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FEE SPEECH: ADVERSE SELECTION AND THE REGULATION OF MUTUAL FUND FEES

Sanjiv Ranjan Das
Rangarajan K. Sundaram

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ABSTRACT

The Investment Advisors Act of 1940 (as amended in 1970) prohibits mutual funds in the US from offering their advisers asymmetric "incentive fee" contracts in which the advisers are rewarded for superior performance via-a-vis a chosen index but are not correspondingly penalized for underperforming it. The rationale offered in defense of the regulation by both the SEC and Congress is that incentive fee structures of this sort encourage "excessive" risk-taking by advisers.

This paper uses an adverse selection model with multiple funds and multiple risky securities to study this issue. We find that incentive funds do, as alleged, lead to more (and suboptimal) risk-taking than do symmetric "fulcrum fees." Nevertheless, from the more important *welfare* angle, we find that investors may be strictly better off under asymmetric incentive fee structures. Thus, there appears to be little justification for this legislation.

Sanjiv Ranjan Das
Harvard University
Graduate School of Business
Soldiers Field
Boston, MA 02163
and NBER
sdas@hbs.edu

Rangarajan K. Sundaram
New York University
Stern School of Business
44 West 4th Street, MEC 9th Floor
New York, NY 10012
rsundara@stern.nyu.edu

1 Introduction

Permissible fee structures in the US mutual fund industry are laid out in the 1970 Amendment to the Investment Advisers Act of 1940.[1] The Act allows mutual funds and their investment advisers to enter into performance-based compensation contracts only if they are of the "fulcrum" variety, that is, ones in which the adviser's fee is symmetric around a chosen index, decreasing for underperforming the index in the same way in which it increases for outperforming it. Thus, while the Act does not rule out "fraction of funds" fees in which advisers are paid a fixed percentage of the total funds under management, it does prohibit so-called "incentive fee" contracts in which advisers receive a base fee plus a bonus for exceeding a benchmark index.[2]

The rationale most commonly offered in defense of the regulation is that incentive fees with their option-like payoff structures encourage investment advisers to take "excessive" amounts of risk by protecting them from the negative consequences of their actions. A recent paper by Das and Sundaram [3] suggests that there might be less to this argument than meets the eye. Das and Sundaram argue that since rational investors would react to an increase in the fund's riskiness by reducing the amount they invest in the fund, incentive fees need not necessarily lead to higher volatility of returns in equilibrium. Second, they suggest that incentive fees with their asymmetric payoff patterns may enable better risk-sharing between investors and investment advisers; thus, from the more relevant *welfare* standpoint, superior equilibrium outcomes may result under incentive fees. Both points are borne out in their model: incentive fees do provide higher equilibrium expected utilities to both investors and investment advisers than do fulcrum fees, and they do so at a lower level of equilibrium volatility.

A scenario not considered by Das and Sundaram [3] is the possibility of *adverse selection*, that is, of the presence of investment advisers of varying degrees of ability at generating returns. Intuition suggests an important role may be played by adverse selection in this setting. By definition, low-ability advisers will find it more difficult to beat the returns on the benchmark index than their higher-ability counterparts; under a fulcrum fee regime, therefore, the former will also have a greater downside exposure. As a consequence, the use of fulcrum fees may make it easier to separate high- and low-ability advisers in equilibrium, and this could potentially lead to improved outcomes for the investor. This paper examines the formalization of this argument, and in particular, the presence of adverse selection as a possible rationale for existing regulations.

A central aspect of the modeling process is the question of who determines the form of the compensation contract. The traditional principal/agent approach assigns this decision to the principal (i.e., the investor, in his role as fund shareholder). We abandon this approach, and instead assume that the choice of fee structure is made by the investment adviser. Two reasons underlie this decision.

The first, and more prosaic, one is that if the investor were, in fact, in control of the form of the compensation contract, restrictions on the possible forms of this contract can hurt, but certainly cannot enhance, investor welfare. Thus, the regulation would be superfluous, and even self-defeating. It is only when the choice of contract is effectively beyond the investor's purview that the need for

[1] For an outline of the history of this legislation, see Das and Sundaram [3] or Lin [16].

[2] We stress the point that incentive fees (or "performance fees" as they have also been called) are necessarily *asymmetric*; they reward good performance without penalizing poor performance.

legislative protection could arise.

A more important reason is that it may simply be a more appropriate assumption in the context of mutual funds that advisers, and not investors, decide the structure of the compensation contract. In principle, a fund is controlled by its shareholders (and, indeed, is required to have "outsiders" comprise at least 40% of its board). In practice, nonetheless, the relationship between a fund and its advisers tends to be extremely close. Indeed, most management companies are responsible for establishing the funds that they advise.

Our final model then has the following form. There are investment advisers of differing abilities who choose fee structures to signal their abilities. Upon observing these fee structures, investors make their investment decisions. The advisers then choose portfolios, allocating the amount invested between the available securities, and the participants' final rewards are realized. We compare equilibria of this game when fee structures are restricted to being of the fulcrum form to those that arise when fees are of the incentive form.[3]

Our main findings are easily summarized. We do find that asymmetric incentive fees encourage the adoption of more risky portfolios than fulcrum fees. Indeed, when faced with fulcrum fees, advisers with poor information-generating ability in our model select only moderately risky portfolios, but even such advisers switch to extreme portfolios in an incentive fee regime. Nonetheless, measured in welfare terms, we find that investors may be strictly better off under an incentive fee regime (we identify conditions under which this is always the case; see Section 5). Thus, we find little justification for the existing regulations; indeed, it appears to be easier to make a case for the opposite requirement that only incentive fees be employed.

Our results are more intuitive than might appear at first blush. In the presence of adverse selection, the fee structure chosen by the investment adviser plays two roles. On the one hand, it acts as a signal of the adviser's ability at generating rewards. On the other hand, it performs a risk-sharing function, determining the final distribution of rewards between the adviser and investor. Thus, in a separating equilibrium, the fee structure adopted by the high-ability adviser must be such that (a) it does not pay for the low-ability adviser to imitate this structure, and (b) the resulting net-of-fees returns to the investor provides at least as much utility as the investor could obtain from the low-ability adviser. The first of these two properties, that of separation, is evidently facilitated more by a fulcrum fee structure than an incentive fee structure, since the presence of downside risk in the former makes mimicking a more expensive proposition for the low-ability adviser. The second property is more complex: the utility the investor can obtain from an adviser of either sort depends on the risk-sharing properties of the fee structure in place as well as the portfolio composition induced by the fee structure for that adviser.

Now, when risk-sharing considerations are not very important (say, the investor is risk-neutral), the first aspect of the fee predominates, and one would expect in this case that, by facilitating separation, a fulcrum fee regime would enable the informed adviser to absorb more of the surplus from the investor than would an incentive fee regime. Thus, from the standpoint of investor welfare, the incentive fee structure should be preferable, and this is, in fact, what we find (see Proposition 5.3).

[3] The benchmark portfolio in either case is taken as exogeneously given, and does not constitute a strategic choice in our model. This is broadly consistent with observed reality. Funds that use fulcrum fees in practice tend to take as the benchmark a widely-recognized index (for example, the S&P 500), rather than develop benchmarks of their own. See Das and Sundaram [3] or Lin [16].

The introduction of risk-sharing considerations, makes the comparison more subtle and the results a little more ambiguous. Although the asymmetric nature of incentive fees does enable superior risk-sharing in general (as Das and Sundaram [3] have argued), this effect is diluted by the fact that incentive fee structures also encourage the adoption of extreme portfolios, which are suboptimal from the viewpoint of a risk-averse investor. Thus, while incentive fees continue to dominate for some parametrizations of the model, we find that fulcrum fees may become preferrable under others.

Before getting to the body of our paper, two aspects of our model bear emphasis. First, our model may be thought of as a principal/agent setting in which the agent (rather than the principal) chooses the fee structure, and the principal responds with the choice of resources to be invested with the agent. Such models do not appear to have been investigated in any depth in the literature; we think they might be of use in studying compensation structures in contexts beyond those of the current paper. Second, our use of the fee or compensation structure as a mechanism for signalling ability also appears novel.[4] It appears to us that this is an important function played by compensation structures in many practical instances.

The remainder of this paper is organized as follows. Section 2 indicates the related literature. Section 3 describes our model and provides formal definitions of the various fee structure restrictions. Section 4 describes equilibria in the model and derives some preliminary results, while Section 5 compares equilibrium outcomes under different fee structure restrictions. Section 6 concludes. The Appendices contain proofs omitted in the main body of the text.

2 The Related Literature

This section provides a brief discussion of the theoretical literature on compensation structures in the mutual fund industry. The presentation here is meant to be indicative of the work that has been done in this area and is not intended as a survey of the field.

Broadly speaking, there are two branches to the literature on mutual fund compensation. On the one hand are the papers that take a partial equilibrium approach and examine the reaction of managers to a *ceteris paribus* change in the fee structure. On the other hand are the papers that adopt a "full" equilibrium approach, solving for compensation structures as part of an equilibrium. Papers falling into the first group include Davanzo and Nesbit [4], Ferguson and Lestikow [5], Goetzman, Ingersoll, and Ross [6], Grinblatt and Titman [9], Grinold and Rudd [10], and Kritzman [13]. Those falling into the second group include Das and Sundaram [3], Heinkel and Stoughton [11], Huddart [12], and Lynch and Musto [17]. Finally, there is the recent paper of Admati and Pfleiderer [1] which combines aspects of both approaches.

Of the first category of papers, the most comprehensive analyses are those in Grinblatt and Titman [9] and Goetzmann, Ingersoll, and Ross [6]. Grinblatt and Titman assume that managers can risklessly capture the value of any options implicit in their payoff structure by hedging in their personal portfolios. This enables the use of results from option pricing theory in characterizing

[4] The literature does, of course, contain many other signalling mechanisms of interest. For example, Leland and Pyle [15] consider a setting where an entrepreneur signals the quality of his project through his proportion of ownership in it, while in a model closely related to our own, Huddart [12] examines two money managers who signal their private information by choosing portfolios to which they commit.

the fee maximizing level of risk for any given contract structure. Among other things, Grinblatt and Titman show that for certain classes of portfolio strategies, adverse risk-sharing incentives are avoided when the penalties for poor performance outweigh the rewards for good performance. Goetzmann, Ingersoll, and Ross [6] are concerned with "high watermark" contracts of the sort frequently used by hedge funds in which the adviser receives a proportion of the fund return each year in excess of the portfolio's previous high water mark, i.e., the maximum share value since inception of the fund. The authors provide a closed-form solution for the value of such contracts, and show, among other things, that such contracts are valuable to the advisers and, ipso facto, represent a claim on a significant portion of investor wealth.

