1} = A_t$ . His lifetime utility is*

$$E(A_t, w) = \frac{1 - e^{-\theta(w + (1 - \beta)A_t - \tau)}}{\theta(1 - \beta)}. \quad (7)$$

2. *An unemployed individual who starts with assets  $A_t$ , pays taxes  $\tau$  in every period, applies for a job offering wage  $w$  and queue length  $q$  and earns benefit  $z$  in every period that he fails to get a job, has optimal consumption  $c_t^u = \beta\psi + (1 - \beta)A_t + z - \tau$ , where  $\psi \in (0, w - z)$  is implicitly defined by*

$$0 = \mu(q) \frac{1 - e^{-\theta(w - z - \psi)}}{\theta} + (1 - \mu(q)) \frac{1 - e^{\theta(1 - \beta)\psi}}{\theta}, \quad (8)$$

so  $A_{t+1} = A_t - \psi$ . His lifetime expected utility is

$$J(A_t, w, q) = \frac{1 - e^{-\theta(\psi + (1 - \beta)A_t + z - \tau)}}{\theta(1 - \beta)}. \quad (9)$$

3. *A nonparticipating worker, who starts period  $t$  with assets  $A_t$ , pays taxes  $\tau$ , earns income  $x$ , and receives unemployment benefit  $z$  in every period, has optimal consumption  $c_t^n = x + z + (1 - \beta)A_t - \tau$ , so  $A_{t+1} = A_t$ . His lifetime utility is*

$$N(A_t, x_i + z) = \frac{1 - e^{-\theta(x_i + (1 - \beta)A_t + z - \tau)}}{\theta(1 - \beta)}. \quad (10)$$

Because  $R\beta = 1$ , workers who face no uncertainty — employed and nonparticipating workers — consume their current net income and interest from savings, maintaining a constant asset level. Unemployed workers consume their current income and dissave a constant amount  $\psi$  of their assets each period, resulting in decreasing asset and consumption levels while unemployed. This reflects the fact that each additional period of unemployment is a bad shock. Upon finding a job, consumption jumps up.

Equation (9) implies that all workers have the same von Neumann-Morgenstern preferences over wage-queue combinations, regardless of their asset levels. This is related to the second part of Proposition 2, which states a similar result in the static model,<sup>10</sup> and verifies our hypothesis that workers apply for the same wage in every period.

---

<sup>10</sup>This feature of our search environment is not shared by the search and bargaining models of Diamond, Mortensen, and Pissarides, even if workers have CARA preferences. This is because, here, the preferences of workers over *lotteries* determine wages, whereas with bargaining what matters is the marginal utility of consumption, which always varies with wealth when agents are risk-averse.

### 5.3 Analysis

Using the value functions derived above, we can define an equilibrium. Let  $\mathcal{P}$  be the set of market participants.

**Definition 4** *A steady state equilibrium is an allocation  $\{\mathcal{K}, \mathcal{W}, Q, U, E, J, N, \mathcal{P}\}$  s.t.:*

1. **[Profit Maximization]**  $\forall w, k$

$$\frac{\eta(Q(w))(f(k) - w)}{1 - \beta(1 - \eta(Q(w)))} - k \leq 0, \quad (11)$$

*with equality if  $k \in \mathcal{K}$  and  $w \in \mathcal{W}(k)$ .*

2. **[Optimal Application]**  $\forall w, U(\cdot) \geq J(\cdot, w, Q(w))$  and  $Q(w) \geq 0$  with complementary slackness;  $U(\cdot) = \sup_{w' \in \bar{\mathcal{W}}} J(\cdot, w', Q(w'))$  or  $U(\cdot) = N(\cdot, z)$  if  $\bar{\mathcal{W}} = \emptyset$ .
3. **[Optimal Participation]**  $i \in \mathcal{P}$  if and only if  $N(\cdot, x_i + z) \leq U(\cdot)$ .

This is a generalization of the definition of equilibrium in the static model. To understand the expression for firm profits, observe that from free entry, the expected present value a vacant firm's profits equals the cost of creating a new vacancy,  $k/(1 - \beta)$ . The value of a vacant firm comes from the possibility of creating a job, yielding profit  $f(k) - w$  for the infinite future, and the continuation value if it fails to create a job. Then in steady state, if  $k \in \mathcal{K}$ ,  $w \in \mathcal{W}(k)$ , and  $q = Q(w)$ ,  $k = \eta(q)(f(k) - w) + (1 - \eta(q))\beta k$ .

Next, Optimal Application and Participation use the fact that the relationship between  $U$ ,  $J$ , and  $N$  does not depend on asset levels  $A$ . Also, notice that if there is a unique queue length  $q$  in steady state, then the end-of-period unemployment rate is

$$u = (1 - p) + p \frac{\delta(1 - \mu(q))}{\delta + \mu(q)}, \quad (12)$$

where  $p \equiv \int_{i \in \mathcal{P}} di$ . The first term is the fraction of non-participants, who are counted as unemployed. The second is the fraction of unemployed participants, a group consisting of new births and previously participating workers who have not find a job yet.

#### Proposition 7

1. *If  $\{\mathcal{K}, \mathcal{W}, Q, U, E, J, N, \mathcal{P}\}$  is an equilibrium, then any  $k^* \in \mathcal{K}$ ,  $w^* \in \mathcal{W}(k^*)$ , and  $q^* = Q(w^*)$  solves:*

$$0 = h(\psi^*) \equiv \max_{k, w, q} \mu(q) \frac{1 - e^{-\theta(w - z - \psi^*)}}{\theta} + (1 - \mu(q)) \frac{1 - e^{\theta(1 - \beta)\psi^*}}{\theta}, \quad (13)$$

$$\text{subject to } \frac{\eta(q)(f(k) - w)}{1 - \beta(1 - \eta(q))} = k, \quad (14)$$

$E$ ,  $J$ , and  $N$  are defined by (7), (9), and (10); and  $U(A) = J(A, w^*, q^*)$ . Conversely, if  $\{k^*, w^*, q^*, \psi^*\}$  solves the above program, then there exists an equilibrium  $\{\mathcal{K}, \mathcal{W}, \mathcal{Q}, U, E, J, N, \mathcal{P}\}$  such that  $k^* \in \mathcal{K}$ ,  $w^* \in \mathcal{W}(k^*)$ , and  $q^* = \mathcal{Q}(w^*)$ .

