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SECURITY IN A LIFE-CYCLE ECONOMY

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Government Debt and Social Security in
a Life-Cycle Economy
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ABSTRACT

This paper develops a tractable overlapping generations model that is useful for analyzing

both the short and long run impact of fiscal policy and social security. It modifies the Blanchard

(1985)/Weil (1987) framework to allow for life/cycle behavior. This is accomplished by introducing

random transition from work to retirement, and then from retirement to death. The transition

probabilities may be picked to allow for realistic average lengths of life, work and retirement. The

resulting framework is not appreciably more difficult to analyze than the standard Cass/Koopmans

one sector growth model: Besides the capital stock, there is only one additional state variable: the

distribution of wealth between workers and retirees. Under reasonable parameter values,

government debt and social security have significant effects on capital intensity.

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1. Introduction

This paper develops a new kind of overlapping generations growth model and then uses the framework to analyze the economic impact of government debt and social security. Individuals within the framework exhibit life cycle behavior. Further, they can have realistic average lengths of life, work and retirement. The framework is useful for analyzing both the short run and long run effects of policy. At the same time, however, it is very tractable. It is not appreciably more complex to analyze than either the conventional Diamond (1965) two period overlapping generations growth model or the widely-used Cass/Koopmans (1965) representative agent paradigm.

The obstacle to overcome in working with overlapping generations models is the heterogeneity implied by the age structure of the population. Individuals of different ages vary both in the level of wealth and in the composition of wealth between human and non-human sources. Because they have different horizons, they also have different marginal propensities to consume. In general, therefore, it is not possible to derive simple aggregate consumption and savings functions [Modigliani (1966)]. The standard two period overlapping generations model avoids the aggregation problem by imposing extreme restrictions on demographic structure.

Blanchard (1985) makes substantial progress toward developing a tractable overlapping generations framework with a reasonable demographic structure by assuming that individuals face a constant probability of death each period. This restriction, also employed by Yaari (1965), makes individual horizons finite in a way that permits simple aggregation of consumption behavior. In a similar spirit, Weil (1987) proposes a manageable overlapping generations setup where individuals live forever, but a new cohort of infinitely-lived people is born each period. With either framework it is possible to study the impact of policies that redistribute wealth between generations. In both setups, the demographic structure makes government bonds net wealth for the current population, as in the classic Diamond (1965) framework.

Neither the Blanchard framework nor the Weil framework, however, captures life-cycle behavior. Within both frameworks individuals currently alive are identical except for their respective levels of non-human wealth. They all have identical marginal propensities to consume. There is no "saving for retirement." It is therefore not possible to use these frameworks to study the impact of policies that redistribute between workers and retirees, such as social security and medicare. Nor is possible to study the impact of demographic changes, such as the aging of the population. Finally, omitting life-cycle considerations may lead to understating the impact of government debt and deficits. For example, Romer (1989) presents some numerical simulations that suggest that government debt has only minor effects on real activity in the Blanchard/Weil framework[1]. Adding life-cycle factors is likely to enhance the impact, for two reasons. First, having a retirement period raises the fraction of government bonds that are net wealth to those currently alive since it shortens the horizon over which the current work force is liable for future taxes. Second, having retirees as well as workers implies that a rise in government debt will redistribute wealth from a low propensity to consume group (workers) to a high propensity to consume one (retirees).[2]

To introduce life-cycle factors but maintain tractability, I make two kinds of modifications of the Blanchard/Weil framework.[3] First, I introduce two stages of life: work and retirement. I then impose a constant transition probability per period for a worker into retirement, as well as a constant probability per period of death for a retiree. Second, I employ a class of non-expected utility preferences proposed by Farmer (1990) that generate certainty-equivalent decision rules in the presence of income risk. With these two modifications it is possible to derive aggregate consumption/savings relations for workers and for retirees. It is also possible to express the current equilibrium values of all the endogenous variables as functions of just two predetermined variables: the capital stock and the distri-

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[1] Romer (1989) argues that the welfare effects of a rise in government debt may be large, even if the impact on aggregate activity is small.

[2] Aiyagari and Gertler (1985) illustrate how government deficits may redistribute wealth between workers and retirees in a two period overlapping generations model.

[3] For an early attempt to embed life-cycle behavior in a growth model, see Tobin (1967.)

bution of non-human wealth between retirees and workers. In effect, the model captures life-cycle behavior by having only one more predetermined variable than in conventional one sector growth frameworks [see e.g., Barro and Salai-I-Martin (1995)].

The framework is not a substitute for large scale numerical overlapping generations models that are employed for policy analysis [e.g., Auerbach and Kotlikoff (1987), Hubbard and Judd (1987)]. On the other hand, because it permits realistic average periods of work and retirement, the model is useful for quantitative policy analysis in way that complements the use of large scale models. The advantage of this framework is its parsimonious representation, which makes it possible to clearly understand the factors that underlie particular results. Further, because of its parsimony, it is straightforward to integrate this life cycle setup into existing growth and business cycle models in order to study a much broader set of issues than are discussed here.

One issue from which the paper abstracts is the possibility that inter-generational caring, as formalized by Barro (1974), could effectively transform the life-cycle individuals of the model economy into infinitely-lived households. When individuals have infinite horizons (and there are no other frictions), the Ricardian Equivalence Theorem applies, implying that both government debt and social security are neutral. The recent behavior of the U.S. economy, however, suggests that it is still worth studying the life-cycle approach. Figure 1 shows the sharp rise in both the ratio of government debt to GDP and the ratio of social security and medicare to GDP that occurred in the last fifteen years or so. Accompanying this expansive fiscal policy was a sharp decline in the net private saving rate and a secular rise in the ex post real interest rate (measured as the difference between the one year government bond rate and the ex post inflation rate).[4] While it may be possible to reconcile these phenomena with a representative agent paradigm, a life-cycle setup seems the natural place to start.

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[1] See Gokhale, Kotlikoff and Sablehaus (1996) for an analysis of the declining saving rate. These authors emphasize the role of transfers to the elderly, particularly medicare.

3

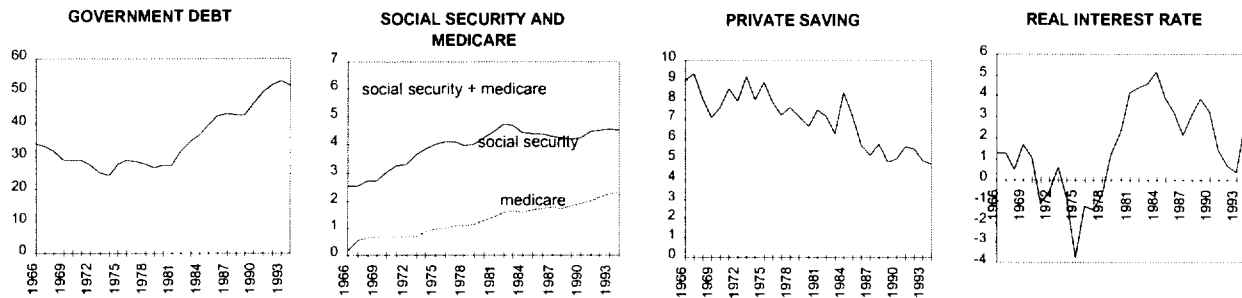| GOVERNMENT DEBT | SOCIAL SECURITY AND MEDICARE | PRIVATE SAVING | REAL INTEREST RATE |

Figure 1.1: Debt Social Security Saving and Interest

Section 2 introduces the key assumptions and then derives an aggregate consumption function for an economy with no government policy. Section 3 embeds the consumption function in a one sector growth model, and then illustrates how life cycle factors affect the equilibrium, both qualitatively and quantitatively. Section 4 adds government policy. It then explores the impact of shifts in government debt, social security and government consumption, again both qualitatively and quantitatively. Section 5 describes the effects of policy for the open economy. Finally, concluding remarks are in section 6.

2. The Aggregate Consumption Function

In this model, individuals have finite lives and they evolve through two distinct stages of life: work and retirement. To derive a tractable aggregate consumption function and at the same time permit realistic (average) lengths of work and retirement, I make three kinds of assumptions. These assumptions involve: (1) population dynamics; (2) insurance arrangements; and (3) preferences.

Population dynamics are as follows: Each individual is born a worker. Conditional on being a worker in the current period, the probability of remaining one in the next period is ω. Conversely, the probability of retiring is $1 - \omega$. To facilitate aggregation, I assume that the transition probability ω is independent of the worker's employment tenure. The average time in the labor force for an individual is thus $\frac{1}{1-\omega}$.