Of the second group of papers, the only one that focusses on the regulatory issue that concerns us in this paper is Das and Sundaram [3], which we have already described above. We discuss some of the other papers in more detail below.

Heinkel and Stoughton [11] and Lynch and Musto [17] aim to explain the predominance of fraction-of-funds fee arrangements in the money management industry (including, but not only, mutual funds). Heinkel and Stoughton employ a two-period model with moral hazard and adverse selection. They show that the equilibrium set of contracts in their model features a smaller performance-based fee in the first period than in a first-best contract. They suggest that this reduced emphasis on the performance component in the first period is analogous to the lack of a performance-based fee in many parts of the asset-management industry. Lynch and Musto [17] examine a moral hazard model in which the manager's effort is observable by the investor, but is not contractable. They focus on identifying conditions under which different fee structures predominate.

Huddart [12] studies a model similar to our own in which multiple fund managers of differing abilities compete. Unlike us, Huddart is not concerned with fee structures and assumes fees are exogeneously fixed at some proportion of assets under management. Signalling of abilities in Huddart's model is accomplished using portfolio choices. However, Huddart does show that the adoption of an incentive fee can mitigate undesirable reputation effects and make investors better off.

Admati and Pfleiderer [1] consider a scenario where the fund manager has superior information to the investor and faces a fulcrum fee structure. They examine if there are any conditions under which the manager would pick the investor's most desired portfolio (i.e., the portfolio that the investor would have chosen had he been possessed of the same information as the manager). The fundamental difference between their analysis and ours is that Admati and Pfleiderer examine the desirability of benchmarking *within* a fulcrum fee structure; they do not consider incentive fee structures. We, on the other hand, take benchmarking as a given and compare the effects of different fee structures on equilibrium payoffs. Secondly, given their motivation, Admati and Pfleiderer are not explicitly concerned with determining equilibrium fee structures and portfolios. Thus, for example, they take the amount invested with the manager as exogeneous; they also compute the investor's most desired portfolio by using gross returns rather than returns net of the manager's fees.

3 The Model

We study a model with two fund managers/investment advisers and a representative investor. One of the advisers, whom we shall refer to as the "informed" adviser, is assumed to have superior ability at generating information concerning returns on the model's risky securities. The other adviser lacks such ability and is termed "uninformed." An adviser's type is private information and is not observable by the investor; rather, the investor must infer this information from the actions of the advisers. The advisers are both assumed to be risk-neutral, and have as their objective the maximization of expected fees received.[5] The reservation utility levels of the informed and uninformed adviser are denoted by $\bar{\pi}_I$ and $\bar{\pi}_N$, respectively.

The investor, a representative stand-in for a large number of identical investors, has an initial wealth of w_0 (normalized to \$1 in the sequel). The investor's objective is to maximize the utility $U(\tilde{w})$ of terminal wealth $\tilde{w}$ at the end of the model's single period. We assume that U has a mean-variance form given by

$$U(\tilde{w}) \;=\; E(\tilde{w}) - \frac{1}{2}\gamma V(\tilde{w}), \tag{3.1}$$

where $E(\cdot)$ and $V(\cdot)$ represent, respectively, the expectation and variance operators, and $\gamma > 0$ is a parameter indicating the investor's aversion to variance.

The Sequence of Events

Events in our model evolve as follows. The investment advisers move first and simultaneously announce their fee structures. After observing these fee structures, the investor decides with which adviser to invest; for analytic simplicity, we assume that the investor must invest with only a single adviser. Next, the informed adviser receives information concerning the return distribution on the risky securities; the uninformed adviser receives no information at this stage. Lastly, the advisers decide on their portfolio compositions, and final rewards are realized. Our objective in this paper is to examine whether restrictions on the advisers' freedom to set fees—specifically, requiring that the advisers only use "fulcrum" fees—can enhance the welfare of the investor. The remainder of this section discusses the components of this model in greater detail.

Securities and Returns Distributions

There are three securities in our model, a riskless security and two risky securities. The net return on the riskless security is normalized to zero. The "true" joint return distribution on the two risky securities is either Π_1 or Π_2. The informed adviser gets to know which of the two distributions represents the true distribution prior to making his investment decision. The uninformed adviser only knows the prior probabilities ζ and $(1-\zeta)$ of the two distributions.[6]

Table 1 describes the specific structure we adopt concerning these returns distributions. As the table indicates, we take the gross returns on each security to follow a Binomial process in which

[5] The assumption of risk-neutrality is made primarily in the interests of analytic tractability. However, the intuition behind our results appears compelling, and we do not think they would be qualitatively altered if the advisers were risk-averse, at least for moderate degrees of risk-aversion.

[6] As will be apparent, it is not necessary for our results that the informed adviser learn the true state of the world with certainty; rather, it suffices that he receive an informative signal. In the latter case, the numbers in Table 1 should be interpreted as posterior probabilities of the outcomes conditional on the signal.

Table 1: Returns Distributions on the Risky Securities

The gross returns on each of the two risky securities can take on either the high value H or the low value L. We assume $H > 1 > L$ and $H + L > 2$. The joint distribution of the two returns is given by either Π_1 or Π_2. The distributions are assumed equiprobable: the prior probability of Π_1 is $\zeta = 1/2$. The table below describes the probabilities of various outcomes under Π_1 and Π_2. The first entry in each outcome corresponds to the return on the first security, and the second to that on the second security. The probabilities in the table are assumed to satisfy the following conditions: (i) $p + 2q + r = 1$, (ii) $p, q, r > 0$, and (iii) $p > r$.

Outcome	Probability under Π_1	Probability under Π_2
(H, H)	q	q
(H, L)	p	r
(L, H)	r	p
(L, L)	q	q

the security returns either H or L. Under the joint distribution Π_1, security 1 returns H with a strictly higher probability than does security 2, but the reverse is true under Π_2; thus, it is valuable information to know which of these represents the true distribution.

Fees

The fees charged by an adviser may depend on the realized returns r_p on the adviser's portfolio, as well as on the realized returns r_b on a "target" or "benchmark" portfolio. The fees, denoted $F(r_p, r_b)$, are assumed to be received at the end of the period, and are deducted from the gross returns r_p on the adviser's portfolio. Thus, given the fee structure F and realized returns r_p and r_b, the net-of-fees return to the investor is $r_p - F(r_p, r_b)$.

The distribution of returns $\tilde{r}_p$ on the adviser's portfolio depends on the composition of this portfolio. We discuss the imperatives that go into the construction of this portfolio below. The benchmark portfolio is exogeneously given, and is taken to be a portfolio consisting of half a unit each of the two risky securities.

The Investor's Decision

The fee structure plays a dual role in our model. On the one hand, it performs a risk-sharing function: each given fee structure (together with the adviser's portfolio choice) implies a particular division of returns between the adviser and the investor. On the other hand, it also has potential informational content: the selection of a fee structure may also send a signal to the investor about the type of the adviser. In a *separating* fee profile, the informed adviser chooses a fee F that (a) would guarantee him at least his reservation utility if the investor were to invest with him, but one that (b) the uninformed adviser would not wish to mimic, because it would put him below his reservation utility level even if he were to be successful in attracting the investor's dollar. A fee profile is *pooling* if it is not separating.

Taking both factors into account, the investor in our model decides on the choice of adviser to invest with. If the fee profile chosen by the advisers is separating, the investor compares the utility level obtained by investing with the informed adviser to that from investing with the uninformed adviser, and selects the adviser who delivers the higher utility. This choice is not, of course, a trivial one. Computing net-of-fees returns requires predicting the portfolio that will be chosen by the adviser, which will, in turn, depend on the fee structure that was committed to. Moreover, even though the informed adviser will always be able to generate higher total returns, the fees charged by the advisers may make the net-of-fees returns from the uninformed adviser more attractive.

The investor's choice problem becomes a little more complex if the fees chosen are pooling. In this circumstance, the investor assumes that each adviser is informed with probability 1/2 and uninformed with probability 1/2, and assigns the dollar to the adviser whose net-of-fees returns (computed under this assumption) are more attractive. Finally, in all cases, we assume that if the investor finds the advisers equally attractive, then he randomizes between them, so each adviser receives the dollar with probability 1/2.

The Advisers' Portfolio Choices

The final move in our model is made by the advisers selecting their portfolios. The informed adviser can condition this choice on the information he receives concerning the "true" state of the world. The uninformed adviser, not being privy to this information, must choose the same portfolio in both states. In choosing their portfolios, advisers take as given the fee structure choices made earlier, and choose an allocation between the three securities that will maximize their expected fees.

We consider two situations: where the adviser is not permitted to use levered strategies, and where such strategies are allowed. In the former case, the sum total of the investment in the two risky securities cannot exceed the initial asset value of a dollar. In the latter case, we assume that there is a pre-specified ceiling on the extent of leveraging permitted; i.e., there is $a^{\max} > 1$ such that the total amount invested in the two risky securities cannot exceed $a^{\max}$. Finally, we also assume that short positions are not permitted in either of the risky securities. This last assumption is made purely for expositional convenience; our results remain unaffected if it is replaced by a ceiling on the maximum size of short positions allowed.

Fee Structures of Special Interest

Our objective in this paper is to examine whether restrictions on the form of the fee structures $F(r_p, r_b)$ that can be used by the advisers in the first step can increase the investor's level of equilibrium utility in this model. We are especially interested in this context in the class of *fulcrum fees*, which existing regulation requires mutual funds to use. Such fees are defined by a symmetry requirement: they must increase for outperforming the benchmark returns in the same way that they decrease for underperforming it. We restrict attention to linear fulcrum fees, which are, by far, the most common type used in practice. Such fees are described by

$$F(r_p, r_b) = b_1 r_p + b_2(r_p - r_b), \tag{3.2}$$

where b_1 and b_2 are non-negative constants denoting, respectively, the base fee and the performance adjustment component. When $b_2 = 0$, the fees are simply a constant fraction b_1 of the total returns r_p; such fees are called "flat fees" or "fraction-of-funds" fees.