2.  $i \in \mathcal{P}$  if and only if  $x_i \leq \psi$ .

3. An equilibrium exists. All equilibria have a common, uniquely defined,  $\psi^*$ .

As in the static model, we call a  $\{k^*, w^*, q^*, \psi^*\}$  that solves this program an equilibrium.

The first part of this proposition is the analog of Proposition 1 in the static model (for the case where  $z < \bar{z}$ ) and its proof is omitted. Unemployed workers choose  $w$  and  $q$  to maximize  $J(A, w, q)$ , subject to firms making zero profits. From equation (9), this is equivalent to maximizing dissavings while unemployed,  $\psi$ , subject to (8), so that the rate of dissavings must be optimal, given the wage  $w$  and hiring probability  $\mu(q)$ .

To understand the second part of the proposition, notice from Lemma 1 that asset levels do not affect the comparison of expected utilities of searching and nonparticipating workers. Therefore, there will exist a cutoff level of outside productivity,  $\bar{x}$  such that all workers with higher outside productivity do not participate. This is in turn given by setting the utility of an unemployed participating worker,  $U(A)$ , equal to the utility of the marginal nonparticipant  $N(A, \bar{x} + z)$ , which implies  $\bar{x} = \psi$ . The proof of the third part of the Proposition is available on request. Note that the uniqueness of  $\psi^*$  does not guarantee the uniqueness of equilibrium investments, wages, and queue lengths. It simply states that all equilibria give a common level of utility to participating workers.

**Proposition 8** Consider two CARA utility functions with coefficients  $\theta_1$  and  $\theta_2$ , and two unemployment benefits  $z_1$  and  $z_2$ . Let  $\{k_i, w_i, q_i, \psi_i\}$  be an equilibrium when risk aversion is  $\theta_i$  and UI is  $z_i$ ,  $i \in \{1, 2\}$ . If  $z_1 \leq z_2$  and  $\theta_1 \geq \theta_2$ , and at least one of the inequalities is strict, then  $k_1 < k_2$ ,  $w_1 < w_2$ ,  $q_1 < q_2$ , and  $\psi_1 < \psi_2$ .

This proposition is once again proved using revealed preference arguments along the lines of Proposition 2 (proof available upon request). Figure 2 again gives the intuition. An increase in UI makes workers' indifference curves everywhere flatter, shifting the point of tangency to the right, raising wages, queue lengths, and capital. Similarly, when workers are more risk-averse, wages are lower, queues are shorter, and firms invest less. The added feature is the participation margin. A higher level of UI discourages search, since the threshold level of home production  $\psi$  is a decreasing function of the level of UI. Similarly, an increase in risk-aversion encourages nonparticipation, since this activity is riskless.

## 5.4 Risk-Aversion, UI and Output

Output is equal to the expected present value of market output produced by a particular worker during her lifetime, net of any investment costs that are incurred on her behalf:<sup>11</sup>

$$Y(q, k, p) \equiv p \frac{\mu(q)f(k) - \beta\mu(q)k - (1 - \beta)k/q}{(1 - \beta)(1 - \beta(1 - \mu(q)))} = p \frac{\mu(q)w}{(1 - \beta)(1 - \beta(1 - \mu(q)))} \quad (15)$$

where we have simplified the expression using (14). Market output is an increasing function of the participation rate  $p$ . The three terms in the numerator represent the present value of output that the worker produces, the rental cost of capital while she is producing, and the rental cost of capital to maintain  $1/q$  vacancies while she is unemployed.

We begin by showing that in the absence of “conventional” moral hazard, i.e. if all workers participate, productive efficiency requires a UI system.

**Proposition 9** *Assume that  $G$  is dirac at 0, so all workers participate. Let  $\{q^e, k^e\}$  be an efficient allocation, and define  $w^e$  by (4) and  $\psi^e$  by (8). Then  $(z^e, \tau^e)$  is efficient if  $\tau^e = uz^e$  and  $z^e \in [0, w^e)$  satisfies*

$$w^e \equiv \frac{1 - \beta(1 - \mu(q^e))}{1 - \beta} \cdot \frac{e^{\theta(w^e - z^e - \beta\psi^e)} - 1}{\theta} \quad (16)$$

The proof is in Appendix B and the intuition is exactly the same as Proposition 6. In the risk-neutral limit ( $\theta = 0$ ),  $z^e = \tau^e = 0$ , but when agents are risk-averse, efficient UI is strictly positive.

To get a sense of the importance of the effects highlighted in this model, we undertake a simple calibrations exercise. We take a period to be a year and set the discount factor to  $\beta = 1/R = 0.94$ , while the birth rate is  $\delta = 0.01$ . The production function is  $f(k) = 10k^{0.5}$ . We set  $\eta(q) = 1 - e^{-0.15q}$ , which implies that unemployed workers are hired with probability less than 0.15 per year. Although such low job finding probabilities do not fit the data, this is the only way to generate reasonable unemployment rates in a model without any job destruction. Finally, we consider values of the coefficient of absolute risk aversion ranging from 0 to 100. These correspond to coefficients of *relative* risk aversion for unemployed workers with no assets and no UI between 0 and 2.5.