4

Once an individual retires the death clock begins to tick. Conditional on being retired in the current period, the probability of surviving to the next is γ and, conversely, the probability of death next period is $1 - \gamma$. The survival probability γ is independent of age, again to facilitate aggregation. The average retirement period is thus $\frac{1}{1-\gamma}$.

I assume that the population grows at the net rate n. In particular, each period $(1 - \frac{\omega}{1+n})N_t$ new workers are born, where

$$N_t = (1 + n)N_{t-1} \tag{2.1}$$

Given that the number of people in each cohort is large, the total work force at time t is N_t, and the number of retirees is $(\frac{1-\omega}{1+n-\gamma})N_t{}^5$. The ratio of retirees to workers, ψ, is thus

$$\psi \equiv \frac{1 - \omega}{1 + n - \gamma} \tag{2.2}$$

Since this ratio is fixed, both the work force and the number of retirees grow at the aggregate population growth rate n.

For simplicity there is no aggregate risk. The only risks individuals face throughout their lifetimes are idiosyncratic: A worker faces a potential loss of wage income. A retiree faces an uncertain time of death. These risks, however, may complicate both the derivation and aggregation of individual decision rules. It is for this reason that I make assumptions about availability of insurance markets and about preferences:

To eliminate the impact of uncertainty about time of death, I introduce a perfect annuities market, following Yaari (1965) and Blanchard (1985). The annuities market provides perfect insurance against this kind of risk. Under the arrangement each retiree effectively turns over his wealth to a mutual fund that invests the proceeds. The fraction γ of those that survive to the next period receive all the returns, while the (estates of) the fraction $1 - \gamma$ who die receive nothing. Each surviving retiree receives a return that is proportionate

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[5]The workforce at t equals $\sum_{s=0}^{\infty} \omega^s(1 - \frac{\omega}{1+n})N_{t-s} = (1 - \frac{\omega}{1+n})N_t \sum_{s=0}^{\infty}(\frac{\omega}{1+n})^s = N_t$. The retiree population equals $\sum_{s=0}^{\infty} \gamma^s(1 - \omega)N_{t-s} = \frac{1-\omega}{1+n-\gamma}N_t$.

to his initial contribution of wealth to the mutual fund. Thus, for example, if R is the gross return per dollar invested by the mutual fund, the gross return on wealth for a surviving retiree is R/γ.

To address the risk of uncertainty of retirement, I restrict preferences. In principle, I could also introduce an insurance market that mitigates the idiosyncratic retirement risk. I refrain from doing so, however, because my objective is to have a framework that captures life-cycle behavior. A perfect insurance market against loss of labor income would smooth income perfectly across work and retirement. In the absence of such a market, all wage income accrues to workers. This latter scenario is clearly a better approximation of the life cycle. Thus, I do not permit an insurance market for wage income.

I address the problem of retirement risk by assuming that individuals have preferences that separate risk aversion from intertemporal substitution. In particular, I employ a special class of CES non-expected utility functions proposed by Farmer (1990) that restrict individuals to be risk neutral with respect to income risk, but allow for an arbitrary intertemporal elasticity of substitution.[6] Since the degree of income risk here is artificial in the sense that it is generated by the assumption of constant probability of transition into retirement, it seems reasonable to mitigate the impact of this income variation by assuming risk neutrality.

Let V_t^z be an individual worker's value function, where the superscript z = w,r indicates whether the individual is a worker (w) or a retiree; let C_t be consumption; and let β be a subjective discount factor. Then, preferences are given by:

$$V_t^z = [(C_t)^\rho + \beta^z E_t \{V_{t+1} \mid z\}^\rho]^{1/\rho} \tag{2.3}$$

with

$$\beta^w = \beta;$$

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[6]For a generalization of the preferences described by equation (2.3) to a broad family of CES non-expected utility functions, see Weil (1990).

$$\beta^r = \beta \cdot \gamma$$

and where $E_t\{V_{t+1} \mid z\}$ is the expectation of the value function next period, conditional on the person being type z at time t and being alive at t+1; i.e.

$$E_t\{V_{t+1} \mid w\} = \omega V_{t+1}^w + (1 - \omega)V_{t+1}^r$$
$$E_t\{V_{t+1} \mid r\} = V_{t+1}^r$$

The retiree's effective discount factor is $\beta\gamma$ and not β since his probability of surviving until the next period is γ.

These preferences generate certainty-equivalent decisions rules in the face of idiosyncratic income risk, in contrast to standard Von-Neumann/Morgernstern utility functions. Roughly speaking, because preferences are over the mean of next period's value function, individuals only care about the first moment of expected income in deriving their decision rules.[7] On the other hand, they do care about smoothing consumption over time. The curvature parameter ρ introduces a smooth trade-off for individuals between consuming today versus consuming tomorrow. In analogy to the standard case, the desire to smooth consumption implies a finite intertemporal elasticity of substitution σ, given by $\sigma = \frac{1}{1-\rho}$.

2.1. Consumption by Retirees

Retirees consume only out of asset income. They have no labor income, though later I allow for social security. In general, one can index each retiree by the time he was born j and the time he left the labor force k. Ultimately, it will not be necessary to keep track of how assets and consumption are distributed among retirees over j and k. Under my assumptions one can simply aggregate across different cohorts.

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[7]Since retirees do not face any income risk, they behave as if they had standard Von-Neumann/Morgenstern preferences.

Let A_t^{rjk} and C_t^{rjk} be (non-human) assets at the beginning of time t and consumption at t, respectively, of a retired person who was born at time j and left the labor force at time k; and let R_t be the gross return on assets from period t-1 to t. For a retiree at t who participates in a perfect annuities market and survives until at least t+1, assets evolve according to:

$$A_{t+1}^{rjk} = (R_t/\gamma)A_t^{rjk} - C_t^{rjk} \tag{2.4}$$

The retiree chooses consumption and asset accumulation to maximize (2.3) subject to (2.4) and a terminal condition that requires him to pay off all debts. The consumption Euler equation for the retiree yields (see the appendix):

$$C_{t+1}^{rjk} = (R_{t+1}\beta)^\sigma C_t^{rjk} \tag{2.5}$$

Consumption at t+1 for a surviving retiree is certain since there is no aggregate risk.

Let $\epsilon_t\pi_t$ be the retiree's marginal propensity to consume out of wealth (mpcw), where π_t is the mpcw for a worker. (I choose this notation since the ratio of the two mpcw's, ϵ_t, has an important role in the model.) The retiree's decision rule for consumption is given by

$$C_t^{rjk} = \epsilon_t\pi_t(R_t/\gamma)A_t^{rjk} \tag{2.6}$$

where $\epsilon_t\pi_t$ obeys the following nonlinear first order difference equation:

$$\begin{aligned}
\epsilon_t\pi_t &= 1 - (R_{t+1}^{\sigma-1}\beta^\sigma\gamma)\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}} \tag{2.7} \\
&= 1 - \beta\gamma, \text{ if } \sigma = 1
\end{aligned}$$

Because the retiree does not face any income uncertainty (thanks to the annuity market), the solution for $\epsilon_t\pi_t$ is similar to the case with standard CES preferences. The retiree's

mpcw varies only with the interest rate, and is constant when $\sigma = 1$ (which corresponds to logarithmic preferences). Further, as in Yaari (1967) and Blanchard (1985), the likelihood of death raises the marginal propensity to consume. In this respect, the finite expected horizon influences the retiree's consumption decision.

An exact solution to the nonlinear difference equation for $\epsilon_t \pi_t$ is not feasible. However, using the methods of Campbell and Shiller (1988), one can find an approximate solution via loglinearization. Eventually I will follow this course.

2.2. Consumption by Workers

Each worker supplies one unit of labor inelastically. Workers consume and save out of assets and wage income, W_t. Let the superscript wj denote a worker born in period j. As with retirees, my assumptions will permit aggregation over different cohorts.