A second class of fees of importance are (asymmetric) *incentive fees*. Like fulcrum fees, incentive fees are described by two parameters b_1 and b_2, with b_1 denoting the base fee level, and b_2 the performance adjustment component. However, unlike fulcrum fees, the performance adjustment component must remain non-negative, and the total fee is given by

$$F = b_1 r_p + b_2 \max\{r_p - r_b, 0\}. \tag{3.3}$$

As we mentioned in the opening remarks to this paper, the existing legislation on mutual fund fees is explicitly motivated by fear of the consequences of *incentive* fee structures, in particular of the adverse risk-taking such contract forms may encourage.[7] In the remainder of this paper, we examine the extent to which these fears are justified by comparing the set of equilibrium outcomes that result under fulcrum fees to those that obtain under incentive fees, with particular emphasis on the investor's welfare under the two regimes.[8]

4 Equilibrium

This section describes the optimization problems whose solutions identify the equilibrium outcomes under the two fee regimes described in the previous section. Sections 4.1–4.2 deal with fulcrum fees, while Sections 4.3–4.4 handle incentive fees. The results presented here are used in Section 5 to compare the equilibrium payoffs of the investors and the advisers under the two regimes. Since the focus of this paper is on separating equilibria, we avoid unnecessary details in the presentation and study only that case in this section. For completeness, Appendix B looks at pooling equilibria also; it is shown there that pooling equilibria never exist under incentive fees, and generally do not exist under fulcrum fees either.

[7] It is important to note that the rationale offered for the prohibition is theoretical, rather than empirical, in nature. That is, the ban on incentive fees is motivated more by concerns about the inherent nature of incentive fee contracts than by any actual evidence of abuse.

[8] While this limited comparison clearly suffices for our purposes, our decision to focus on just incentive and fulcrum fee structures (rather than examine general unrestricted fee structures) also stems from a practical consideration. Incentive fee structures are commonly used in relationships between investors and investment advisers where they are legal (e.g., in hedge funds). In contrast, as is well known, unrestricted equilibrium contracts in principal/agent models often take on unrealistically complex and unintuitive forms.

4.1 Portfolio Choices under Fulcrum Fees

As the first step in identifying equilibria under fulcrum fees, we identify the portfolios that would be chosen by the two advisers in the last step of the game given an arbitrary fulcrum fee (b_1, b_2). Some new notation will simplify this process. We will denote a typical portfolio choice for either adviser by $(\alpha_0, \alpha_1, \alpha_2)$, where α_0 represents the amount invested in the riskless security, and α_1 and α_2 represent, respectively, the amounts invested in the first and second risky securities. Of course, we must have $\alpha_0 + \alpha_1 + \alpha_2 = 1$, and, since short selling of the risky securities is prohibited, $\alpha_1, \alpha_2 \geq 0$. Moreover, since $a^{\max} \geq 1$ represents the maximum amount that may be invested in the two risky securities combined, we must also have $\alpha_1 + \alpha_2 \leq a^{\max}$. In the interests of notational simplicity, we will write a for $a^{\max}$ throughout this section.

Proposition 4.1 *Let any fulcrum fee (b_1, b_2) be given. Under the fee structure (b_1, b_2):*

1. *The informed adviser will choose the portfolio $(1-a, a, 0)$ if state 1 were to occur, and the portfolio $(1-a, 0, a)$ if state 2 were to occur.*
2. *Any portfolio of the form $(1-a, m, a-m)$ for $m \in [0, a]$ is optimal for the uninformed adviser.*

Proof See Appendix A.1. □

In words, Proposition 4.1 states that given any fulcrum fee structure, the informed adviser will always choose an extreme portfolio, while the uninformed adviser is indifferent between any combination of the two risky securities. This result is intuitive. Since the informed adviser receives the information in advance about which security will be the higher-performing one, his expected fee is maximized by investing the maximum amount in that security and nothing in the other security. On the other hand, the uninformed adviser has no particular information and, since both securities have identical a priori return charactersitics, no particular grounds for preferring one security to the other.

Given these portfolio choices and the information in Table 1, we can compute the ex-ante distribution of returns to the informed adviser and the investor that would arise if the investor were to choose the informed adviser. Denoting these returns by F_I and Y_I, respectively, we have

$$F_I = \begin{cases} b_1(aH + 1 - a) + b_2(aH + 1 - a - H), & \text{w.p. } q \\ b_1(aH + 1 - a) + b_2(aH + 1 - a - (H + L)/2)), & \text{w.p. } p \\ b_1(aL + 1 - a) + b_2(aL + 1 - a - (H + L)/2)), & \text{w.p. } r \\ b_1(aL + 1 - a) + b_2(aL + 1 - a - L), & \text{w.p. } q \end{cases} \tag{4.1}$$

$$Y_I = \begin{cases} (1 - b_1)(aH + 1 - a) - b_2(aH + 1 - a - H), & \text{w.p. } q \\ (1 - b_1)(aH + 1 - a) - b_2(aH + 1 - a - (H + L)/2)), & \text{w.p. } p \\ (1 - b_1)(aL + 1 - a) - b_2(aL + 1 - a - (H + L)/2)), & \text{w.p. } r \\ (1 - b_1)(aL + 1 - a) - b_2(aL + 1 - a - L), & \text{w.p. } q \end{cases} \tag{4.2}$$

Since the uninformed adviser is indifferent between all portfolios of the form $(1-a, m, a-m)$ for $m \in [0, a]$, we may assume without loss that he picks the portfolio among these that maximizes the investor's expected utility. (This would also maximize the chance of his receiving the investment.) A simple computation shows that this occurs when the adviser selects the risky securities in the same proportions as the market portfolio, that is, the optimal portfolio is $(1-a, a/2, a/2)$. (This is intuitive.) Under these proportions, the distribution of returns to the uninformed adviser and investor, denoted F_N and Y_N respectively, are given by

$$F_N = \begin{cases} b_1(1-a+aH)+b_2(a-1)(H-1), & \text{w.p. } q \\ b_1(1-a+a(H+L)/2)+b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ b_1(1-a+a(L+H)/2)+b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ b_1(1-a+aL)+b_2(a-1)(L-1), & \text{w.p. } q \end{cases} \tag{4.3}$$

$$Y_N = \begin{cases} (1-b_1)(1-a+aH)-b_2(a-1)(H-1), & \text{w.p. } q \\ (1-b_1)(1-a+a(H+L)/2)-b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ (1-b_1)(1-a+a(L+H)/2)-b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ (1-b_1)(1-a+aL)-b_2(a-1)(L-1), & \text{w.p. } q \end{cases} \tag{4.4}$$

When we wish to emphasize the dependence of these returns on the fee structure, we shall write $F_I(b_1, b_2)$, $Y_I(b_1, b_2)$, etc. The expectations of these variables will be denoted $E[\cdot]$ (for example, $E[F_I(b_1, b_2)]$), and the variances by $V[\cdot]$ (for instance, $V[Y_I(b_1, b_2)]$). Using this notation, conditional on knowing the adviser's identity, and given (b_1, b_2), the expected utility of the investor from investing with the informed adviser is

$$E[U_I(b_1, b_2)] = E[Y_I(b_1, b_2)] - \frac{1}{2}\gamma V[Y_I(b_1, b_2)]. \tag{4.5}$$

Similarly, conditional on knowing the adviser's identity, the expected utility to the investor from investing with the uninformed adviser is

$$E[U_N(b_1, b_2)] = E[Y_N(b_1, b_2)] - \frac{1}{2}\gamma V[Y_N(b_1, b_2)]. \tag{4.6}$$

4.2 Separating Equilibrium under Fulcrum Fees

For an equilibrium in this model to be separating, it must satisfy two conditions: (i) the fee structure chosen by the informed adviser must be one that the uninformed adviser would not wish to mimic, and (ii) the investor receives at least as much expected utility from investing with the informed adviser as he could from investing with the uninformed adviser. Thus, identifying a separating equilibrium requires a two step procedure. First, we look at the maximum utility the investor could obtain from the uninformed adviser, subject to the latter receiving at least his reservation expected fee level. That is, we solve:

$$\begin{aligned} \text{Maximize} \quad & E[U_N(b_1, b_2)] \\ \text{subject to} \quad & E[F_N(b_1, b_2)] \geq \bar{\pi}_N \\ & b_1, b_2 \geq 0 \end{aligned} \tag{4.7}$$

Let EU_N^* denote the maximized value of the objective function in this problem. In the second step, we look for the fee structure that maximizes the expected fee of the informed adviser subject to two constraints: providing the investor with at least his "reservation" utility level EU_N^*, and ensuring the non-mimicking condition.

$$
\begin{aligned}
\text{Maximize} \quad & E[F_I(b_1, b_2)] \\
\text{subject to} \quad & E[U_I(b_1, b_2)] \ \geq \ EU_N^* \\
& E[F_N(b_1, b_2)] \ \leq \ \bar{\pi}_N \\
& b_1, b_2 \ \geq \ 0
\end{aligned}
\tag{4.8}
$$

Let EF_I^* denote the maximized value of the objective function in (4.8), and EU_I^* the expected utility of the investor in a solution. If there is a solution to (4.8) which satisfies $EF_I^* \geq \bar{\pi}_I$, then a separating equilibrium exists in this model; if not, then no separating equilibrium exists.[9,10]

4.3 Portfolio Choices under Incentive Fees

Identifying equilibrium outcomes under incentive fees involves the same steps as under fulcrum fees. We begin by identifying the equilibrium portfolio choices of the two advisers for any choice of incentive fee (b_1, b_2).

Proposition 4.2 *Let any incentive fee (b_1, b_2) be given. Under the fee structure (b_1, b_2):*

1. *The informed adviser will choose the portfolio $(1-a, a, 0)$ if state 1 were to occur, and the portfolio $(1-a, 0, a)$ if state 2 were to occur.*
2. *If $b_2 > 0$, then the uninformed adviser will choose either the portfolio $(1-a, a, 0)$ or the portfolio $(1-a, 0, a)$. If $b_2 = 0$, then any portfolio of the form $(1-a, m, a-m)$ for $m \in [0, a]$ is optimal for the uninformed adviser.*

Proof See Appendix A.2. □

Proposition 4.2 summarizes, in a sense, the main argument behind existing regulations on fee structures that allowing for incentive fees will lead to "excessive" amounts of risk. Under fulcrum fees, the informed adviser chooses an extreme portfolio, but, as we saw, the most reasonable choice of portfolio for the uninformed adviser was one that held the risky assets in the same proportions as the benchmark. Under incentive fees, however, the uninformed adviser also always takes an extreme portfolio. Moreover, unlike the informed adviser's choice, this could be the "wrong" extreme

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[9] For arbitrary values of the parameters, separating equilibria need not, of course, exist (for instance, existence will fail if $\bar{\pi}_I$ is very large relative to the expected returns on the portfolio). For reasonable parameter values, however, separating equilibria will typically exist.