Table 1 shows the results. An increase in the coefficient of absolute risk aversion from 0 to 10 has little effect on output, because the changes in the equilibrium allocation, as compared with the risk-neutral case, are in the neighborhood of the output-maximizing allocation. However, when risk-aversion is higher, the loss in output associated with lack of insurance is much larger. It increases from 7% when  $\theta = 20$  to 27% when  $\theta = 40$ ,

<sup>11</sup>Another definition of output is the cross-sectional output per capita in a given period, which is a decreasing function of the population growth rate, since primarily young workers are unemployed. Our measure of output corresponds more closely to the analysis in Section 3.

Coeff. of ARA ( $\theta$ )	0	10	20	40	100
Capital ( $k$ )	0.216	0.197	0.167	0.100	0.028
Wage ( $w$ )	0.232	0.222	0.204	0.158	0.084
Queue ( $q$ )	3.85	2.61	1.58	0.65	0.20
Dissavings ( $\psi$ )	0.158	0.133	0.104	0.063	0.027
Coeff. of RRA ( $\theta\beta\psi$ )	0.00	1.25	1.96	2.37	2.54
Unemp. Rate ( $u$ )	7.2%	6.5%	6.0%	5.6%	5.4%
Output ( $Y$ )	0.158	0.156	0.147	0.116	0.063
Efficient UI ( $z^e$ )	0.000	0.088	0.125	0.160	0.193

Table 1: Calibration results for different coefficients of absolute risk aversion, without worker nonparticipation. Parameter details are in the text.

and to 60% when  $\theta = 100$ . The last row gives the level of efficient UI, which increases the level of output back to the risk-neutral level, 0.158. In this calibrated economy the introduction of UI can increase the level of output by up to 250%.

We next consider the effect of UI on workers' participation. When  $G$  is a diffuse distribution, increasing UI discourages search, so the net effect on output is generally ambiguous. If the elasticity of participation with respect to the unemployment benefit is low relative to risk aversion, the conclusions of Proposition 9 will be qualitatively unchanged. However, the highest level of output (that of the risk-neutral economy) will no longer be attainable because UI will reduce participation. Also, output-maximizing will generally not entail decentralizing  $\{q^e, k^e\}$ , since in the neighborhood of this allocation, UI achieves only a second order increase in output, while it results in a first order decrease in participation. On the other hand, if the elasticity of participation is high, output might be maximized for a negative UI, i.e. taxing unemployment and nonparticipation.

To see whether this is likely to be the case, we return to the calibrations. For this purpose we have to choose  $G$ . Atkinson and Micklewright (1991) conclude in their survey article that a ten percentage point increase in the replacement ratio  $z/w$  raises the average unemployment duration by about six percent, though some estimates are as large as ten percent. To be conservative in calculating the benefits of UI, we assume that this semi-elasticity is ten percent. Part of the effect of UI on unemployment is captured by "market generated moral hazard", since an increase in UI leads workers to prefer longer job queues, and hence longer unemployment durations. The remainder of the effect comes through some workers' decision not to participate. We calibrate the distribution function  $G$  accordingly.

Table 2 shows the equilibrium of the model in this case. For low levels of risk-aversion, the output-maximizing level of UI is significantly lower. Indeed, for risk-neutral workers,

Coeff. of ARA ( $\theta$ )	0	10	20	40	100
Output-Maximizing UI ( $z$ )	$\leq 0$	0.011	0.098	0.153	0.193
Capital ( $k$ )	—	0.200	0.207	0.212	0.216
Wage ( $w$ )	—	0.224	0.227	0.230	0.232
Queue ( $q$ )	—	2.73	3.15	3.55	3.85
Dissavings ( $\psi$ )	—	0.128	0.073	0.040	0.018
CRRA ( $\theta(\beta\psi + z - \tau)$ )	0	1.31	3.20	7.21	19.6
Unemp. Rate ( $u$ )	—	6.9%	9.3%	10.8%	12.2%
Participation Rate ( $p$ )	—	99.7%	97.4%	95.9%	94.6%
Output ( $Y$ )	—	0.156	0.154	0.152	0.150

Table 2: Output-maximizing UI for different coefficients of absolute risk aversion, with worker nonparticipation. Parameter details are in the text.

the output-maximizing level of UI is typically *negative*, as noted above. Similarly, when the coefficient of absolute risk aversion is 10, accounting for conventional moral hazard reduces the output maximizing level of UI by a factor of 8, from 0.088 to 0.011. But, for higher levels of risk-aversion, moral hazard has little effect on the output-maximizing UI. This is despite the fact that the reduction in search effort raises the unemployment rate significantly. For example, when the coefficient of absolute risk aversion is 40, with no UI the unemployment rate is 5.6%. With efficient UI and no conventional moral hazard, the unemployment rate rises to 7.2% (the risk-neutral level). Conventional moral hazard raises the output-maximizing unemployment rate by another fifty percent, to 10.8%, nearly double the level with no UI. Still, (market) output is about 30% higher than without any UI. Since our measure of output excludes the value of home-production, this exaggerates the cost of nonparticipation, making our estimates conservative.

## 6 Evidence and Extensions

### 6.1 Empirical Evidence

The main contribution of our paper is theoretical: it solves a dynamic general equilibrium search model with risk-aversion and precautionary savings, and provides analytical solutions for the variables of interest. In the process, we highlighted a novel effect: UI can improve the composition of jobs and increase output. Calibrations indicate that this effect may be quantitatively important. Further empirical research is beyond the scope of the current paper, but in this section we report some related evidence, suggestive for some of the forces emphasized in this paper.

Acemoglu (1997) analyzes the impact of unemployment benefits on the composition

of jobs and labor productivity across different U.S. states. He classifies jobs into good (high wage) and bad (low wage) jobs using inter-occupation and inter-industry wage differentials after controlling for observable human capital variables. Although part of these differences are certainly explained by compensating wage differentials and unobserved heterogeneity, previous research has suggested that these wage differentials are also partly related to rents and desirability of jobs (e.g. Krueger and Summers, 1988, but see also Murphy and Topel, 1987). For example, Holzer et al (1991) show that high wage jobs attract more job applicants, and Katz and Summers (1989) find that high wage sectors also have higher capital-labor ratios, which is similar to our model’s setup where firms offering higher wages also choose higher  $k$ .