For a cohort j worker, assets evolve according to

$$A_{t+1}^{wj} = R_t A_t^{wj} + W_t - C_t^{wj} \tag{2.8}$$

The worker chooses consumption and asset accumulation to maximize (2.3), subject to (2.8) and to the constraints that become operative once he retires, described in the previous section. The first order necessary condition for the worker's consumption/saving choice yields (see the appendix)

$$\omega C_{t+1}^{wj} + (1 - \omega)\Lambda_{t+1} C_{t+1}^{rj(t+1)} = (R_{t+1}\Omega_{t+1}\beta)^\sigma C_t^{wj} \tag{2.9}$$

where $\Lambda_{t+1} \equiv \frac{\partial V_{t+1}^r / \partial C_{t+1}^r}{\partial V_{t+1}^w / \partial C_{t+1}^w}$ is the marginal rate of substitution of consumption across work and retirement, and where Ω_{t+1} is a factor that weights the gross return R_{t+1} and is given by

$$\Omega_{t+1} = \omega + (1 - \omega)\epsilon_{t+1}^{\frac{1}{1-\sigma}} \tag{2.10}$$

In calculating the net marginal gain utility from saving, $R_{t+1}\Omega_{t+1}\beta[\omega C_{t+1}^{wj} + (1-\omega)\Lambda_{t+1} C_{t+1}^{rj(t+1)}]^{\rho-1}$,

the worker takes into account the likelihood he may retire in the next period. Because he is risk neutral, only the mean of the distribution of next period's (utility-weighted) consumption influences this calculation.

As with a retiree, the worker's consumption depends upon his wealth. Wealth for a worker, however, includes his discounted stream of labor income, i.e., his human wealth H_t^j, in addition to current non-human assets $R_t A_t^{wj}$. Accordingly, the worker's decision rule for consumption is

$$C_t^{wj} = \pi_t(R_t A^{wj} + H_t^j) \tag{2.11}$$

where the mpcw, π_t, is given by

$$
\begin{aligned}
\pi_t &= 1 - (R_{t+1}\Omega_{t+1})^{\sigma-1}\beta^\sigma \frac{\pi_t}{\pi_{t+1}} \\
&= 1 - \beta, \text{ if } \sigma = 1
\end{aligned}
\tag{2.12}
$$

Further,

$$H_t^j = \sum_{v=0}^{\infty} \frac{W_{t+v}}{\prod_{z=1}^{v} R_{t+z}\Omega_{t+z}/\omega} \tag{2.13}$$

Life cycle factors are present in two main ways. First, the finite expected horizon induces the worker to consume out of wealth at a faster pace relative to the infinite horizon case. The presence of Ω_{t+1} in the expression for π_t captures this phenomenon. Note that Ω_{t+1} varies positively with ϵ_{t+1}, the ratio of a retiree's mpcw to a worker's mpcw (see equation (2.7)). Further, since $\epsilon_{t+1} > 1$, it follows that $\Omega_{t+1} > 1$.[8] In turn, $\Omega_{t+1} > 1$, implies that π_t is larger than would be the case if the worker lived forever. Intuitively,

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[8]It is straightforward to show analytically that, in the steady state, $\epsilon_t > 1$ (see footnote 11 in the next sub-section). Numerical simulations show that this inequality also holds outside the steady state for reasonable parameter values.

10

the likelihood that the worker will retire and thus begin consuming out of wealth at a higher rate induces him to consume at a higher rate today. Because he wants to smooth consumption over time, he draws down his wealth at a faster rate than would be the case if he were infinitely-lived.

Second, relative to the case of infinite life, the worker applies a higher discount rate to future labor income. In the infinite horizon, the relevant discount rate is the return on assets, R_{t+1}. In the life cycle setting here it is $R_{t+1}\Omega_{t+1}/\omega$, as equation (2.13) indicates. Since $\Omega_{t+1} > 1$ and $\omega < 1$, it follows that $R_{t+1}\Omega_{t+1}/\omega > R_{t+1}$.[9] This enlarged discount rate is due to two distinct factors. First, the likelihood that the worker may retire induces him to place a lower weight on future income: hence the presence in the denominator of the effective discount factor of ω, the probability of remaining in the labor force in the next period. Second, the presence of Ω_{t+1} in the numerator reflects the fact that, within the life-cycle framework, consumption out of a dollar of wealth early in life can be smoothed over more periods than out of dollar late in life. Everything else equal, the marginal utility gain from a unit of wealth for a retired person is less than for a worker, since the former consumes out of wealth at a faster pace than does the latter.

As with the retiree, an exact solution for the consumption decision is not possible, except in the case of a unit intertemporal elasticity of substitution, where π_t is constant. Thus, I will also have to find an approximate solution via loglinearization for the worker's decision rule.

2.3. Aggregate Consumption and the Distribution of Wealth

I now aggregate across individual retirees to obtain a consumption function for retirees, and do the same across individual workers to obtain a consumption function for this group. Combining the two relations then yields an aggregate consumption function. Because aggregate consumption will depend the distribution of non-human wealth between the two

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[9]Hayashi (1982), for example, estimates a higher discount rate for human wealth than for non-human wealth.

11

groups, I also characterize how this distribution evolves.

Since the marginal propensity to consume out of wealth, $\epsilon_t \pi_t$, is the same for all retirees, one can simply sum (2.6) across individual retirees to obtain an aggregate relation. Let A_t^r be the total non-human assets that retirees carry from period t-1 into period t. The aggregate gross return on this wealth is R_t since each retiree at t-1 earns a return R_t/γ, but only the fraction γ of these individuals survive. Thus, total wealth available to retirees at t is $R_t A_t^r$, implying that aggregate retiree consumption, C_t^r is given by

$$C_t^r = \epsilon_t \pi_t R_t A_t^r \tag{2.14}$$

Though it differs from that of retirees', individual workers also have an identical marginal propensity to consume out wealth, π_t, implying that one can similarly aggregate consumption across workers. Let A_t^w be the total non-human assets that workers carry from period t-1 into period t and let H_t be the total human wealth of the current work force. Summing (2.11) over individual workers, therefore, implies that total worker consumption at t, C_t^w is given by

$$C_t^w = \pi_t(R_t A_t^w + H_t) \tag{2.15}$$

with

$$
\begin{aligned}
H_t &= \sum_{v=0}^{\infty} \frac{N_{t+v} W_{t+v}}{\prod_{z=1}^{v}(1+n)R_{t+z}\Omega_{t+z}/\omega} \\
&= N_t W_t + \frac{H_{t+1}}{(1+n)R_{t+1}\Omega_{t+1}/\omega}
\end{aligned}
\tag{2.16}
$$

Equation (2.16) indicates that H_t is a discounted sum of the economy-wide wage bill at each point in time, $N_{t+\tau} W_{t+\tau}$. Note that the discount rate that is applied to the aggregate wage bill is the product of the gross population growth rate $1 + n$ and the rate at which individual workers discount their labor income, $R_{t+\tau}\Omega_{t+\tau}/\omega$. The factor $1+n$ augments the

12

discount rate because, with finite lives, the share of the total wage bill of those currently alive declines over time as the labor force grows. In total, therefore, three distinct factors arise in the life cycle setting presented here that raise the discount rate on future labor income. They are: (1) finite expected time in the labor force (reflected by the presence of ω in the discount rate); (2) greater utility gains from smoothing expenditures out of wealth early in life than late in life (reflected by the presence of Ω_{t+1}.); and (3) growth of the labor force (reflected by the presence of $(1+n)$).

Let A_t denote aggregate assets and λ_t denote the share of assets held by retirees, i.e., $\lambda_t \equiv A_t^r/A_t$ and $(1-\lambda_t) = A_t^w/A_t$. Then, to obtain an aggregate consumption function, simply add (2.6) and (2.15):[10]

$$C_t = \pi_t\{[1 + (\epsilon_t - 1)\lambda_t]R_t A_t + H_t\} \tag{2.17}$$

Clearly, a novel feature of equation (2.17) is the presence of the share of non-human wealth held by retirees, λ_t. Since ϵ_t exceeds unity (i.e., retirees have a higher marginal propensity to consume than workers), a rise in λ_t raises aggregate consumption demand. An implication is that social security will influence aggregate consumption and saving. Also, because the effective discount rate used to measure aggregate human wealth exceeds the market interest rate, government debt and deficits will influence consumption demand. I defer a comprehensive discussion of fiscal policy and social security until section 4, however.

To characterize how λ_t evolves over time, note first that total assets accumulated by retirees from period t to t+1 depends both on the saving of current retirees at t and the assets of workers at t who switch into retirement at t+1. That is,

$$\lambda_{t+1}A_{t+1} = \lambda_t R_t A_t - C_t^r + (1-\omega)[(1-\lambda_t)R_t A_t + W_t - C_t^w]$$

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[10]Because of the life-cycle effects present, equation (2.17) does not impose the counterfactual relation between aggregate consumption growth and real interest rates that arise in a representative agent economy. For discussion of the implications for aggregate consumption of life-cycle behavior, see Clarida (1991) and Gali (1990).