[10] Note that in a separating equilibrium only one fund (namely, that run by the informed adviser) will remain in the market. The other, unable to meet its reservation fee level, will exit. However, it is the threat of competition offered by the uninformed adviser that drives the equilibrium.

portfolio; from an a priori standpoint, the portfolios $(1-a,a,0)$ and $(1-a,0,a)$ are both optimal for the uninformed adviser, but, obviously, the second one is an inferior choice if state 1 were the true state, while the first one is inferior if state 2 were the true state. Thus, by protecting him from the downside consequences of his actions, incentive fees encourage the adviser to take on extreme positions even in the absence of any information to justify those positions.

However, the important issue is not one of which fee structure encourages most risk-taking but which is best (welfare maximizing) for the investor. This requires us to identify the equilibria of the model with incentive fees. To this end, we first identify the returns that arise from each adviser. To maintain a distinction between the equilibrium outcomes under incentive fees and fulcrum fees, we will amend the notation of the previous subsection as follows: we will denote by G the fee received by the adviser under an incentive fee, by X the returns to the investor, and by EV the expected utility of the investor.

If the investor were to invest with the informed adviser, the *ex ante* distribution of returns G_I and X_I to the adviser and investor, respectively, are given by

$$G_I = \begin{cases} b_1(aH+1-a)+b_2(aH+1-a-H), & \text{w.p. } q \\ b_1(aH+1-a)+b_2(aH+1-a-(H+L)/2)), & \text{w.p. } p \\ b_1(aL+1-a), & \text{w.p. } r \\ b_1(aL+1-a), & \text{w.p. } q \end{cases} \tag{4.9}$$

$$X_I = \begin{cases} (1-b_1)(aH+1-a)-b_2(aH+1-a-H), & \text{w.p. } q \\ (1-b_1)(aH+1-a)-b_2(aH+1-a-(H+L)/2)), & \text{w.p. } p \\ (1-b_1)(aL+1-a), & \text{w.p. } r \\ (1-b_1)(aL+1-a), & \text{w.p. } q \end{cases} \tag{4.10}$$

Now suppose the investor chooses the uninformed adviser. Consider first the case where $b_2 > 0$, and assume, without loss, that the uninformed adviser picks the portfolio $(1-a,a,0)$. In this case, the distribution of returns G_N and X_N to the two parties are

$$G_N = \begin{cases} b_1(1-a+aH)+b_2(a-1)(H-1), & \text{w.p. } q \\ b_1(aH+1-a)+b_2(aH+1-a-(H+L)/2)), & \text{w.p. } z \\ b_1(aL+1-a), & \text{w.p. } z \\ b_1(aL+1-a), & \text{w.p. } q \end{cases} \tag{4.11}$$

$$X_N = \begin{cases} (1-b_1)(1-a+aH)-b_2(a-1)(H-1), & \text{w.p. } q \\ (1-b_1)(aH+1-a)-b_2(aH+1-a-(H+L)/2)), & \text{w.p. } z \\ (1-b_1)(aL+1-a), & \text{w.p. } z \\ (1-b_1)(aL+1-a), & \text{w.p. } q \end{cases} \tag{4.12}$$

On the other hand, if $b_2 = 0$, then any portfolio of the form $(1-a,m,a-m)$ for $m \in [0,a]$ is optimal. In this case, we may assume that the adviser picks the portfolio that maximizes the investor's utility, since this maximizes in turn his chances of attracting the investment. A simple calculation shows that this portfolio is $(1-a, a/2, a/2)$, so the returns that arise are:

$$G_N = \begin{cases} b_1(1-a+aH) + b_2(a-1)(H-1), & \text{w.p. } q \\ b_1(1-a+a(H+L)/2) + b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ b_1(1-a+a(L+H)/2) + b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ b_1(1-a+aL), & \text{w.p. } q \end{cases} \tag{4.13}$$

$$X_N = \begin{cases} (1-b_1)(1-a+aH) - b_2(a-1)(H-1), & \text{w.p. } q \\ (1-b_1)(1-a+a(H+L)/2) - b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ (1-b_1)(1-a+a(L+H)/2) - b_2(a-1)((H+L)/2-1), & \text{w.p. } z \\ (1-b_1)(1-a+aL), & \text{w.p. } q \end{cases} \tag{4.14}$$

Once again, when we wish to emphasize the dependence of any of these quantities on the fee structure, we will write $X_I(b_1, b_2)$, $G_I(b_1, b_2)$, etc. Now, note that if the investor chooses the informed adviser, then, conditional on knowning the adviser's type, the investor's ex-ante expected utility is

$$E[V_I(b_1, b_2)] = E[X_I(b_1, b_2)] - \frac{1}{2}\gamma V[X_I(b_1, b_2)].$$

Similarly, the ex-ante expected utility from investing with the uninformed adviser (again, conditional on knowing the adviser's type) is

$$E[V_N(b_1, b_2)] = E[X_N(b_1, b_2)] - \frac{1}{2}\gamma V[X_N(b_1, b_2)].$$

4.4 Separating Equilibrium under Incentive Fees

The first step in identifying a separating equilibrium is identifying the maximum utility the investor could receive from the uninformed adviser subject to the adviser receiving at least his reservation utility level. That is, we solve

$$\begin{array}{lrcl} \text{Maximize} & E[V_N(b_1, b_2)] & & \\ \text{subject to} & E[G_N(b_1, b_2)] & \geq & \bar{\pi}_N \\ & b_1, b_2 & \geq & 0 \end{array} \tag{4.15}$$

Let EV_N^* denote the maximized value of the objective function in this problem. In the second step, we look for the fee structure that maximizes the expected fee of the informed adviser subject to two constraints: providing the investor with at least his "reservation" utility level EV_N^*, and ensuring the non-mimicking condition.

$$\begin{array}{lrcl} \text{Maximize} & E[G_I(b_1, b_2)] & & \\ \text{subject to} & E[V_I(b_1, b_2)] & \geq & EV_N^* \\ & E[G_N(b_1, b_2)] & \leq & \bar{\pi}_N \\ & b_1, b_2 & \geq & 0 \end{array} \tag{4.16}$$

In the next section, we use these optimization problems to obtain and compare equilibrium outcomes under the two fee regimes.

5 Comparison of Equilibrium Outcomes

Our comparison of equilibrium outcomes under the different fee regimes proceeds in two steps. In Section 5.1, we examine the case of a risk-neutral investor, i.e., one for whom the variance-aversion coefficient γ is zero. There are no relevant risk-sharing features to the fee structure in this case; thus, the analysis here enables us to focus on just the separation aspects of the fee structure. We show in this case (see Proposition 5.3 below) that incentive fees unambiguously dominate fulcrum fees from the standpoint of investor welfare.

Then, in Section 5.2, we consider the case of a risk-averse investor. It becomes relevant in this case that incentive fees encourage even the uninformed adviser to choose extreme portfolios. As a consequence, the results become more ambiguous; indeed, for some parametrizations, fulcrum fees outcomes dominate incentive fee outcomes.

In summary, the main point made by these results is that there appears little justification for the regulatory requirements in place today. Indeed, it seems easier to make a case for the reverse regulation requiring only the use of incentive fees.

5.1 A Risk-Neutral Investor

We begin with identifying separating equilibrium outcomes under fulcrum fees. Some new notation will be helpful in this process. First, we will denote by R_N and R_I, respectively, the returns on the portfolios chosen by the uninformed and informed adviser in a fulcrum fee regime, and by R_B the return on the benchmark portfolio. We will also use EF_I^* and EU_I^* to denote the utilities of the informed adviser and the investor in a separating equilibrium. Finally, as in Section 4.2, EU_N^* will denote the investor's "reservation utility" level defined via the problem (4.7).

Proposition 5.1 *If $\bar{\pi}_N < E(R_N - R_B)$, then the equilibrium outcomes under a fulcrum fee regime are as follows:*

$$EF_I^* = \frac{E(R_I - R_B)}{E(R_N - R_B)}\bar{\pi}_N. \tag{5.1}$$

$$EU_I^* = ER_I - EF_I^*. \tag{5.2}$$

$$EU_N^* = ER_N - \bar{\pi}_N. \tag{5.3}$$

If $\bar{\pi}_N \geq E(R_N - R_B)$, then these outcomes become

$$EF_I^* = E(R_I - R_N) + \bar{\pi}_N. \tag{5.4}$$

$$EU_I^* \;=\; ER_N - \bar{\pi}_N. \tag{5.5}$$

$$EU_N^* \;=\; ER_N - \bar{\pi}_N. \tag{5.6}$$

Remark Note that when $\bar{\pi}_N < E(R_N - R_B)$, the investor receives strictly more in the separating equilibrium than his "reservation" level EU_N^*. (This follows from $E(R_I) > E(R_N)$.) Thus, even under risk-neutrality, the informed adviser cannot always obtain all the gains from trade and reduce the investor to his reservation utility level. □

Proof When the investor is risk-neutral, we must have $EU_N + EF_N = ER_N$ and $EU_I + EF_I = ER_I$. Using the first of these expressions, it is immediate that any solution to the investor's "reservation utility" problem (4.7) results in $EU_N^* = ER_N - \bar{\pi}_N$, whence (5.3) and (5.6) follow. To see the rest of the proposition, we (i) substitute this and the identity $EU_I = [E(R_I) - EF_I]$ into the separation problem (4.8), and (ii) use the full forms for the expected fees $EF_I = b_1 E(R_I) + b_2 E(R_I - R_B)$ and $EF_N = b_1 E(R_N) + b_2 E(R_N - R_B)$. After some rearranging, the separation problem (4.8) now becomes

$$\begin{array}{lll} \text{Maximize} & b_1\, E(R_I) + b_2\, E(R_I - R_B) & \\ & & \\ \text{subject to} & b_1\, E(R_N) + b_2\, E(R_N - R_B) \;\leq\; \bar{\pi}_N & \\ & b_1\, E(R_I) + b_2\, E(R_I - R_B) \;\;\;\leq\; \bar{\pi}_N + E(R_I - R_N) & \\ & b_1, b_2 \;\geq\; 0 & \end{array} \tag{5.7}$$

When $b_2 = 0$, the first constraint imposes an upper bound on b_1 of $\bar{\pi}_N / E(R_N)$, while the second constraint imposes an upper-bound on b_1 of $[\bar{\pi}_N + E(R_I - R_N)]/E(R_I)$. A simple calculation shows that the first of these bounds is always smaller than the second whenever $\bar{\pi}_N \leq E(R_N)$. This latter inequality must, of course, always hold in any sensible definition of the problem (the reservation fee cannot be greater than the total expected returns). Thus, at $b_2 = 0$, the second constraint is always slack.