Acemoglu then investigates how legislated changes in state level replacement ratios for low wage workers, between 1983 and 1993, affect the industrial and occupational composition of jobs among noncollege graduates. Controlling for state and time effects, he finds that a state that increases its replacement ratio experiences an increase in the unemployment rate, but also a relative increase in the number of workers in high wage occupations and industries, and both higher labor productivity and higher labor productivity growth. In the case of industries, this is only caused by a larger decline in the employment of low wage industries than high wage industries. In the case of occupations, the results are stronger — there is actually an increase in the number of high wage jobs, despite the decline in overall employment. For example, a 10 percentage point increase in the replacement ratio increases the number of high wage jobs by 1.3%, suggesting that UI may have an important effect on the types of jobs that are created.

## 6.2 Extensions

To explore the robustness of our model, we briefly discuss three variants in which all of our main results obtain.

1. It is conceptually easy to allow firms to hire multiple workers. Firms choose capital  $k$  ex ante, and output is  $y = F(k, l)$  where  $l$  is the number of workers that the firm hires. The distribution of the number of hires is an increasing function of the queue length, and the probability that a worker is hired is a decreasing function of the queue. In tighter labor markets firms attract fewer workers on average, so invest less in capital.

2. Our discussion has concentrated on the risk of not getting a job. However, careers also differ in other dimensions of risk, such as the possibility of layoff, the variability of wage-tenure profiles, and the likelihood of promotion. Many discussions of UI emphasize the riskiness of these episodes, and subsequent spells of unemployment. Our qualitative results will continue to hold in this case. Risk-aversion implies that workers will require compensation to accept jobs with high rates of separation, and this cost increase distorts the balance between different sectors. UI encourages workers to take more risks, and



firms to create riskier jobs (see Barlevy, 1997, for some evidence). Because it restores balance in the production side, moderate UI raises output.

3. Our stylized model assumes that when workers find unemployment risk more costly, firms accept additional vacancy risk and therefore reduce their ex ante capital investments. In practice, firms do maintain vacancies for long periods of time (e.g. Myers and Creamer, 1967). However, the reason is typically not that they are unable to locate a worker, but because they are unable to locate a *satisfactory* worker. For example, using Dutch data, van Ours and Ridder (1993) estimate that receiving applications takes about three weeks on average, but selecting the qualified applicant from among those takes on average fifteen weeks.

A firm may have some latitude in choosing the *specificity* of its investment. For example, it can design a secretarial job which most workers with a high school diploma could fill, or it can open a higher productivity job with more specific requirements, such as familiarity with a range of software, experience in the same line of business, and so on, which would be harder to fill. In this environment, if workers wish to avoid unemployment, firms will accommodate them by making less specific investments.

A slight variant of our model easily deals with this issue. Each firm chooses ex ante the specificity of the tasks that future employees will perform, a number  $\alpha$ . The matching probabilities for the worker and firm  $\mu(q, \alpha)$  and  $\eta(q, \alpha)$  are decreasing functions of  $\alpha$ , so an increase in specificity raises unemployment and vacancy duration. Offsetting this, more specific jobs are more productive, so the output of a match  $\phi(\alpha)$  is increasing in specificity  $\alpha$ . An increase in risk aversion reduces the specificity of jobs offered in equilibrium, while an increase in UI raises specificity. Once again, an intermediate level of UI maximizes output and utility. These effects may become more pronounced if workers also have to make ex ante investments: in the presence of UI, workers may choose to specialize their skills and wait for suitable jobs, and firms would create them. Without UI, firms would not demand and workers would not invest in specialized skills (see Grossman and Shapiro, 1982, for a partial equilibrium analysis of a similar model).

In practice, the impact of labor market tightness on job specificity may be more important than its impact on physical capital investments. For example, Murnane and Levy (1996) present case study evidence on recruitment practices of firms. In the tight labor markets of the late 1960s, Ford Motor Company recruited basically any able-bodied worker who applied. In contrast, automobile manufacturers in the high unemployment market of the 1980s attracted a large number of applicants for each position and were extremely selective, using a series of interviews and tests to select the workers who were skilled and suited for the jobs. Consistent with our story, these new manufacturing jobs appear to be of higher quality, require more specific skills, and yield higher productivity.

## 7 Conclusion

This paper has constructed a general equilibrium model of search with risk-aversion. An increase in risk-aversion induces workers to seek lower wage jobs with less unemployment risk. Firms cater to these preferences by creating jobs with lower wages and lower capital intensity. UI has the reverse effects due to market generated moral hazard: insured workers want to seek riskier jobs and the market once again caters to these preferences by creating the desired jobs, so UI increases wages and reduces employment.

Our framework, however, also points to a novel effect of UI on productivity. The conventional wisdom emphasizes the trade-off between output and the risk-sharing provided by UI. We show that general equilibrium interactions in the labor market can reverse this wisdom. UI induces firms to invest more in capital, and in moderation, raises output. Because the distortion in the production side of the economy is due to incomplete insurance and risk-aversion, UI is precisely the right tool to deal with this market failure. In fact, we establish that, despite the potential non-convexities in the economy, there exists a level of UI which restores output to its maximum level. It is useful to reiterate that despite the beneficial effects of UI, our model does not make clear policy recommendations because it does not explain why the private sector cannot offer UI. One can argue that adverse selection as in Rothschild and Stiglitz (1976) might prevent this, but more analysis would be required to reach a definite conclusion.

Our analysis also contains methodological improvements over the existing search literature. We solve a dynamic general equilibrium model with risk-aversion, incomplete insurance markets, and endogenous wages, productivity, consumption, and savings. To our knowledge, this is the only paper to accomplish this task. The real advantage of our framework, however, is that all the results can be obtained using a static model, which can easily be applied to other problems in search theory and labor economics.