The last term in brackets is assets accumulated by workers at t for t+1. The fraction $1 - \omega$ of these assets accrues to retirees at t+1, reflecting the fraction of time t workers that leave the labor force in the subsequent period. Given that total assets held by workers at t+1, $(1 - \lambda_{t+1})A_{t+1}$, equals assets carried by workers at t into t+1 times the fraction ω that remain in the work force:

$$\lambda_{t+1} = \omega(1 - \epsilon_t \pi_t)\lambda_t R_t \frac{A_t}{A_{t+1}} + (1 - \omega) \qquad (2.18)$$

Up to this point I have determined aggregate consumption and the distribution of non-human wealth between workers and retirees, taking as given the paths of aggregate assets A_t, the interest rate R_t, and the wage rate W_t. The next section closes the model by adding production.

3. Dynamic Equilibrium and Steady State

I now embed the life-cycle framework of the previous section within a one sector growth economy. The economy is closed and growth is exogenous. Also, there is no government policy. Sections 4 and 5 relax these assumptions.

3.1. Production, Resource Constraints and Equilibrium

The economy is competitive. Firms employ capital and labor to produce output using a constant returns to scale technology. There are no adjustment costs. Think of individuals as renting their capital to firms. Let Y_t be output, X_t be the state of technology, and K_t be capital. Then aggregate output is given by the following Cobb-Douglas technology

$$Y_t = (X_t N_t)^\alpha K_t^{1-\alpha} \qquad (3.1)$$

where the parameter α is the labor share. Technology is labor augmenting and grows exogenously, as follows:

14

$$X_{t+1} = (1 + x)X_t \tag{3.2}$$

Capital depreciates at the rate δ.

I can now characterize the behavior of wages, rents, and asset supplies (non-human wealth). Equation (3.1) implies that W_t and R_t are given by

$$W_t = \alpha \frac{Y_t}{N_t} \tag{3.3}$$

$$R_t = (1 - \alpha)\frac{Y_t}{K_t} + (1 - \delta) \tag{3.4}$$

Capital is the only vehicle for saving. (In the next section I add government debt). Therefore, the net supply of assets in this case equals the capital stock:

$$A_t = K_t \tag{3.5}$$

Since the economy is closed, and since there is no government, the capital stock evolves according to

$$K_{t+1} = Y_t - C_t + (1 - \delta)K_t \tag{3.6}$$

Definition 1 (Competitive Equilibrium): A competitive equilibrium is a sequence of endogenous predetermined variables $\{K_{t+1}, \lambda_{t+1}\}$ and a sequence of endogenous variables $\{\pi_t, \epsilon_t, \Omega_t, H_t, C_t, W_t, R_t, A_t\}$ that satisfy equations (3.6), (2.18), (2.12), (2.7), (2.10), (2.16), (2.17), (3.3), (3.4), and (3.5), given the sequence of the exogenous predetermined variables $\{N_{t+1}, X_{t+1}\}$ specified by (2.1) and (3.2), and given the initials values of all the predetermined variables, $K_t, \lambda_t, N_t,$ and X_t.

15

In the conventional one sector growth model the capital stock, K_t, is the only endogenous variable. In the life-cycle economy here, the share of non-human wealth accruing to retirees, λ_t, is also a endogenous state variable. The distribution of wealth matters here because the mpcw differs between retirees and workers. In general, it is possible to express all the endogenous variables as functions of the two predetermined variables K_t and λ_t. As a matter of practice it is possible to reduce the model to a system of five simultaneous difference equations in the two predetermined variables, K_t and λ_t, and three "forward looking jump variables", H_t, π_t, and ϵ_t. While this system is too cumbersome to solve analytically, it is very easy to solve numerically. We do so in the next section, after including fiscal policy and social security. It is also very easy to solve for the steady state, which we do in the next sub-section.

3.2. Steady State

Analysis of the steady state provides some flavor for how the life-cycle aspects of the model affect behavior. In the steady state, all quantity variables grow at the exogenously given rate of growth of the effective labor force, $X_t N_t$, which is equal to $(1+x)(1+n) \approx 1+x+n$. Because there is growth in the steady state, it is convenient to normalize certain variables relative to output. We use lower case variables to denote the value of a variable relative to output. In particular $k = \frac{K}{Y}$ is the steady state capital output ratio and $h = \frac{H}{Y}$ is the ratio of human wealth to current output.

Let $\Gamma(R,\Omega)$ be steady state human wealth, H_t^{ss}, divided by its value that would arise for a infinitely lived representative agent economy, H_t^{*ss}, holding constant the path of future labor income. Thus, for example, $(1 - \Gamma(R,\Omega)) \times 100$, is the percent decline in human wealth that arises from using the discount rate on labor income that applies in the life cycle economy instead of the rate that is relevant for the representative agent economy. $\Gamma(R,\Omega)$ is given by[11]

---

[11] Given that H_t grows at the gross rate $(1+x)(1+n) \approx 1+x+n$ in the steady state, $H_t^{ss} = W_t + H_t^{ss}(1+x)(1+n)/[(1+n)R\Omega/\omega]$ and $H_t^{*ss} = W_t + H_t^{*ss}(1+x)(1+n)/R$. Thus, $H_t^{ss} = W_t R/[R - (1+x)\omega/\Omega]$ and $H_t^{*ss} = W_t R/[R - (1+x+n)]$.

$$\Gamma(R, \Omega) \equiv \frac{H_t^{ss}}{H_t^{*ss}} \equiv \frac{R - (1 + n + x)}{R - (1 + x)\omega/\Omega} \qquad (3.7)$$

Then, it is convenient to express the steady state as a system of seven nonlinear equations that determine seven variables: $k, \lambda, h, R, \pi, \epsilon$, and Ω. The equations for k, λ, and h hold for a pure laissez-faire economy (and thus require some modification when fiscal policy and social security become operative):

$$
\begin{aligned}
(x + n + \delta)k &= 1 - \pi\{[1 + (\epsilon - 1)\lambda]Rk + h\}, \qquad (3.8) \\
\lambda &= \psi \cdot \frac{1 + n - \gamma}{1 + x + n - \gamma\omega(R\beta)^\sigma}, \\
h &= \Gamma(R, \Omega) \cdot \frac{R\alpha}{R - (1 + x + n)}
\end{aligned}
$$

where ψ is the ratio of workers to retirees, defined by equation (2.2). The relations for R, π, ϵ, and Ω hold across policy regimes:

$$
\begin{aligned}
R &= (1 - \alpha)k^{-1} + 1 - \delta, \qquad (3.9) \\
\pi &= 1 - (R\Omega)^{\sigma-1}\beta^\sigma, \\
\epsilon\pi &= 1 - R^{\sigma-1}\beta^\sigma\gamma, \\
\Omega &= \omega + (1 - \omega)\epsilon^{\frac{1}{1-\sigma}}
\end{aligned}
$$

The relation for k in (3.8) is the steady state version of the resource constraint (3.6), after using (2.17) to eliminate consumption and after normalizing by output. The left side is steady state investment per unit of output. The right is steady state saving per unit of output. The other relations in (3.8) are the steady state counterparts of (2.18), and (2.16) (divided by Y). The relations in (3.9) are the steady state counterparts of (3.4), (2.12),

(2.7), and (2.10).

Life cycle factors are present in three distinct ways. First, k depends inversely on λ, which in turn depends on the age structure of the population implied by the retiree to worker ratio, ψ. Since $\epsilon > 1$, a rise in the share of nonhuman wealth held by the retirees raises consumption demand, and thus lowers steady state saving and investment.[12] Second, the finiteness of life raises the mpcw for workers as well as for retirees. Because, $\Omega > 1$ and $\gamma < 1$, both π and $\epsilon\pi$ exceed the mpcw that would arise with infinite life, $1 - R^{\sigma-1}\beta^{\sigma}$. This enhanced propensity to consume works to reduce k. Third, h is only the fraction $\Gamma(R,\Omega)$ of its value that would arise in the representative agent case, $\frac{R\alpha}{R-(1+x+n)}$, due to the higher discount rate that applies in the life cycle economy. While the first two factors work to reduce k, this third factor instead works to raise k, since it lowers consumption demand by workers.

It follows that the equilibrium need not correspond to the case with infinitely lived households. Instead, aggregate consumption and saving depend on labor force and population dynamics, as they do in a conventional life cycle setting. Also transfers between workers and retirees will matter since the two groups have different marginal propensities to consume. Finally, debt and deficits will matter since workers discount (net) future wage income at a rate greater than the market interest rate. In the next section we will illustrate these propositions with quantitative examples.