When $b_1 = 0$, the first constraint imposes an upper-bound on b_2 of $\bar{\pi}_N / E(R_N - R_B)$, while the second constraint imposes an upper-bound of $[\bar{\pi}_N + E(R_I - R_N)]/E(R_I - R_B)$. The first of these bounds is smaller if, and only if, $\bar{\pi}_N < E(R_N - R_B)$, which may or may not hold.

Summing up, therefore, there are two possibilities. If $\bar{\pi}_N < E(R_N - R_B)$, the second constraint is always slack. An easy computation shows that the maximum in problem (5.7) occurs in this case when $b_1 = 0$ and $b_2 = \bar{\pi}_N / E(R_N - R_B)$. This leads to the equilibrium payoffs (5.1)–(5.2). In the second case, when $\bar{\pi}_N \geq E(R_N - R_B)$, the second constraint is binding at a maximum. One solution to the problem (there are many) is $b_1 = [\bar{\pi}_N - E(R_N - R_B)]/E(R_B)$ and $b_2 = 1 - b_1$. This leads (as do all solutions) to the payoffs (5.4)–(5.5). □

To derive the corresponding result for incentive fees, some more notation is unfortunately necessary. For a given value of a, let

$$\begin{array}{lllllll} y_h & = & aH + 1 - a & \quad & x_h & = & H \\ y_{hl} & = & a(H + L)/2 + 1 - a & \quad & x_{hl} & = & (H + L)/2 \\ y_l & = & aL + 1 - a & \quad & x_l & = & L \end{array} \tag{5.8}$$

These terms have a simple interpretation: y_h and y_l are simply the set of possible outcomes on the adviser's portfolio if the adviser chooses an extreme portfolio; while the outcomes y_h, y_{hl}, and y_l are the possible outcomes if the adviser picks the portfolio $(1-a, a/2, a/2)$. Similarly, x_h, x_{hl}, and x_l are the possible outcomes on the benchmark portfolio. Now define

$$T \;=\; \left(\frac{y_h - y_l}{y_h - x_{hl}}\right)[q(y_h - x_h) + z(y_h - x_{hl})]. \tag{5.9}$$

Recall that the expected fees and utilities under an incentive fee regime are denoted EG and EV, respectively. We will denote by $\mathcal{R}_\mathcal{I}$ and $\mathcal{R}_\mathcal{N}$ the returns on the portfolios of the informed and uninformed adviser, respectively, under incentive fees. As earlier, R_B will denote the returns on the benchmark portfolio. The following result provides the counterpart of Proposition 5.1 for an incentive fee regime:

Proposition 5.2 *If $\bar{\pi}_N < T$, the equilibrium outcomes under an incentive fee regime are*

$$EG_I^* \;=\; \left[\frac{q(y_h - x_h) + p(y_h - x_{hl})}{q(y_h - x_h) + z(y_h - x_{hl})}\right]\bar{\pi}_N. \tag{5.10}$$

$$EV_I^* \;=\; E(\mathcal{R}_\mathcal{I}) - \mathcal{E}\mathcal{F}_\mathcal{I}^*. \tag{5.11}$$

$$EV_N^* \;=\; E(\mathcal{R}_\mathcal{N}) - \bar{\pi}_\mathcal{N}. \tag{5.12}$$

If $\bar{\pi}_N \geq T$, then these outcomes become

$$EG_I^* \;=\; E(\mathcal{R}_\mathcal{I} - \mathcal{R}_\mathcal{N}) + \bar{\pi}_\mathcal{N} \tag{5.13}$$

$$EV_I^* \;=\; E(\mathcal{R}_\mathcal{N}) - \bar{\pi}_\mathcal{N} \tag{5.14}$$

$$EV_N^* \;=\; E(\mathcal{R}_\mathcal{N}) - \bar{\pi}_\mathcal{N}. \tag{5.15}$$

Proof The "reservation utility" EV_N^* is equal to $E(\mathcal{R}_\mathcal{N}) - \bar{\pi}_\mathcal{N}$ for the same reason as in Proposition 5.1. Thus, we only have to show that the remaining values follow from the separation problem (4.16). To this end, writing X^+ for $\max\{X, 0\}$, note that the problem may, once again, be written as

$$\begin{array}{lrcl}
\text{Maximize} & b_1\, E(\mathcal{R}_\mathcal{I}) + \lfloor_\in \mathcal{E}[(\mathcal{R}_\mathcal{I} - \mathcal{R}_\mathcal{B})^+] & & \\
\text{subject to} & b_1\, E(\mathcal{R}_\mathcal{I}) + \lfloor_\in \mathcal{E}[(\mathcal{R}_\mathcal{I} - \mathcal{R}_\mathcal{B})^+] & \leq & \bar{\pi}_N + E(\mathcal{R}_\mathcal{I} - \mathcal{R}_\mathcal{N}) \\
 & b_1\, E(\mathcal{R}_\mathcal{N}) + \lfloor_\in \mathcal{E}[(\mathcal{R}_\mathcal{N} - \mathcal{R}_\mathcal{B})^+] & \leq & \bar{\pi}_N \\
 & b_1, b_2 \;\geq\; 0 & &
\end{array} \tag{5.16}$$

Now, some simple calculation using (4.9)-(4.14) reveals the following values for the inputs into this problem:

$$\begin{array}{lcl} E(\mathcal{R}_{\mathcal{I}}) & = & (p+q)\,y_h + (q+r)\,y_l \\ E(\mathcal{R}_{\mathcal{N}}) & = & (y_h + y_l)/2 \\ E(R_{\mathcal{B}}) & = & (x_h + x_l)/2 \\ E[(\mathcal{R}_{\mathcal{I}} - \mathcal{R}_{\mathcal{B}})^{+}] & = & q(y_h - x_h) + p(y_h - x_{hl}) \\ E[(\mathcal{R}_{\mathcal{N}} - \mathcal{R}_{\mathcal{B}})^{+}] & = & q(y_h - x_h) + z(y_h - x_{hl}) \\ E(\mathcal{R}_{\mathcal{I}} - \mathcal{R}_{\mathcal{N}}) & = & (p-z)(y_h - y_l) \end{array}$$

Thus, the first two constraints in the separation problem may be expanded as

$$b_1[(p+q)y_h + (q+r)y_l] + b_2[q(y_h - x_h) + p(y_h - x_{hl})] \;\leq\; \bar{\pi}_N + (p-z)(y_h - y_l).$$

$$b_1[(y_h + y_l)/2] + b_2[q(y_h - x_h) + z(y_h - x_{hl})] \;\leq\; \bar{\pi}_N.$$

When $b_2 = 0$, the first constraint implies an upper-bound on b_1 of $[(p-z)(y_h - y_l) + \bar{\pi}_N]/[(p+q)y_h + (q+r)y_l]$, while the second implies an upper-bound of $2\bar{\pi}_N/[(y_h + y_l)]$. The first of these bounds is larger than the second one if, and only if, $\bar{\pi}_N \leq (y_h + y_l)/2 = E(\mathcal{R}_{\mathcal{N}})$. This must, evidently, hold in any sensible definition of the problem (the reservation expected fee cannot exceed gross expected returns), so the first constraint is always slack when $b_2 = 0$.

When $b_1 = 0$, the two constraints imply upper-bounds on b_2 of, respectively, $[(p-z)(y_h - y_l) + \bar{\pi}_N]/[q(y_h - x_h) + p(y_h - x_{hl})]$, and $\bar{\pi}_N/[q(y_h - x_h) + z(y_h - x_{hl})]$. The first of these bounds is larger than the second if, and only if,

$$\bar{\pi}_N \;<\; \left(\frac{y_h - y_l}{y_h - x_{hl}}\right)[q(y_h - x_h) + z(y_h - x_{hl})]. \tag{5.17}$$

Thus, when (5.17) is satisfied, the first constraint is always slack. In this case, the solution to the optimization problem is determined from the second constraint holding with equality, and given the linear structure of the problem, must lie at either $b_1 = 0$ or at $b_2 = 0$. A comparison of the values of the objective function at these extremes requires considerable algebraic manipulation. The details, which are in Appendix C, eventually establish that the maximum lies at $b_1 = 0$, which yields the values (5.10)–(5.11) for the expected utilities.

When (5.17) is violated, however, the first constraint holds with equality at the optimum. A simple computation now reveals that the resulting expected utilities are as in (5.13)–(5.14), completing the proof of the proposition. □

The payoffs in Propositions 5.1 and 5.2 easily enable us to prove the following result, the centerpiece of this subsection:

Proposition 5.3 *There is an interval of parameter values under which the investor's expected utility under an incentive regime is strictly higher than under a fulcrum fee regime. However, there are no* *parameter values under which the investor's expected utility under a fulcrum fee regime dominates that under an incentive fee regime.*

Proof We first show that incentive fee outcomes for the investor could dominate fulcrum fee outcomes. To this end, note that:

1. The reservation utility levels are the same in the two cases.
2. Under incentive fees, the investor does strictly better than this reservation level whenever $\bar{\pi}_N < T$.
3. Under fulcrum fees, the investor receives only his reservation level whenever $\bar{\pi}_N \geq E(R_N - R_B)$.

Thus, to show the desired result, it suffices to show that there exist parametrizations of the problem which satisfy $E(R_N - R_B) < \bar{\pi}_N < T$. This is easy. A simple computation shows that $E(R_N - R_B) = (a-1)(x_h + x_l - 2)$. For $a = 1$, this quantity is zero, while, of course, even in this case, T is strictly positive (see expression (5.9)). Thus, one can always find parameter values with $\bar{\pi}_N$ lying in the required interval.

The other part is considerably more involved. We will proceed in two steps. First, we will show that there are no parametrizations such that (i) the investor receives his reservation utility under incentive fees, but (ii) receives strictly more than his reservation utility under fulcrum fees. After this, we will complete the proof by showing that whenever the investor receives strictly more than his reservation utility under both regimes, the equilibrium utility level is always larger under incentive fees.