## Appendix A: Proofs of The Main Results

**Proof of Proposition 1:** We prove this proposition in the following order: Step 1 establishes that any equilibrium solves the constrained optimization problem, Step 2 shows that any allocation that solves this program is part of an equilibrium, and Step 3 proves existence.

**Step 1.** Let  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  be an equilibrium allocation with  $k^* \in \mathcal{K}$ ,  $w^* \in \mathcal{W}(k^*)$ , and  $q^* = Q(w^*)$ . We must prove that  $\{k^*, q^*, w^*\}$  solves the constrained optimization problem. First, Profit Maximization ensures that  $\{k^*, w^*, q^*\}$  satisfies constraint (2). Next, Optimal Application implies

$$U = \mu(q^*)u(A + w^*) + (1 - \mu(q^*))u(A + z) \geq u(A + z),$$

so  $w^* \geq z$ , satisfying constraint (3).

Suppose now that another a triple  $\{k, w, q\}$  satisfies constraint (3) and achieves a higher value of the objective,

$$\mu(q)u(A + w) + (1 - \mu(q))u(A + z) > U.$$

We will prove that it must violate constraint (2). Since  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  is an equilibrium, Optimal Application implies

$$\mu(Q(w))u(A + w) + (1 - \mu(Q(w)))u(A + z) \leq U.$$

Since  $w \geq z$  by (3), these inequalities imply  $\mu(q) > \mu(Q(w))$ , so  $q < Q(w)$ . Thus

$$\eta(q)(f(k) - w) - k < \eta(Q(w))(f(k) - w) - k \leq 0,$$

where the weak inequality exploits Profit Maximization. Therefore,  $\{k, w, q\}$  violates (2), and is not in the constraint set.

**Step 2.** We now prove by construction that for any solution  $\{k^*, w^*, q^*\}$  to the constrained maximization problem, there is an equilibrium  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  with  $\mathcal{K} = \{k^*\}$ ,  $\mathcal{W}(k^*) = \{w^*\}$ , and  $Q(w^*) = q^*$ . Set

$$U = \mu(q^*)u(A + w^*) + (1 - \mu(q^*))u(A + z),$$

and let  $Q(w)$  satisfy

$$U = \mu(Q(w))u(A + w) + (1 - \mu(Q(w)))u(A + z),$$

or  $Q(w) = 0$  if there is no solution to the equation, which in particular will be the case if  $w < z$ . It is immediate that  $\{\mathcal{K}, \mathcal{W}, Q, U\}$  satisfies Optimal Application.

We now show that it also satisfies Profit Maximization. Suppose to the contrary that some triple  $\{k', w', Q(w')\}$  violates Profit Maximization, so

$$\eta(Q(w'))(f(k') - w') - k' > 0.$$

One implication of this is that  $Q(w') > 0$ . Since the previous paragraph argued that  $Q(w') = 0$  for all  $w' < z$ , this implies  $w' \geq z$ . Another implication is that we can choose  $q' < Q(w')$  such that

$$\eta(q')(f(k') - w') - k' = 0.$$

By the construction of  $Q(\cdot)$ ,  $q' < Q(w')$  and  $w' \geq z$  imply

$$U < \mu(q')u(A + w') + (1 - \mu(q'))u(A + z).$$

We have shown that  $\{k', w', q'\}$  satisfies both constraints and yields a higher value of the objective than  $\{k^*, w^*, q^*\}$ , a contradiction.

**Step 3.** For existence, we must consider two separate cases. The more interesting possibility is that  $z < \bar{z}$ . This ensures that the constraint set is nonempty; for example,  $\{k, w, q\} = \{\bar{k}, \bar{z}, \infty\}$  satisfies both constraints. The constraint set is also compact and the objective function (1) is continuous. This implies that a solution to the constrained maximization problem exists, and by Step 2 above, this solution is an equilibrium.

We also note in passing that if  $z < \bar{z}$ , there is no equilibrium in which  $\mathcal{K} = \bar{\mathcal{W}} = \emptyset$ . If there were, a firm could enter at a wage of  $w = \frac{1}{2}(z + \bar{z})$ , using capital  $k$  satisfying  $w = f(k) - kf'(k)$ . Optimal Application implies that it would attract an infinite queue, and so would hire a worker with probability 1. Its profits would be  $k(f'(k) - 1) > 0$  (since  $w < \bar{z}$ ,  $k < \bar{k}$ , and  $f'(k) > 1$ ), violating Profit Maximization.

The other possibility is  $z > \bar{z}$ , which implies that the constraint set of the maximization problem is empty. Step 1 implies that if an equilibrium exists,  $\mathcal{K} = \bar{\mathcal{W}} = \emptyset$ . We show that one does. Consider a firm that deviates from the proposed equilibrium, buying capital  $k > 0$  and posting a wage. If  $w < z$ , optimal application implies  $Q(w) = 0$ , so  $\eta(Q(w)) = 0$  as well. Thus the deviation is unprofitable, leaving the firm with profit  $-k$ . On the other hand, if  $w \geq z > \bar{z} = f(\bar{k}) - \bar{k}$ , the firms profit are

$$\mu(Q(w))(f(k) - w) - k < f(k) - w - k < (f(k) - k) - (f(\bar{k}) - \bar{k}) < 0$$

by concavity of  $f$  and  $f'(\bar{k}) = 1$ , so the deviation is again unprofitable. ■

**Proof of Proposition 2:** We begin by introducing a critical property, *Revealed Preference*: if  $\{w_i, q_i, k_i\}$  is the equilibrium with UI  $z_i$ , preferences  $u_i$ , and assets  $A_i$ , then

$$\mu(q_1)(u_1(A_1 + w_1) - u_1(A_1 + z_1)) \geq \mu(q_2)(u_1(A_1 + w_2) - u_1(A_1 + z_1))$$

Because  $\{w_2, q_2, k_2\}$  is an equilibrium with  $z_2, u_2$ , and  $A_2$ , (2) must be satisfied. Then if this inequality were false,  $\{w_1, q_1, k_1\}$  would not maximize (1) with  $z_1, u_1$ , and  $A_1$  —  $\{w_2, q_2, k_2\}$  would do better. We exploit Revealed Preference throughout this Appendix.