3.3. Quantitative Properties

To illustrate how the model captures life-cycle behavior, this section reports the results from some simple numerical examples. I choose the following values for the exogenous parameters: $\beta = .96$, $\delta = .1$, $\sigma = .25$, $\alpha = .667$, $\omega = .977$, $\gamma = .9$, $x = .01$, $n = .01$. To select parameters, I used Auerbach and Kotlikoff (1987) for guidance. These authors assumed that individuals worked from age 21 to 65, and then live in retirement from 66 to

---

[12]One can prove $\epsilon > 1$ by contradiction: Suppose that $\epsilon \leq 1$. Then equation (3.9) implies $\gamma \geq \Omega^{\sigma-1} = [\omega + (1-\omega)\epsilon^{\frac{1}{1-\sigma}}]^{\sigma-1}$. But since $\gamma < 1$, this condition can only hold if $\epsilon > 1$, which gives rise to a contradiction.

75. Therefore, I chose values of ω and γ which imply that individuals work on average for 45 years and are in retirement for 10 years.

Table 1 reports the values of the steady state variables, along with the value of $\Gamma(R, \Omega)$, the ratio of human wealth in the life cycle economy to human wealth in the representative agent economy (see equation (3.7)). Since the net interest rate (3.1 percent) exceeds the net growth rate (2 percent), the economy is dynamically efficient

k	λ	h	R	π	ϵ	Ω	Γ
2.541	0.160	8.345	1.031	0.061	2.130	1.040	0.185

Table 1: Steady State for Laissez-Faire Economy

Figure 3.1 illustrates how varying the demographic structure affects the steady state outcome. Specifically, it considers the impact on the capital stock per unit of effective labor, $\frac{K}{XN}$ ($= k^{\frac{1}{1-\alpha}}$), and on R of increasing the retiree/worker ratio, φ, by raising the average length of retirement, $\frac{1}{1-\gamma}$ [see equation (2.2)]. As ψ increases, $\frac{K}{XN}$ rises and R falls: Raising the average length of retirement induces individuals to save. Raising ψ, further, eventually pushes the economy into an inefficient steady state.

Figure 3.2 illustrates how life cycle factors may influence the dynamic behavior of the model by showing the impact of a redistribution of wealth from workers to retirees.[13] Suppose at time 0, the share of non-human wealth held by retirees, λ_t, doubles (e.g., due to a lump sum tax and transfer scheme). This redistribution produces a rise in consumption and a decline in saving, that comes about because retirees have a higher mpcw than workers (see Table 1). As a consequence, $\frac{K}{XN}$ initially declines, reaching a trough about 3 percent below the steady state. The decline in capital, however, produces a rise in R that stimulates saving, which moves the economy back to its long run equilibrium.

---

[13]To compute dynamics, I loglinearize the model described in Definition 1 around its steady state to generate a system of linear difference equations. Then to solve the system I simply use the formulas in Blanchard and Kahn (1981). This approach is similar in spirit to King, Plosser and Rebelo (1988). The Gauss program that I used for all the computations is available upon request.
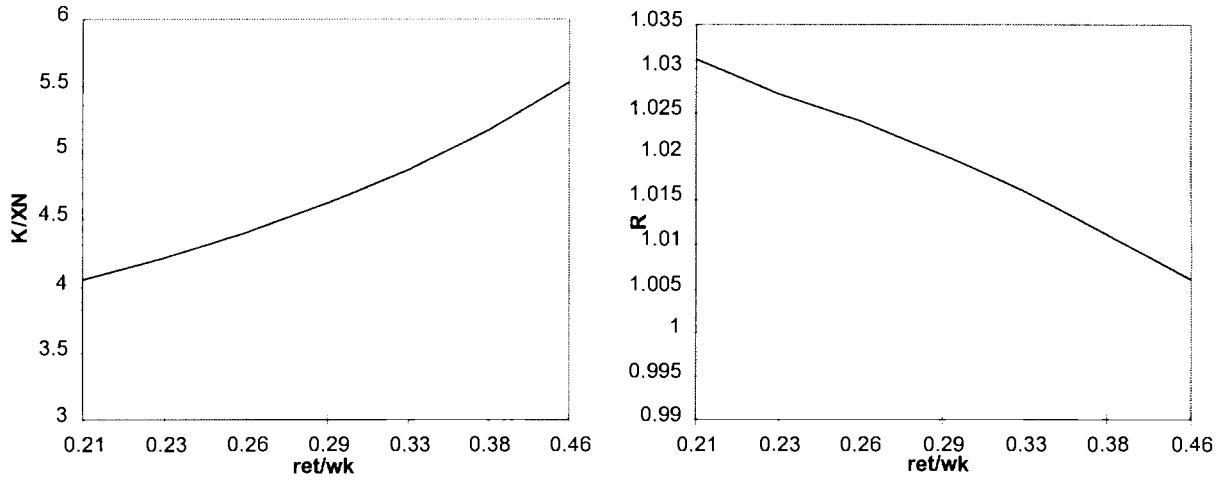
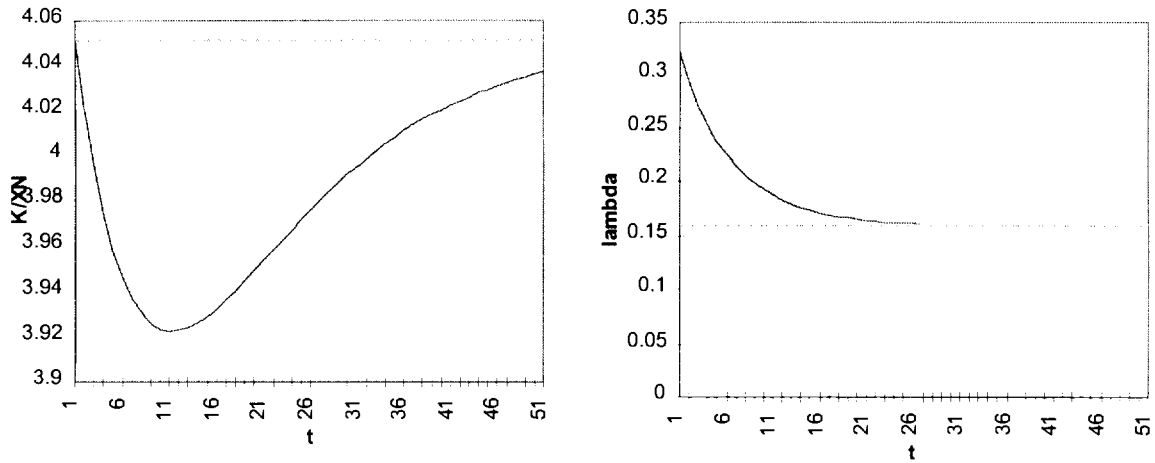Figure 3.1: Steady State Impact of Retiree-Worker Ratio

Figure 3.2: Adjustment to Wealth Redistribution

20

4. Fiscal Policy and Social Security

In this section we introduce fiscal policy and social security. We then use the model to perform a variety of policy experiments.

The government consumes G_t each period. It also pays retirees a total of E_t in social security benefits. To finance its expenditures, the government levies a total of T_t in lump sum taxes on workers and it also issues one period government bonds, B_{t+1}. Each period, therefore, the government satisfies,

$$B_{t+1} = R_t B_t + G_t + E_t - T_t \qquad (4.1)$$

Assuming that the government eventually must pay its debt, iterating equation (4.1) forward yields the following intertemporal budget constraint:[14]

$$R_t B_t = \sum_{v=0}^{\infty} \frac{T_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{G_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{E_{t+v}}{\prod_{z=1}^{v} R_{t+z}} \qquad (4.2)$$

Equation (4.2) states simply that the discounted stream of taxes must equal the current value of outstanding government debt plus stream of government expenditures. A key features of this constraint is that the per period discount rate the government uses is the riskless rate $R_{t+\tau}$, which in general is below the discount rate that individual workers apply to future net earnings streams.

4.1. Effects of Fiscal Policy and Social Security on the Consumption Function

Every retiree still consumes the fraction $\epsilon_t \pi_t$ of his wealth each period. However, his wealth now includes a discounted stream of social security benefits. Let S_t be the sum across retirees alive at t of the capitalized value of social security benefits. Then it is

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[14]The intertemporal budget constraint holds when the economy is dynamically efficient, i.e., when the interest rate exceeds the growth rate of the economy, making it infeasible to simply roll over the debt. Since the parameter values I employ imply dynamic efficency (see table 3), equation (4.2) applies for this analysis.

straightforward to show that, with social security, total consumption by retirees becomes

$$C_t^r = \epsilon_t \pi_t [R_t A_t^r + S_t] \tag{4.3}$$

where equation (2.7) still governs $\epsilon_t \pi_t$. S_t is given by

$$
\begin{aligned}
S_t &= \sum_{v=0}^{\infty} \frac{E_{t+\nu}}{\prod_{z=1}^{v}(1+n)R_{t+z}/\gamma} \\
&= E_t + \frac{S_{t+1}}{(1+n)R_{t+1}/\gamma}
\end{aligned}
\tag{4.4}
$$

Since total social security payments are distributed among a retiree population that grows at the net rate n, the gross retiree growth rate 1+n enters the discount factor.