To see the first part, note that under incentive fees, we have $EV_I^* = EV_N^*$ only if $\bar{\pi}_N > T$, while under a fulcrum fee, we have $EU_I^* > EU_N^*$ only if $\bar{\pi}_N < E(R_N - R_B)$. Thus, to rule out this case, it suffices to show that we must always have

$$T \geq E(R_N - R_B). \tag{5.18}$$

Now $R_N = 1 - a + aR_B$ (see expressions (4.3) and (4.4)), so $E(R_N - R_B) = (a-1)(x_h + x_l - 2)/2$. Thus, (5.18) is equivalent to

$$\left(\frac{y_h - y_l}{y_h - x_{hl}}\right)[q(y_h - x_h) + z(y_h - x_{hl})] \geq \frac{1}{2}(a-1)(x_h + x_l - 2). \tag{5.19}$$

Now $y_h - y_l = a(x_h - x_l)$, and $(x_h - x_l) > (x_h - x_l - 2)$. So it is easily seen that a sufficient condition for (5.19) to hold is

$$2aq(y_h - x_h) \geq (a - 1 - 2az)(y_h - x_{hl}). \tag{5.20}$$

Since $z = (p+r)/2$ and $p + 2q + r = 1$, we must have $q + z = 1/2$. Therefore, $2az = a - 2aq$, so (5.20) holds if and only if

$$2aq(y_h - x_h) \geq (2aq - 1)(y_h - x_{hl}), \tag{5.21}$$

or $2aq(x_h - x_{hl}) \leq (y_h - x_{hl})$. By definition, $y_h = ax_h$, and we must, of course, have $q < 1/2$. So

$$2aq(x_h - x_{hl}) \;<\; a(x_h - x_{hl}) \;=\; y_h - ax_{hl} \;<\; y_h - x_{hl}.$$

Thus, (5.21) holds, and the first step is proved. It remains to be shown that the investor's utility EV_I^* under incentive fees must be larger than that under fulcrum fees EU_I^* when both lie strictly above the reservation utility level. This is the same thing as showing that the following two conditions hold simultaneuously:

$$\bar{\pi}_N \;<\; \min\{E(R_N - R_B), T\}.$$

$$EG_I^* \;>\; EF_I^*.$$

If the first inequality holds, then the second inequality, when expanded, amounts to

$$\left(\frac{q(y_h - x_h) + p(y_h - x_{hl})}{q(y_h - x_h) + z(y_h - x_{hl})}\right) \;>\; \left(\frac{(p+q)y_h + (q+r)y_l - (x_h + x_l)/2}{(a-1)(x_h + x_l - 2)/2}\right).$$

Since $z = (p+r)/2$, and $p+2q+r = 1$, it follows that $q+z = 1/2$. Using this fact, the numerator of the RHS expression can be rewritten as $(a-1)(x_h + x_l - 2)/2 + (p-z)(y_h - y_l)$. Cross-multiplying and rearranging the terms, the required inequality holds if, and only if,

$$(y_h - y_l)[q(y_h - x_h) + z(y_h - x_{hl})] \;<\; \frac{1}{2}(a-1)(y_h - x_{hl})(x_h + x_l - 2).$$

Using $q + z = 1/2$ again, we have $q(y_h - x_h) + z(y_h - x_{hl}) = [(y_h - x_{hl}) - q(x_h - x_l)]/2$. Some rearrangement now shows that the required inequality holds if, and only if,

$$(y_h - x_{hl})[y_h - y_l - (a-1)(x_h + x_l - 2)] \;<\; q(y_h - y_l)(x_h - x_l). \tag{5.22}$$

Now, $(y_h - y_l) = a(x_h - x_l)$ and $(x_h + x_l - 2) < (x_h - x_l)$, so

$$\begin{aligned}(y_h - y_l) - (a-1)(x_h + x_l - 2) \;&>\; a(x_h - x_l) - (a-1)(x_h - x_l) \\ &=\; (x_h - x_l).\end{aligned}$$

Therefore, the LHS of (5.22) is always strictly larger than $(y_h - x_{hl})(x_h - x_l)$. Since $x_{hl} = (x_h + x_l)/2$ and $q < 1/2$, a further computation also establishes that $(y_h - x_{hl}) > q(y_h - y_l)$. This means the inequality in (5.22) can never hold. Thus, it can never be the case that $EF_I^* < EG_I^*$ when $\bar{\pi}_N < \min\{E(R_N - R_B), T\}$, completing the proof of the proposition. □

Table 2: Separating Equilibrium Outcomes in a Fulcrum Fee Regime

This table presents values of several variables in the separating equilibrium outcome under a fulcrum fee regime. The variance-aversion parameter of the investor is fixed at $\gamma = 2$, and the probabilities p, q, and r of Table 1 are fixed at 0.50, 0.15, and 0.20, respectively. In the table:(i) H and L refer to the returns on the risky securities, (ii) $\bar{\pi}_N$ is the reservation expected fee level for the uninformed adviser, (iii) $a^{\max}$ refers to the maximum amount that may be invested in the two risky assets combined, (iv) EF_I^* is the maximized value of the informed adviser's expected fee in problem (4.8), (v) EU_I^* is the value of the investor's expected utility in this solution, and (vi) EU_N^* is the maximized investor utility in the problem (4.7).

$H = 1.20$, $L = 0.90$, $\bar{\pi}_N = 0.01$			
$a^{\max}$	EF_I^*	EU_I^*	EU_N^*
1.00	0.0550	1.0334	1.0334
1.50	0.0370	1.0830	1.0536

$H = 1.45$, $L = 0.95$, $\bar{\pi}_N = 0.25$			
$a^{\max}$	EF_I^*	EU_I^*	EU_N^*
1.00	0.3250	0.9383	0.9383
1.50	0.3493	1.0488	1.0488

$H = 1.50$, $L = 0.90$, $\bar{\pi}_N = 0.01$			
$a^{\max}$	EF_I^*	EU_I^*	EU_N^*
1.00	0.1000	1.1634	1.1634
1.50	0.0235	1.2547	1.2332

$H = 1.40$, $L = 0.99$, $\bar{\pi}_N = 0.25$			
$a^{\max}$	EF_I^*	EU_I^*	EU_N^*
1.00	0.3115	0.9371	0.9371
1.50	0.3333	1.0419	1.0419

5.2 A Risk-Averse Investor

When we move to a variance-averse investor, the risk-sharing aspects of the fee structure also become important. In particular, while asymmetric incentive fees do enable better risk-sharing in general (as argued by Das and Sundaram [3]), they also lead to more extreme portfolios, which are suboptimal choices from the investor's standpoint. The implied trade-off means that incentive fees could now become worse than fulcrum fees for some parametrizations, and this is, in fact, what we find. The remainder of this subsection elaborates on this point, begining with a fulcrum fee regime.

Separating Equilibria under Fulcrum Fees

Even with a variance-averse investor, neither of the optimization problems (4.7) and (4.8) that define a separating equilibrium under fulcrum fees presents any special obstacles. The first one has a particularly simple structure: the objective function is quadratic and strictly concave in (b_1, b_2) and the constraints are linear in (b_1, b_2). Thus, the Kuhn–Tucker first-order conditions are necessary

Table 3: Separating Equilibrium Outcomes in an Incentive Fee Regime

This table presents values of several variables in the separating equilibrium outcome under an incentive fee regime. The variance-aversion parameter of the investor is fixed at $\gamma = 2$, and the probabilities p, q, and r of Table 1 are fixed at 0.50, 0.15, and 0.20, respectively. In the table:(i) H and L refer to the returns on the risky securities, (ii) $\bar{\pi}_N$ is the reservation expected fee level for the uninformed adviser, (iii) $a^{\max}$ refers to the maximum amount that may be invested in the two risky assets combined, (iv) EG_I^* is the maximized value of the informed adviser's expected fee in problem (4.16), (v) EV_I^* is the value of the investor's expected utility in this solution, and (vi) EV_N^* is the maximized investor utility in the problem (4.15).

$H = 1.20$, $L = 0.90$, $\bar{\pi}_N = 0.01$			
$a^{\max}$	EG_I^*	EV_I^*	EV_N^*
1.00	0.0143	1.0630	1.0334
1.50	0.0137	1.0870	1.0501

$H = 1.45$, $L = 0.95$, $\bar{\pi}_N = 0.25$			
$a^{\max}$	EG_I^*	EV_I^*	EV_N^*
1.00	0.3143	0.9383	0.9383
1.50	0.3391	1.0503	1.0225

$H = 1.50$, $L = 0.90$, $\bar{\pi}_N = 0.01$			
$a^{\max}$	EG_I^*	EV_I^*	EV_N^*
1.00	0.0143	1.1996	1.1634
1.50	0.0136	1.2456	1.2302

$H = 1.40$, $L = 0.99$, $\bar{\pi}_N = 0.25$			
$a^{\max}$	EG_I^*	EV_I^*	EV_N^*
1.00	0.3046	0.9371	0.9371
1.50	0.3384	1.0340	1.0240

and sufficient to obtain a maximum. The second problem is a little more involved: although it has a linear objective function, it also has two constraints, one linear and one quadratic, apart from the non-negativity requirements.

Consequently, for specific parameterizations, these problems are easily solved. (General solutions are not easy to obtain in closed-form because of the large number of parameters involved.) Table 2 presents the equilibrium utility levels of the informed adviser and the investor, and the endogeneously determined equilibrium "reservation" utility level of the investor, for a range of parameter values.

Two aspects of these solutions are worth noting. First, it is easy to see that as the reservation fee level $\bar{\pi}_N$ of the uninformed adviser increases, it must be the case that the investor's expected utility *falls* in the separating equilibrium, but the informed adviser's expected fee *rises*. (This is not reflected in the table since we do not hold all the other parameters fixed when varying $\bar{\pi}_N$.) Indeed, the greater the reservation utility of the uninformed adviser, the lower the maximum utility the investor can obtain from the uninformed adviser. This means in the informed adviser's optimization problem, both constraints have weakened: the increase in $\bar{\pi}_N$ relaxes the non-mimicking constraint, while the decrease in EU_N^* relaxes the investor's reservation utility constraint.

Second, note from the table that the presence of leverage does not always have a beneficial effect on the payoffs of the informed adviser; in particular, it is possible that as $a^{\max}$ increases, the equilibrium expected utility of the informed adviser could fall, while that of the investor increases. This apparently unintuitive phenomenon has a simple explanation. Lacking information favoring either security, the uninformed adviser selects the risky securities in the same proportions as the benchmark portfolio. Consequently (see (4.4)), when there is no leverage, his fees involve no performance-adjustment component, but this is not true in the presence of leverage. Thus, leverage increases the set of payoff patterns that the uninformed adviser can generate, and, in turn, strengthens the effect of the non-mimicking constraint in the separation problem (4.8). This could evidently have the effect of making the informed adviser worse off, and the investor better off.