1. Take  $u_1$  more risk-averse than  $u_2$ . Revealed Preferences implies

$$\begin{aligned}\mu(q_1) (u_1(A + w_1) - u_1(A + z)) &\geq \mu(q_2) (u_1(A + w_2) - u_1(A + z)) \\ \mu(q_2) (u_2(A + w_2) - u_2(A + z)) &\geq \mu(q_2) (u_2(A + w_1) - u_2(A + z))\end{aligned}$$

Multiply these inequalities and simplify:

$$\begin{aligned}(u_1(A + w_1) - u_1(A + z)) (u_2(A + w_2) - u_2(A + z)) & \quad (A1) \\ \geq (u_1(A + w_2) - u_1(A + z)) (u_2(A + w_1) - u_2(A + z)). & \end{aligned}$$

Suppose  $w_1 > w_2 > z$ . By the intermediate value theorem, there exists a  $\lambda \in (0, 1)$  with

$$u_2(A + w_2) \equiv \lambda u_2(A + w_1) + (1 - \lambda)u_2(A + z). \quad (A2)$$

Since  $u_1$  is a strictly concave transformation of  $u_2$ ,

$$u_1(A + w_2) > \lambda u_1(A + w_1) + (1 - \lambda)u_1(A + z). \quad (A3)$$

Eliminating  $u_2(A + w_2)$  and  $u_1(A + w_2)$  from (A1) using (A2) and (A3), we obtain:

$$\begin{aligned}\lambda (u_1(A + w_1) - u_1(A + z)) (u_2(A + w_1) - u_2(A + z)) \\ > \lambda (u_1(A + w_1) - u_1(A + z)) (u_2(A + w_1) - u_2(A + z)),\end{aligned}$$

a contradiction, proving  $w_2 \geq w_1$ . By (4),  $k_2 \geq k_1$ . Then the optimal capital choice relationship  $\eta(q^*)f'(k^*) = 1$  defines an increasing relation between  $q$  and  $k$ ,  $q_2 \geq q_1$ . And both inequalities are strict if and only if  $w_2 > w_1$ , which is true because constraint set and the objective function are smooth, all these inequalities are strict (see Edlin and Shannon, 1996).

2. Next consider the effect of assets. We show that with decreasing absolute risk aversion (DARA), there is a strictly concave transformation  $v$  such that  $u(A_1 + c) \equiv v(u(A_2 + c))$  for  $A_1 < A_2$  and for all  $c$ . That  $v$  exists and is  $C^2$  follows from the assumptions on the utility function. To prove that  $v$  is strictly concave, twice differentiate its definition with respect to  $c$ :

$$\begin{aligned}u'(A_1 + c) &\equiv v'(u(A_2 + c))u'(A_2 + c), \\ u''(A_1 + c) &\equiv v'(u(A_2 + c))u''(A_2 + c) + v''(u(A_2 + c))u'(A_2 + c)^2.\end{aligned}$$

Use the first relationship to eliminate  $v'$  from the second one:

$$\frac{u''(A_1 + c)}{u'(A_1 + c)} = \frac{u''(A_2 + c)}{u'(A_2 + c)} - \frac{v''(u(A_2 + c))u'(A_2 + c)^2}{u'(A_1 + c)}$$

Since  $A_1 < A_2$  and  $u$  has DARA,  $-\frac{u''(A_1+c)}{u'(A_1+c)} > -\frac{u''(A_2+c)}{u'(A_2+c)}$ . Thus the last term above must be positive, which can only be the case if  $v''$  is negative, proving  $v$  is concave. The arguments with CARA and IARA are analogous. Having established this transformation, the first part of this Proposition completes the proof.

3. Finally, establish the effects of UI. Analogous with (A1), Revealed Preference implies

$$(u(A + z_1) - u(A + z_2))(u(A + w_1) - u(A + w_2)) \geq 0.$$

As  $z_1 < z_2$ , we have  $w_1 \leq w_2$ . The rest of the argument follows as in Part 1 to establish  $w_2 > w_1$ ,  $q_2 > q_1$ , and  $k_2 > k_1$ . ■

**Proof of Proposition 4:** Let  $u(x) = ax + b$ . The objective (1) becomes a linear transformation of  $\mu(q)w$ . Eliminate  $w$  using the constraint, noting that  $\eta(q) \equiv q\mu(q)$ . The resulting unconstrained maximization problem is identical to the output maximization problem,  $\max_{q,k} Y(q, k)$ . ■

**Proof of Proposition 6:** We first prove that  $(z^e, \tau^e)$  exists. The government budget constraint defines  $\tau^e$  as a continuous function of  $z$ , mapping  $[0, (1 - \mu(q^e))w^e]$  into itself. Next, observe that the RHS of (5) is continuous and decreasing in  $z$ . Also, concavity of  $u$  implies  $u(A_0 - \tau + w^e) > u(A_0 - \tau) + u'(A_0 - \tau + w^e)w^e$ , so the right hand side of (5) is bigger than  $w^e$  when  $z$  is zero, and it is equal to  $0 < w^e$  when  $z = w^e$ . Thus (5) defines  $z^e$  as a continuous function of  $\tau$ , mapping  $[0, w^e]$  into itself. Finally, existence is guaranteed by Brouwer's fixed point theorem.