Every worker still consumes the fraction π_t of his wealth. There are two adjustments to a worker's wealth, however. First, wealth now includes the value of social security payments that the worker can expect once he retirees. Second, the measure of human wealth is now net of a discounted stream of taxes. Let S_t^w be the sum across workers alive at t of the capitalized future social security benefits they can expect during retirement. Then, it is straightforward to show that, with taxes and social security, total consumption by workers becomes

$$C_t^w = \pi_t [A_t^w + H_t + S_t^w] \tag{4.5}$$

where equation (2.12) still governs π_t.

The new relation for H_t is obtained simply by amending equation (2.16) to allow for taxes:

$$
\begin{aligned}
H_t &= \sum_{v=0}^{\infty} \frac{N_{t+v}W_{t+v} - T_{t+v}}{\prod_{z=1}^{v}(1+n)R_{t+z}\Omega_{t+z}/\omega} \\
&= N_t W_t - T_t + \frac{H_{t+1}}{(1+n)R_{t+1}\Omega_{t+1}/\omega}
\end{aligned}
\tag{4.6}
$$

In turn, let $\widehat{S}_t \equiv \frac{S_t}{\psi N_t}$, be the value of social security at t per beneficiary (recall that ψN_t is the number of retirees at t.) Then, S_t^w is given by

$$
\begin{aligned}
S_t^w &= \sum_{v=0}^{\infty} \frac{(1-\omega)\omega^v N_t \left(\frac{\epsilon_{t+1+v}\widehat{S}_{t+1+v}}{R_{t+v}\Omega_{t+v}}\right)}{\prod_{z=1}^{v} R_{t+v}\Omega_{t+v}} \\
&= (1-\omega)N_t\left(\frac{\epsilon_{t+1}\widehat{S}_{t+1}}{R_{t+1}\Omega_{t+1}}\right) + \frac{S_{t+1}^w}{(1+n)R_{t+1}\Omega_{t+1}/\omega}
\end{aligned}
\tag{4.7}
$$

Equation (4.7) requires some interpretation. The expression $(1-\omega)\omega^v N_t\left(\frac{\epsilon_{t+1+v}\widehat{S}_{t+1+v}}{R_{t+v}\Omega_{t+v}}\right)$ is the capitalized value at $t + v$ of social security entitlements to all individuals who were in the work force at t and retire at $t + v + 1$. [15] The expression for S_t^w, therefore, is just the discounted sum of this capitalized value from $v = 0$ to $v = \infty$. It is thus a measure of the aggregate value of social security entitlements to the work force at time t

Combining the new expressions C_t^r and C_t^w yields the new aggregate consumption function :

$$
C_t = \pi_t[(1-\lambda_t)R_t A_t + H_t + S_t^w + \epsilon_t(\lambda_t R_t A_t + S_t)]
\tag{4.8}
$$

Equation (4.8), in conjunction with equations (4.6), (4.7), and (4.4), compactly expresses the impact of fiscal policy and social security on aggregate consumption. Taxes influence consumption demand via the measure of workers' human wealth, H_t. As equation (4.6) indicates, though, the rate at which the work force discounts future taxes, given by $(1 + n)R_t\Omega_t/\omega$ exceeds the rate at which the government can borrow, R_t. Thus, policies which postpone taxes into the future - e.g. current deficits financed by future tax increases - raise H_t, and stimulate consumption demand. Similarly, workers do not fully capitalize

---

[15]To see this, note that $(1-\omega)\omega^v N_t$ is the number of workers from the time t labor force that retires at $t + v + 1$; and $\frac{\epsilon_{t+1+v}\widehat{S}_{t+1+v}}{R_{t+v}\Omega_{t+v}}$ is the value at $t + v$ of a stream of social security payments to an individual that begins in the subsequent period. The ratio of the retiree's to the worker's mpcw, ϵ_{t+1+v} enters this expression since it reflects the value to worker of being able to consuming today out of wealth to be received in retirement.

the stream of taxes associated with anticipated paths of government expenditures (see equation (4.2)), again due to the difference between private and public discount rates. Thus, government expenditures have a greater impact on demand in the life cycle setting here than when households have infinite horizons: Everything else equal, government expenditures crowd out less private consumption.

Finally, even after controlling for the impact of the timing of taxes, social security payments raise consumption demand. Transfers from workers to retirees increase stimulate consumption since the latter have a greater propensity to consume than do the former. Equation (4.8) captures this phenomenon since the propensity to consume out aggregate retiree social security wealth, S_t exceeds the propensity to consume out of aggregate human wealth (which incorporates the taxes on workers used to finance the entitlement payments).

Social security also influences the evolution of the distribution of wealth. Given that retirees receive a total transfer of E_t per period, total retiree non-human wealth evolves according to

$$\lambda_{t+1} A_{t+1} = \lambda_t R_t A_t + E_t - C_t^r + (1 - \omega)[(1 - \lambda_t) R_t A_t + N_t W_t - T_t - C_t^w]$$

It follows that the difference equation for retiree share of financial wealth, λ_{t+1}, is now given by

$$\lambda_{t+1} = \omega(1 - \epsilon_t \pi_t) \lambda_t R_t \frac{A_t}{A_{t+1}} + \omega[E_t - \epsilon_t \pi_t S_t]/A_{t+1} + (1 - \omega) \tag{4.9}$$

4.2. Dynamic Equilibrium and Steady State with Fiscal Policy and Social Security

Non-human wealth now equals the sum of capital and government bonds:

$$A_t = K_t + B_t \tag{4.10}$$

With government consumption, capital evolves according to

$$K_{t+1} = Y_t - C_t - G_t + (1 - \delta)K_t \qquad (4.11)$$

where aggregate output, Y_t still obeys the Cobb-Douglas formulation given by equation (3.1)

Government policy fixes the ratio of government consumption to output, $\bar{g}_t$, the ratio of social security payments to output, $\bar{e}_t$, and the stock of government bonds to output, $\bar{b}_t$. Accordingly,

$$
\begin{aligned}
G_t &= \bar{g}_t Y_t, \qquad (4.12) \\
E_t &= \bar{e}_t Y_t, \\
B_t &= \bar{b}_t Y_t
\end{aligned}
$$

Given the paths of G_t, E_t, and B_t, the government adjusts taxes, T_t, to satisfy its intertemporal budget constraint, given by equation (4.2).

Definition 2 (Equilibrium with Fiscal Policy and Social Security): An equilibrium with fiscal policy and social security is a sequence of endogenous predetermined variables $\{K_{t+1}, \lambda_{t+1}\}$ and a sequence of endogenous variables $\{\pi_t, \epsilon_t, \Omega_t, H_t, C_t, W_t, R_t, A_t, T_t, S_t, S_t^w, \}$ that satisfy equations (4.11), (4.9), (2.12), (2.7), (2.10), (4.6), (4.8), (3.3), (3.4), (4.10), (4.2), (4.4), and (4.7), given the sequences of the exogenous predetermined variables, $\{N_{t+1}, X_{t+1}\}$ specified by (2.1) and (3.2) and of the exogenous policy variables $\{\bar{b}_t, \bar{g}_t, \bar{e}_t\}$ and given the initials values of the predetermined variables, K_t, λ_t, N_t, and X_t.

Fiscal policy and social security add three new endogenous variables, T_t, S_t, and S_t^w and three new exogenous variables, $\bar{b}_t, \bar{g}_t$, and $\bar{e}_t$. As before, K_t and λ_t are the endogenous state variables. The paths of these variables, however, now depend on government policy,

as I will demonstrate later. In practice, it is now possible to reduce the model to a system of seven difference equations, with the two predetermined variables and five jump variables: $H_t, \pi_t, \epsilon_t, S_t$, and S_t^w. Recall that before we could reduce the system to five difference equations in two predetermined variables and three jump variables: H_t, π_t, and ϵ_t. The addition of social security necessitates difference equations for S_t and S_t^w.