Separating Equilibria under Incentive Fees

Turning to incentive fees, now, Problem (4.15) is a little more complex than its counterpart (4.7) in the fulcrum fee case, since there are two possible distributions for G_N and X_N depending on the choice of (b_1, b_2). Thus, a two-step procedure is required, where we first look for the maximum conditional on $b_2 > 0$, and then for a maximum conditional on $b_2 = 0$. A comparison of the maximized utility levels in the two cases then establishes the "reservation" utility level EV_N^* for the second problem.

The added complication is, however, minor; for specific parametrizations, both problems (4.15) and (4.16) are easy to solve. Table 3 presents the equilibrium utility levels of the informed adviser and the investor, and the equilibrium reservation utility level EV_N^* of the investor, for the same range of parameter values as used in Table 2.

Observe that the two properties we described for equilibrium outcomes under a fulcrum-fee regime continue to hold under an incentive fee regime also. It remains true that an increase in $\bar{\pi}_N$ has a positive effect on the informed adviser and a negative effect on the investor; and that in increase in leverage could make the informed adviser worse off, while improving the investor's welfare.

Comparison of Outcomes

A perusal of Tables 2 and 3 immediately establishes that under risk-aversion either fee regime could dominate from the investor's viewpoint. For example, when $H = 1.20$, $L = 0.90$ and $\bar{\pi}_N = 0.010$, the investor is strictly better off under incentive fees than under fulcrum fees for both values of $a^{\max}$; for these parameter values, the informed adviser strictly prefers the fulcrum fee regime. These preferences are reversed when $H = 1.40$, $L = 0.99$, $\bar{\pi}_N = 0.25$, and $a^{\max} = 1.50$. Now, the investor strictly prefers the fulcrum fee regime, while the informed adviser strictly prefers the incentive fee regime.

Indeed, it is even possible that now *both* the investor and the informed adviser strictly prefer the fulcrum fee regime: this happens, for instance, when $H = 1.50$, $L = 0.90$, $\bar{\pi}_N = 0.01$, and $a^{\max} = 1.50$. Finally, the tables also highlight the importance of leverage in these preferences. For example, consider the parameter set $H = 1.50$, $L = 0.90$, and $\bar{\pi}_N = 0.01$. When $a^{\max} = 1.50$, both the investor and the informed adviser strictly prefer the fulcrum fee regime, but the investor's preferences shift in favor of the incentive fee regime if $a^{\max} = 1.0$.

In summary, there is no basis for judging one regime superior to the other from any of the agents' standpoints in this case. Depending on the parameteric structure, regulation requiring the use of fulcrum fees may help investor welfare, but for other parametric structures, this will occur if the use of incentive fees is required.

6 Conclusion

The fee structure adopted by a mutual fund plays three roles: (i) it influences trading behavior and portfolio choice by affecting investment adviser incentives, (ii) it determines the distribution of returns between investor and adviser, and *ipso facto* serves a risk-sharing function, and (iii) it may be used as a device for signalling ability. Our paper describes an equilibrium model of fee structure determination in which all three factors are present.

The focus of our model is on existing regulations on mutual fund fee structures that require mutual funds to compensate their advisers only through the use of "fulcrum" fees, i.e., fees in which the adviser's fee is symmetric with respect to a chosen index, increasing for outperforming the index in the same way in which it decreases for underperforming it. The regulation is explicitly motivated by the fear that asymmetric or option-like "incentive" fee structures (which are commonly used in the hedge fund industry) will hurt investors by inducing advisers to take "excessive" amounts of risk.

In a break from the traditional approach, the choice of fee structure in our model is made not by the investor, but by the investment adviser, who also selects the risk profile of the fund's portfolio. Investors respond to these decisions by making portfolio decisions. In such a scenario, we find that restrictions requiring the fund to use only fulcrum fees are not readily justifiable. Indeed, in many circumstances, we find that investors can be made better off in welfare terms by requiring that only asymmetric incentive fees be used. This result is particularly striking since, as we have mentioned above, the regulation is explicitly motivated by a fear of asymmetric incentive fee structures.

Two novel features of our model bear highlighting. First, our model is akin to a principal-agent game in which the agent sets the compensation contract, and the principal responds by deciding on the amount of resources to be invested with the agent. Such a model is interesting in itself, and would appear to have application beyond the immediate context of the current paper. Second, our model appears to be the first in the Finance literature in which the fee or compensation structure is used as a signalling device. Since fee contracts are easily observed and understood, they form a credible and practical precommittment device, providing a strong implementation basis for the paper.

A Proofs

A.1 Proof of Proposition 4.2

Recall that a denotes $a^{\max}$. Consider the informed adviser first, and assume, without loss, that the adviser has learnt that state 1 will occur, i.e., that the distribution Π_1 represents the true joint distribution. (The proof is analogous if state 2 is the true state.) Suppose the adviser were to pick the portfolio $(\alpha_0, \alpha_1, \alpha_2)$, where $\alpha_0 + \alpha_1 + \alpha_2 = 1$. Then, the distribution of returns $\tilde{r}_p$ on the adviser's portfolio is as follows:

$$\tilde{r}_p = \begin{cases} \alpha_0 + (\alpha_1 + \alpha_2)H, & \text{with probability } q \\ \alpha_0 + \alpha_1 H + \alpha_2 L, & \text{with probability } p \\ \alpha_0 + \alpha_1 L + \alpha_2 H, & \text{with probability } r \\ \alpha_0 + (\alpha_1 + \alpha_2)L, & \text{with probability } q \end{cases} \tag{1.1}$$

Thus, the distribution of $(\tilde{r}_p - \tilde{r}_b)$, the difference in performance between the adviser's portfolio and the benchmark, is given by

$$\tilde{r}_p - \tilde{r}_b = \begin{cases} \alpha_0 + (\alpha_1 + \alpha_2 - 1)H, & \text{with probability } q \\ \alpha_0 + (\alpha_1 - 1/2)H + (\alpha_2 - 1/2)L, & \text{with probability } p \\ \alpha_0 + (\alpha_1 - 1/2)L + (\alpha_2 - 1/2)H, & \text{with probability } r \\ \alpha_0 + (\alpha_1 + \alpha_2 - 1)L, & \text{with probability } q \end{cases} \tag{1.2}$$

¿From (1.1) and (1.2), the expected fee $EF_I(\alpha, b_1, b_2)$ for the adviser, given the portfolio $\alpha = (\alpha_0, \alpha_1, \alpha_2)$, is

$$EF_I(\alpha, b_1, b_2) = b_1 M_1 + b_2 M_2, \tag{1.3}$$

where M_1 and M_2 are given by

$$M_1 = \alpha_0 + \{\alpha_1(p+q) + \alpha_2(q+r)\}H + \{\alpha_1(q+r) + \alpha_2(p+q)\}L$$

$$\begin{aligned} M_2 = {} & \alpha_0 + \{\alpha_1(p+q) + \alpha_2(q+r) - q - (p+r)/2\}H \\ & + \{\alpha_1(q+r) + \alpha_2(p+q) - q - (p+r)/2\}L \end{aligned}$$

It follows easily by checking the partials that this expected fee is maximized at $\alpha_1 = a$ and $\alpha_2 = 0$, that is, by putting the maximum possible into security 1. This proves the first part of Proposition 4.2. To see the other part of the proposition, suppose the uninformed adviser were to pick the portfolio $(\alpha_0, \alpha_1, \alpha_2)$. For notational simplicity, let z denote $(p + r)/2$, where p and r are the probabilities from Table 1. Then, since the two states of the world are equiprobable, the returns (H, L) and (L, H) on the two risky securities each occurs with prior probability z. Thus, the distribution of portfolio returns $\tilde{r}_p$ is

$$\tilde{r}_p = \begin{cases} \alpha_0 + (\alpha_1 + \alpha_2)H, & \text{with probability } q \\ \alpha_0 + \alpha_1 H + \alpha_2 L, & \text{with probability } z \\ \alpha_0 + \alpha_1 L + \alpha_2 H, & \text{with probability } z \\ \alpha_0 + (\alpha_1 + \alpha_2)L, & \text{with probability } q \end{cases} \tag{1.4}$$

while the ex-ante distribution of the difference in returns between the adviser's portfolio and the benchmark is

$$\tilde{r}_p - \tilde{r}_b = \begin{cases} \alpha_0 + (\alpha_1 + \alpha_2 - 1)H, & \text{with probability } q \\ \alpha_0 + (\alpha_1 - 1/2)H + (\alpha_2 - 1/2)L, & \text{with probability } z \\ \alpha_0 + (\alpha_1 - 1/2)L + (\alpha_2 - 1/2)H, & \text{with probability } z \\ \alpha_0 + (\alpha_1 + \alpha_2 - 1)L, & \text{with probability } q \end{cases} \tag{1.5}$$

¿From (1.4) and (1.5), the expected fee $EF_U(\alpha, b_1, b_2)$ is seen to be

$$EF_U(\alpha, b_1, b_2) = b_1 N_1 + b_2 N_2, \tag{1.6}$$

where

$$N_1 = \alpha_0 + (\alpha_1 + \alpha_2)(q + z)(H + L)$$

$$N_2 = \alpha_0 + (\alpha_1 + \alpha_2 - 1)(q + z)(H + L)$$

Since $(q+z)(H+L) > 1$, it follows immediately now that any vector of the form $(1-a, m, a-m)$ for $m \in [0, a]$ constitutes an optimal portfolio for the adviser. □

A.2 Proof of Proposition 4.3

Consider the informed adviser first, and assume without loss that state 1 will occur. Given any portfolio $(\alpha_0, \alpha_1, \alpha_2)$ such that $\alpha_0 + \alpha_1 + \alpha_2 = 1$, the distribution of outcomes $\tilde{r}_p$ on the adviser's portfolio is

$$\tilde{r}_p = \begin{cases} \alpha_0 + (\alpha_1 + \alpha_2)H, & \text{w.p. } q \\ \alpha_0 + \alpha_1 H + \alpha_2 L, & \text{w.p. } p \\ \alpha_0 + \alpha_1 L + \alpha_2 H, & \text{w.p. } r \\ \alpha_0 + (\alpha_1 + \alpha_2)L, & \text{w.p. } q \end{cases} \tag{1.7}$$

while the distribution of the difference $(\tilde{r}_p - \tilde{r}_b)$ is given by

$$\tilde{r}_p - \tilde{r}_b = \begin{cases} \alpha_0(1 - H), & \text{w.p. } q \\ \alpha_0 + (\alpha_1 - 1/2)H + (\alpha_2 - 1/2)L, & \text{w.p. } p \\ \alpha_0 + (\alpha_1 - 1/2)L + (\alpha_2 - 1/2)H, & \text{w.p. } r \\ \alpha_0(1 - L), & \text{w.p. } q \end{cases} \tag{1.8}$$

Under the incentive fee (b_1, b_2), the informed adviser's expected fee $E[G_I(b_1, b_2)]$ is

$$E[G_I(b_1, b_2)] \;=\; b_1\, E[\tilde{r}_p] + b_2\, E[\max\{0, \tilde{r}_p - \tilde{r}_b\}].$$

A simple calculation shows that for any $b_1 > 0$, the first term $b_1 \cdot E[\tilde{r}_p]$ is maximized by choosing the portfolio $(1 - a, a, 0)$, that is, by putting the maximum possible into the first risky security. Since b_2 is non-negative, it suffices to show that the same is true of the term $E[\max\{0, \tilde{r}_p - \tilde{r}_b\}]$ also. To this end, we first identify the terms on the right-hand side of (1.8) that are non-negative. Obviously, this will depend on the relative magnitudes of the quantities α_0, α_1 and α_2.