Now we prove that  $\{w^e, q^e, k^e\}$  is an equilibrium of the model. From Proposition 1, we must show that any alternative allocation  $\{w', q', k'\}$  that gives firms zero profits, must give workers less utility. Note that  $w^e \neq w'$ . Otherwise,  $k^e = k'$  by (4) and  $q^e = q'$  by firms' optimal capital choice  $\eta(q)f'(k) = 1$ , making the two allocations identical.

We begin with the result from Proposition 4. The output maximizing allocation is an equilibrium with risk-neutral workers and  $z = \tau = 0$ , so Proposition 1 implies that risk-neutral workers with no UI weakly prefer  $\{w^e, q^e, k^e\}$  to  $\{w', q', k'\}$ :

$$\mu(q^e)w^e \geq \mu(q')w'. \quad (\text{A4})$$

Next, since the ratio of a nonnegative, strictly concave function to a nonnegative linear function is a nonnegative strictly quasiconcave function,  $\frac{u(A_0 - \tau^e + w) - u(A_0 - \tau^e + z^e)}{w}$  is strictly quasiconcave in  $w$  for  $w > z^e$ . Moreover, one can confirm that (5) is the first order condition from choosing  $w$  to maximize this function. Therefore,

$$\frac{u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)}{w^e} > \frac{u(A_0 - \tau^e + w') - u(A_0 - \tau^e + z^e)}{w'}, \quad (\text{A5})$$

where the strict inequality holds because  $w^e \neq w'$ . Now multiply (A4) and (A5):

$$\mu(q^e) (u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)) > \mu(q') (u(A_0 - \tau^e + w') - u(A_0 - \tau^e + z^e))$$

That is, the worker strictly prefers  $\{w^e, q^e, k^e\}$  to  $\{w', q', k'\}$ . This is an equilibrium, and given the strict inequality, there can be no other equilibrium. ■

**Proof of Lemma 1:** The first and third parts of this Lemma are easily proved. For a worker employed at the wage  $w$  in all future periods, the optimal consumption path is constant, since  $\beta R = 1$ . Thus the budget constraint and transversality condition require

$$c_t^e = w + \frac{R-1}{R}A_t - \tau = w + (1-\beta)A_t - \tau,$$

so  $A_{t+1} = A_t$  and  $c_{t+1}^e = c_t^e$  for all  $t$ . The value of a worker employed at  $w$  is then given by (7). The proof of the third part is identical.

Next consider an unemployed worker. He looks for a job and then chooses his consumption. If he finds a job at time  $t$ , we know from above that he will consume  $w + (1-\beta)A_t - \tau$ . If he fails to find a job, he earns net benefit  $z - \tau$ , and consumes  $c_t^u$ . Conjecture a linear consumption rule as a function of assets, with marginal propensity to consume out of assets equal to  $1-\beta$ . This conjectured consumption function can be written as:  $c_t^u = \beta\psi + (1-\beta)A_t + z - \tau$ , for some unknown parameter  $\psi$ , leaving him with assets  $A_{t+1} = A_t - \psi$  next period. To solve for  $\psi$ , use the Euler equation:

$$e^{-\theta(\beta\psi+(1-\beta)A_t+z-\tau)} = \mu(q)e^{-\theta(w+(1-\beta)A_{t+1}-\tau)} + (1-\mu(q))e^{-\theta(\beta\psi+(1-\beta)A_{t+1}+z-\tau)}$$

Multiply both sides of this by  $e^{\theta(\beta\psi+(1-\beta)A_t+z-\tau)}$  and use  $A_{t+1} = A_t - \psi$  to obtain (8), which implicitly defines  $\psi \in (0, w - z)$  as a function of  $w, q$  and  $z$ .

Using this consumption rule, the value function  $J(A_t, w', q')$  can be written as:

$$J(A_t, w', q') = \mu(q')E(A_t, w') + (1-\mu(q')) \left( \frac{1 - e^{-\theta(\beta\psi+(1-\beta)A_t+z-\tau)}}{\theta} + \beta J(A_t - \psi, w', q') \right). \quad (\text{A6})$$

Note that this equation is written for any  $(w', q')$ , not necessarily for the same  $(w, q)$  that solves (8). We have already solved for  $E(A_t, w')$  in equation (7). Now conjecture the form of  $J$  as in (9) and substitute this into (A6). This yields:

$$J(A_t, w', q') = \mu(q) \frac{1 - e^{-\theta(w+(1-\beta)A_t-\tau)}}{\theta(1-\beta)} + (1-\mu(q)) \frac{1 - e^{-\theta(\beta\psi+(1-\beta)A_t+z-\tau)}}{\theta(1-\beta)}$$

Now eliminate  $\psi$  using its definition in equation (8). The resulting expression reduces to the definition of  $J$  in (9), confirming our conjecture. ■

## References

- [1] **Acemoglu, Daron, 1996** “A Microfoundation for Social Increasing Returns in Human Capital,” *Quarterly Journal of Economics*, 111, 779–804.
- [2] **Acemoglu, Daron, 1997** “Good Jobs versus Bad Jobs: Theory and Some Evidence,” MIT Working Paper.
- [3] **Acemoglu, Daron and Robert Shimer, 1997** “Efficient Wage Dispersion” MIT and Princeton Working Paper.
- [4] **Andolfatto, David and Paul Gomme, 1996** “Unemployment Insurance and Labor-Market Activity in Canada” *Carnegie-Rochester Conference Series on Public Policy*, 44, 47–82.
- [5] **Atkeson, Andrew and Robert Lucas, 1995** “Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance,” *Journal of Economic Theory*, 66, 64–88.
- [6] **Atkinson, Anthony and John Micklewright, 1991** “Unemployment and Compensation and Labor Market Transitions: A Critical Review” *Journal of Economic Literature*, 58, 277–97.
- [7] **Azariades, Costas, 1975** “Implicit Contracts and Underemployment Equilibria” *Journal of Political Economy*, 83, 1183–1202.
- [8] **Baily, Martin Neil, 1974** “Wages and Employment under Uncertain Demand” *Review of Economic Studies*, 41, 37–50.
- [9] **Baily, Martin Neil, 1977** “Unemployment Insurance as Insurance For Workers” *Industrial and Labor Relations Review*, 30, 495–504.
- [10] **Barlevy, Gadi, 1997** “Wage Cyclicity, Search and Matching” Northwestern Mimeo.
- [11] **Burdett, Kenneth, Shouyong Shi, and Randall Wright, 1997** “Pricing with Frictions,” University of Pennsylvania Mimeo.
- [12] **Burtless, Gary, 1993** “Is Unemployment Insurance Ready for the 1990s?” in Paul N. Van De Water (editor), *Social Insurance For the Nineties*, Kendall Hunt Publishing, 1993.
- [13] **Costain, James, 1996**, “Unemployment Insurance in a General Equilibrium Model of Job Search and Precautionary Savings,” University of Chicago Working Paper.
- [14] **Diamond, Peter, 1981** “Mobility Costs, Frictional Unemployment and Efficiency,” *Journal of Political Economy*, 89, 798–812.