Let τ, s, and s^w be the steady state values of $\frac{T_t}{Y_t}, \frac{S_t}{Y_t}$ and $\frac{S_t^w}{Y_t}$. Suppose, further, that $\bar{b}_t, \bar{g}_t$, and $\bar{e}_t$ are fixed at $\bar{b}, \bar{g}$, and $\bar{e}$ in the long run equilibrium. The steady state equilibrium then becomes a system of ten nonlinear equations that determine ten variables. Seven variables are from the steady state system without government policy (see section 3): $k, \lambda, h, \pi, \epsilon, R, \Omega$. Three variables are new: τ, s and s^w. The equations for π, ϵ, R, and Ω are unchanged, and are thus still given by (3.9). The new relations for k, λ, and h are:

$$(x + n + \delta)k = 1 - \pi\{(1 - \lambda)R(k + \bar{b}) + h + s^w + \epsilon[\lambda R(k + \bar{b}) + s]\} - \bar{g}, \quad (4.13)$$

$$\lambda = (\psi + \frac{\omega(\bar{e} - \epsilon\pi s)(k + \bar{b})^{-1}}{1 + n - \gamma}) \cdot \frac{1 + n - \gamma}{1 + x + n - \gamma\omega(R\beta)^\sigma},$$

$$h = \Gamma(R, \Omega)\frac{R(\alpha - \tau)}{R - (1 + x + n)}$$

The equations for the new variables τ, s and s^w are:

$$\tau = [R - (1 + x + n)]\bar{b} + \bar{g} + \bar{e}, \quad (4.14)$$

$$s = \frac{R}{R - \gamma(1 + x)}\bar{e},$$

$$s^w = \Gamma(R, \Omega) \cdot \frac{(1 - \omega)\epsilon}{\Omega} \cdot \frac{s/\psi}{R - (1 + x + n)}$$

In general, because of life cycle factors, k depends on the exogenous policy variables $\bar{b}, \bar{g}$ and $\bar{e}$. Substitute the expression for τ into the equation for h to obtain

26

$$h = \Gamma(R,\Omega) \cdot \frac{R(a - \overline{g} - \overline{e})}{R - (1 + x + n)} - \Gamma(R,\Omega)Rb \tag{4.15}$$

As equation (4.15) indicates, workers only capitalize the fraction $\Gamma(R,\Omega)$ of future tax liabilities associated with government debt (again, because life cycle factors make the workers' discount rate exceed the rate at which the government can borrow). Put differently, the net wealth of government bonds (normalized by output) to the private sector is $[1 - \Gamma(R,\Omega)] \cdot \overline{b}$. A rise in $\overline{b}$ thus stimulates consumption and reduces saving, forcing down k. The effect is magnified, further, to the extent that retirees are holding the bonds, since retirees have a larger mpcw than do workers. The equation for k in (4.13) shows that reallocating wealth in favor of retirees works to reduce the steady state capital intensity.

A rise in $\overline{g}$ also reduces k. In the standard one sector growth model with a representative agent a rise $\overline{g}$ has no impact on k: Consumption drops to fully offset the rise in $\overline{g}$. In the life cycle economy, however, there is less than full crowding out of consumption. As equation (4.15) shows, workers do not fully capitalize the future tax liabilities associated with government expenditures. A rise in $\overline{g}$ therefore reduces net saving in equilibrium, thereby reducing k.

Finally, social security matters. A rise in $\overline{e}$ raises s and s^w. It also reduces h, though not enough to compensate for the gross rise in social security wealth. Because retirees have a higher mpcw than workers, social security unambiguously reduces k. In the next section, I present some numerical simulations to illustrate this prediction as well as the other policy predictions.

4.3. Policy Experiments

This section considers a variety of quantitative policy exercises. I begin by calculating a steady state for baseline value of policy parameters.. The values for the exogenous non-policy parameters are: $\beta = 1$, $\delta = .1$, $\sigma = .25$, $\alpha = .667$, $\omega = .977$, $\gamma = .9$, $x = .01$, $n = .01$. These values are the same as were used for the laissez-faire economy of section 3, except that the discount factor β is raised from .985 to 1. The values for the policy parameters

are: $\bar{g} = .2, \bar{b} = .25, \bar{e} = .02$. The ratio of government consumption to output, $\bar{g}$, is in rough accord with the empirical postwar average. The ratios of government debt to output and of social security to output roughly match their empirical counterparts for the 1970s.

Table 2 reports the values of steady state variables, using the baseline policy parameters as inputs. The addition of government policy raises the net interest rate to 4.9 percent (despite the rise in β from .96 to 1) and, in correspondence, reduces the capital output ratio to 2.2. Interestingly, the fraction of government bonds that is net wealth for the current population is .62 ($= 1 - \Gamma$), which is line with Bernheim's estimate of roughly .5. Finally, the ratio of social security wealth to GDP for this economy is roughly .5 ($= s + s^w$), also a plausible number.

k	λ	h	R	π	ϵ	Ω	Γ	τ	s	s^w
2.230	0.177	4.624	1.049	0.628	2.102	1.039	0.381	0.227	0.149	0.426

Table 2: Steady State with Government Policy

I next consider a variety of policy experiments involving, in turn, government debt, social security and government consumption:

Government Debt. Figure 4.1 reports the steady state impact of varying the ratio of government debt to output on $\frac{K}{XN}$ and R. The impact is substantial. Varying $\bar{b}$ from 0 to 1 leads to a reduction of $\frac{K}{XN}$ by nearly a third and to a roughly four hundred basis point rise in R. The actual variation in $\bar{b}$ for the post-war U.S. economy is much smaller than this, of course. However, the figure also suggests a significant impact of the Reagan "experiment" of raising $\bar{b}$ from .25 to .5 : The interest rate rises by about 100 basis points and $\frac{K}{XN}$ declines about 9 percent. The numbers are reasonable, in light of the actual historical experience.

Figure 4.2 portrays the transition dynamics that result from a rise in government debt. The specific policy experiment is a rise in government debt that is phased in over a period of ten years, meant to approximate the actual buildup of debt that occurred during the Reagan-Bush years (see Figure 2.1). Specifically, $\bar{b}$ increases from .25 to .5 over a ten year
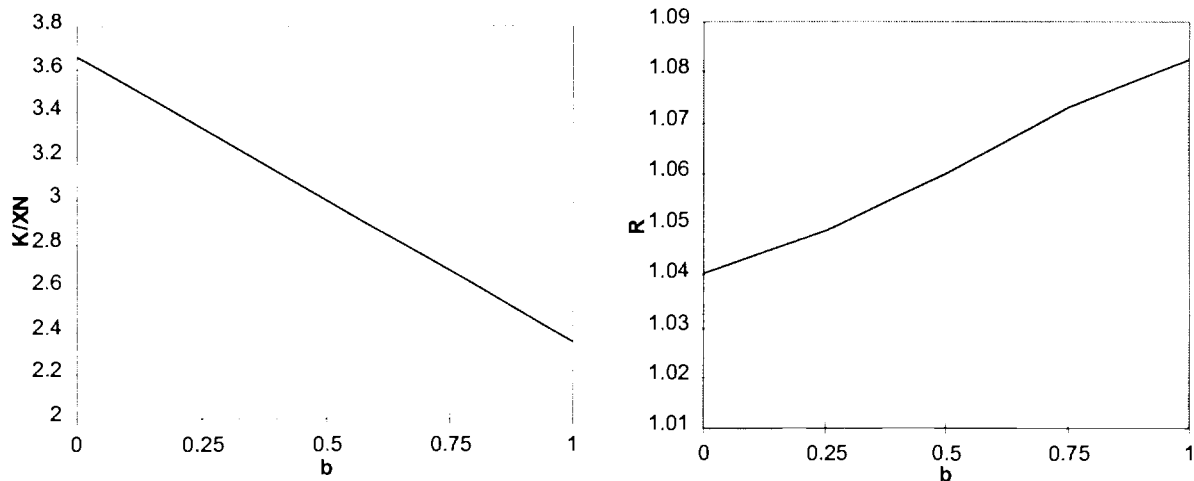
Figure 4.1: Steady State Impact of Government Debt

period, in equal increments each year. I assume, further, that the policy change is fully anticipated.