Consider first the case where $\alpha_0 \leq 0$ and $\alpha_1 \geq \alpha_2$ (so $\alpha_1 \geq (1 - \alpha_0)/2$). In this case, the first term $\alpha_0((1 - H)$ is non-negative, while the last term $\alpha_0(1 - L)$ is always non-positive. The second term is also always non-negative since, using $\alpha_2 = 1 - \alpha_0 - \alpha_1$ and $H + L > 2$, we have

$$\begin{aligned}
\alpha_0 + (\alpha_1 - 1/2)H + (\alpha_2 - 1/2)L &= \alpha_0(1 - L) + (\alpha_1 - 1/2)(H - L) \\
&\geq \alpha_0(1 - L) - \alpha_0(H - L)/2 \\
&= \alpha_0(1 - (H + L)/2) \\
&\geq 0
\end{aligned}$$

This leaves the third term in (1.8). A straightforward calculation shows that there is $\alpha^* \in ((1 - \alpha_0)/2, 1 - \alpha_0)$ such that the term is positive for $\alpha_1 < \alpha^*$ and negative for $\alpha_1 > \alpha^*$. Summing up, therefore, if $\alpha_1 \in ((1 - \alpha_0)/2, \alpha^*]$, then

$$\begin{aligned}
E[r_p - r_b] \;=\; & \alpha_0(q(1 - H) + p + r) + \alpha_1(pH + rL) \\
& + \alpha_2(pL + rH) - (pH + pL + rL + rH)/2
\end{aligned}$$

while, if $\alpha_1 \in (\alpha^*, 1 - \alpha_0]$, then

$$E[r_p - r_b] \;=\; \alpha_0(q(1 - H) + p) + \alpha_1 pH + \alpha_2 pL - p(H + L)/2.$$

A comparison of these terms establishes after a little computation that $E[r_p - r_b]$ is maximized when $\alpha_0 = 1 - a$, $\alpha_1 = a$, and $\alpha_2 = 0$. A second, and easier set of computations, shows that this dominates the best possible outcome if $\alpha_0 > 0$, that is, when there is a positive amount invested in the riskless asset. (This is intuitive. Since the expected return on the first risky asset exceeds that on the riskless asset in state 1, the risk-neutral adviser would never want to invest a positive fraction of his portfolio in the risk-free asset.) This establishes the first part of Proposition 4.3.

To see the second part, note that the ex-ante outcomes for the uninformed adviser are exactly those in (1.7) and (1.8), but with the probabiltiies of the four outcomes being q, z, z, and q, respectively, where $z = (p + r)/2$. Suppose the uninforemd adviser chooses a portfolio $(\alpha_0, \alpha_1, \alpha_2)$. Running through the same set of computations as for the informed adviser (but using the new set of probabilities) easily establishes that the optimal portfolio for the uninformed adviser is to invest as much as possible into one of the two risky stocks. This completes the proof. $\square$

B Pooling Equilibrium

This section describes pooling equilibria under either fee regime. Section B.1 discusses the fulcrum fee case, while Section B.2 looks at incentive fees. The notation used in the two cases is the same as that introduced in Sections 4.1 and 4.3, respectively. We show that pooling equilibria never exist under incentive fees, and, in general, do not exist under fulcrum fees either.

B.1 Pooling Equilibrium under Fulcrum Fees

If a profile of fee structures is pooling, then the investor presumes that each adviser is informed with probability 1/2 and uninformed with probability 1/2. Thus, the investor's expected utility from investing with an adviser who has announced the structure (b_1, b_2) is

$$W(b_1, b_2) = \frac{1}{2} EU_I(b_1, b_2) + \frac{1}{2} EU_N(b_1, b_2), \tag{2.1}$$

The investor compares his expected utility under each announced fee structure using (2.1), and chooses the adviser who offers the higher expected utility. If the expected utility from the two advisers is the same, the investor chooses each adviser with probability 1/2.

For a profile of fee structures to constitute a pooling *equilibrium*, it is necessary that neither adviser can profit from a unilateral change of strategy. For this to be the case, it is necessary that the investor be indifferent between the two chosen fee structures. Otherwise, one of the advisers would be better off using another fee structure (or even withdrawing from the market). As a consequence, it must be the case that each adviser receives the money with probability 1/2 in a pooling equilibrium. To meet the reservation utility constraint, therefore, conditional on receiving the dollar, each of the candidate fee structures must guarantee each adviser an expected fee of at least twice his reservation level.

These conditions severely restrict the candidate fee structures that could constitute pooling equilibria. In particular, neither adviser can set $b_1 > 0$; else the other adviser could mimic the fee structure, but reduce b_1 slightly, thereby making it strictly more attractive to the investor and receiving the dollar with probability one. Analogously, the value of b_2 that is chosen must also be "unimprovable," and so must solve

$$\max_{b_2 \geq 0} W(0, b_2) \tag{2.2}$$

where $W(\cdot)$ is defined in (2.1). It is easy to show that $W(0, b_2)$ is a strictly concave function of b_2 and so has a unique maximum b_2^* on $b_2 \geq 0$. Note that this maximum need not always occur at

$b_2 = 0$. The fulcrum transfers weight from the tails to the center of the reward distribution, and for small values of b_2, this could benefit the variance-averse investor. Of course, if the investor is risk-neutral ($\gamma = 0$), then $b_2 = 0$ is the only solution.

Thus, the only candidate pooling equilibrium under fulcrum fees is where both advisers offer the fee structure $(0, b_2^*)$. For this candidate structure to actually constitute a pooling equilibrium, two additional conditions must be met: (i) under $(0, b_2^*)$, each adviser receives an expected fee of at least twice his reservation utility, conditional on receiving the investment, and (ii) there is no separating equilibrium fee profile which the informed adviser finds preferable. Condition (i) rules out the existence of pooling equilibrium if the investor is risk-averse except in the unintersting case where both advisers have reservation utilities of zero. Indeed, unsurprisingly, the combined conditions appear very difficult to satisfy in general. We tried a vast range of parametrizations, but were not able to unearth a single case where they were satisfied simultaneously.

B.2 Pooling Equilibrium under Incentive Fees

Unlike with fulcrum fees, it is easy to show that no pooling equilibria can exist under incentive fees. Arguing along analogous lines as above, it is seen that any candidate pooling equilibrium must satisfy $b_1 = 0$ and have b_2 equal to the solution to (2.2). However, under incentive fees, the solution to (2.2) is always $b_2 = 0$ (there is no analogous transfer of weight from the tails to the center in this case). Thus, pooling equilibria cannot exist except in the trivial case where $\bar{\pi}_I = \bar{\pi}_N = 0$. In this latter case, separating equilibria do not exist (any choice of non-negative (b_1, b_2) by the informed adviser can be costlessly and profitably mimicked by the uninformed adviser); so the only equilibrium under incentive fees is, in fact, a pooling equilibrium where $b_1 = b_2 = 0$.

C Proof of Proposition 5.2: Details

In the proof of Proposition 5.2, it remains to be shown that when the second constraint in the separation problem is the only binding one, then the optimum occurs at $b_1 = 0$. To see this, note that when the second constraint alone is binding and $b_2 = 0$, then the informed adviser's expected fee is given by

$$G_1 = \left(\frac{(p+q)y_h + (q+r)y_l}{(y_h + y_l)/2} \right) \bar{\pi}_N.$$

while if $b_1 = 0$, this expected fee is

$$G_2 = \left(\frac{q(y_h - x_h) + p(y_h - x_{hl})}{q(y_h - x_h) + z(y_h - x_{hl})} \right) \bar{\pi}_N.$$

Thus, to complete the proof of the proposition it suffices to show that we always have $G_2 \geq G_1$. To this end, note that since $p + 2q + r = 1$, we have $q + z = q + (p + r)/2 = 1/2$. Therefore, some manipulation yields

$$(p+q)y_h + (q+r)y_l \;=\; \frac{1}{2}(y_h + y_l) + (p-z)(y_h - y_l).$$

$$q(y_h - x_h) + z(y_h - x_{hl}) \;=\; \frac{1}{2}[y_h - x_{hl} - q(x_h - x_l)].$$

Thus, $G_1 \leq G_2$ if, and only if,

$$[(y_h + y_l)/2 + (p-z)(y_h - y_l)][q(y_h - x_h) + z(y_h - x_{hl})] \;\leq\; \frac{1}{2}(y_h + y_l)[q(y_h - x_h) + p(y_h - x_{hl})].$$

Multiplying through, cancelling common terms, and rearranging, it can be seen that this inequality holds if, and only if,

$$\begin{aligned} q(y_h - x_h)(y_h - y_l) \;&\leq\; (y_h - x_{hl})[(y_h + y_l)/2 - z(y_h - y_l)] \\ &=\; (y_h - x_{hl})[qy_h + (1-q)y_l]. \end{aligned}$$

Once again, opening out the parentheses, cancelling common terms, and rearranging, this inequality is the same as requiring

$$qy_h(x_{hl} - x_h) \;\leq\; y_l[y_h - qx_h - (1-q)x_{hl}].$$

The LHS is negative always, while the RHS is positive always. Thus, the required inequality holds, completing the proof of the proposition. □

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