- [15] **Diamond, Peter, 1982** "Wage Determination and Efficiency in Search Equilibrium," *Review of Economics Studies* 49, 217–27.
- [16] **Edlin Aaron and Chris Shannon, 1996** "Strict Monotonicity in Comparative Statics," University of California-Berkeley Working Paper.
- [17] **Ehrenberg, Ronald and Ronald Oaxaca, 1976** "Unemployment Insurance, Duration of Unemployment and Subsequent Wage Gain," *American Economic Review*, 104, 1263–96.
- [18] **Gomes, Joao, Jeremy Greenwood and Sergio Rebelo, 1997** "Equilibrium Unemployment," Rochester Working Paper.
- [19] **Gordon, David, 1974** "A Neoclassical Theory of Keynesian Unemployment," *Economic Inquiry*, 12, 431–459.
- [20] **Grossman Sanford and Carl Shapiro, 1982** "A Theory of Factor Mobility" *Journal of Political Economy*.
- [21] **Hansen Gary and Ayse Imrohoroglu, 1992** "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy*, 100, 118–42.
- [22] **Holmstrom, Bengt, 1982** "Moral Hazard in Teams." *Bell Journal of Economics*, 13, 323-340.
- [23] **Holzer Harry, Lawrence Katz and Alan Krueger (1991)**; "Job Queues and Wages" *Quarterly Journal of Economics* Vol 106, pp 739-768.
- [24] **Hopenhayn, Hugo and Juan Pablo Nicolini, 1997** "Optimal Unemployment Insurance," *Journal of Political Economy*, 105, 412–438.
- [25] **Hosios, Arthur, 1990** "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57, 279–298.
- [26] **Katz, Lawrence and Bruce Meyer, 1990** "The Impact of Potential Duration of Unemployment Benefits on the Duration of Unemployment," *Journal of Public Economics*, 41, 45–72.
- [27] **Katz, Lawrence and Lawrence Summers, 1989** "Industry Rents: Evidence and Implications," *Brookings Papers on Economic Activity*, Special Issue, 209–275.
- [28] **Krueger Alan and Lawrence Summers (1988)**; "Efficiency Wages and the Inter-Industry Wage Structure" *Econometrica* Vol 56, 259-93.
- [29] **Marimon Ramon and Fabrizio Zilibotti, 1997** "Unemployment versus Mismatch of Talent," Pompeu Fabra Working Paper.
- [30] **Moen, Espen, 1997** "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

- [31] **Montgomery, James, 1991** "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 106, 163–179.
- [32] **Mortensen, Dale, 1977** "Unemployment Insurance and Job Search Decisions," *Industrial Labor Relations Review*, 30, 505–17.
- [33] **Mortensen, Dale, 1982** "Property Rights and Efficiency in Mating, Racing, and Related Games", *American Economic Review*, 72, 968–979.
- [34] **Mortensen, Dale and Christopher Pissarides, 1994** "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–416.
- [35] **Murnane, Richard and Frank Levy, 1996** *Teaching the Basic New Skills*, New York, The Free Press.
- [36] **Murphy Kevin and Robert Topel (1987)**; "Unemployment, Risk and Earnings: Testing for Equalizing Wage Differences in the Labor Market" in *Unemployment and the Structure of Labor Markets*, edited by Kevin Lang and Jonathan Leonard, Basil Blackwell.
- [37] **Myers, John G. and Daniel Creamer, 1967** "Measuring Job Vacancies: A Feasibility Study in the Rochester, N.Y. Areas" *Conference Board Studies in Business Economics*, 97.
- [38] **Peters, Michael, 1991** "Ex Ante Price Offers in Matching Games: Non-Steady States," *Econometrica*, 59, 1425–1454.
- [39] **Pissarides, Christopher, 1990** *Equilibrium Unemployment Theory*, Oxford: Basil Blackwell.
- [40] **Sattinger, Michael 1990** "Unemployment, the Market for Interviews, and Wage Determination," *Journal of Political Economy*, 98, 356–72.
- [41] **Shavell, Steven and Laurence Weiss, 1979** "The Optimal Payment of Unemployment Benefits over Time," *Journal of Political Economy*, 87, 1347–62.
- [42] **Shimer, Robert, 1996** "Essays in Search Theory," PhD dissertation, Massachusetts Institute of Technology: Cambridge, Massachusetts.
- [43] **Topel, Robert, 1993** "Unemployment and Insurance" in Van De Water op. cit.
- [44] **Topel, Robert and Finis Welch, 1980** "Unemployment Insurance: Survey and Extensions" *Economica* Vol 47, 351-379.
- [45] **Van Ours, J. C. and G. Ridder, 1993**, "Vacancy Durations: Search or Selection" *Oxford Bulletin of Economic Research*, Vol 55, 187-98.