Social Security. Figure 4.3 reports the impact of varying social security benefits from 0 percent to 5 percent of GDP, holding constant the demographic structure, and given a ratio of government debt to GDP equal to .25. The total effect is quite large: The interest rate rises by almost 600 basis points and the capital per efficiency unit of labor falls by nearly forty percent. Note, however, that this experiment assumes that all the rise in social security comes from an increase in benefits per retiree as opposed to a rise in the number of retirees. In results that I do not report here, I show that if the rise in aggregate social security payments comes about because retirees live longer, then the capital stock may actually rise: the longer horizon of retirees works to increase individual saving (see Figure 3.1).[16]

Figure 4.4 reports the dynamic effects of doubling the aggregate level of social security

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[16]In the U.S., social security has risen for three reasons: (1) increased benefit levels; (2) increased life spans; and (3) demographic effects of the baby boom. The model as it stands can capture the first two effects. It is however possible to modify the framework to allow for non-stationary demographics that would capture baby boom effects. I'm currently working on this issue.
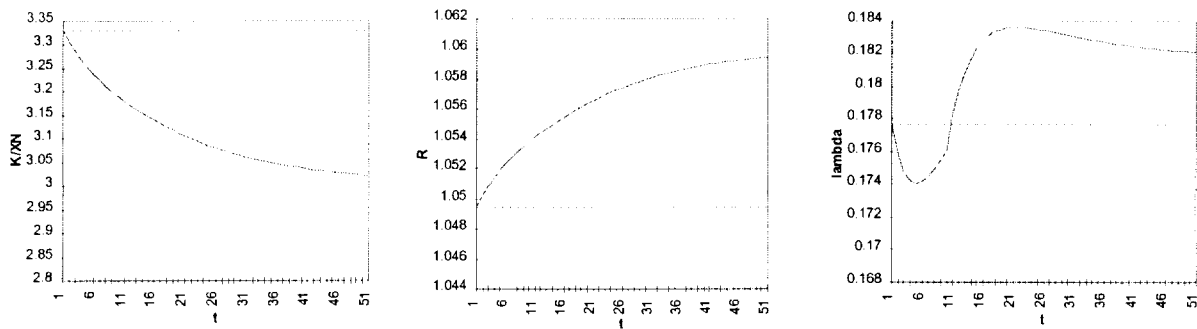
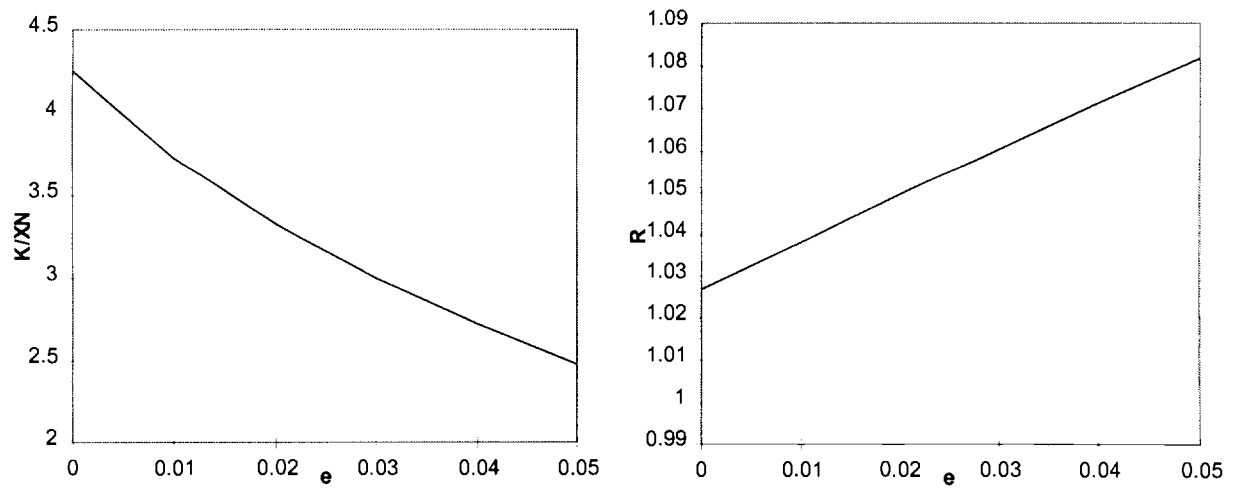Figure 4.2: Adjustment to Rise in Government Debt

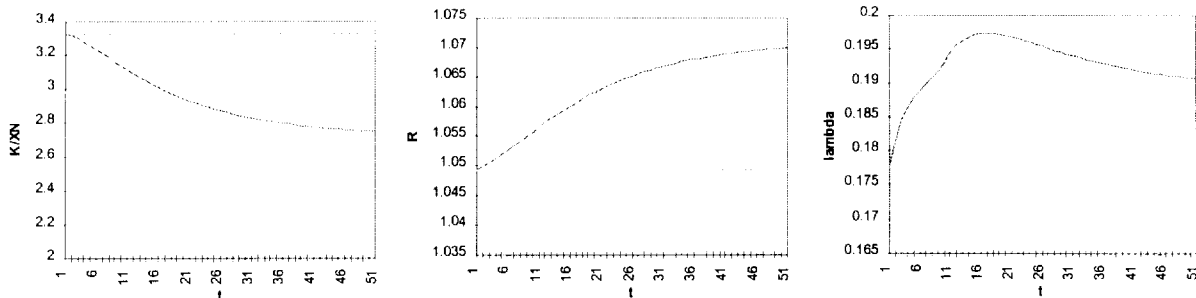Figure 4.3: Steady State Impact of Social Security

30

Figure 4.4: Adjustment to Rise in Social Security

from 2 percent of GDP to 4 percent, smoothly over a ten year period. The experiment roughly captures the behavior of aggregate transfers to the elderly between the 1970s and 1980s to early 1990s. It assumes, however, that all the rise comes from increased benefit levels. One interesting phenomenon is the sharp initial rise in retiree's share of wealth due to the phasing in of increased benefits, which works to speed up the decline in the capital stock.

Government Consumption. Figure 4.5 shows the effect of a rise in the share of government consumption from 20 to 30 percent of GDP. There is a significant decline in the capital stock, since private consumption does not fall sufficiently to offset the rise in g, in contrast to what occurs in the conventional representative agent framework. Barro and Salai-I-Martin (1995) show that, empirically, a rise in g is associated with a decline in the growth rate. One could capture these facts with this model by adjusting it to allow for endogenous growth. Alternatively, with the model as it stands, a rise in g produces a decline in the growth rate along the transition to the new steady state.
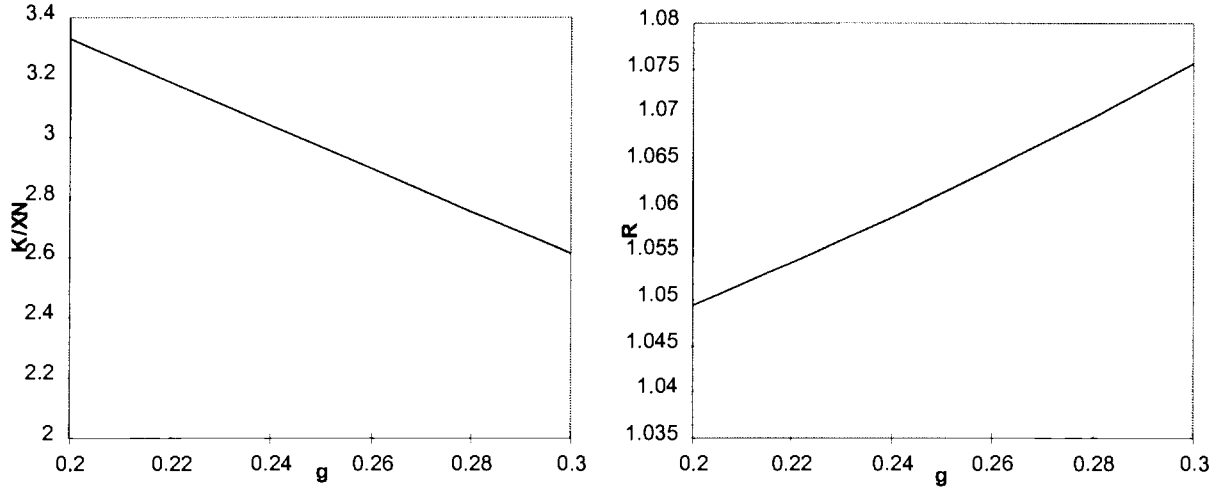
31

Figure 4.5: Steady State Impact of Government Consumption

5. Policy Effects in an Open Economy

It is easy to extend the analysis to the case of a small open economy. Let F be the economy's net foreign asset position. Assuming that it is possible to borrow and lend abroad at the world interest rate, $\overline{R}$, the economy-wide resource constraint becomes

$$K_{t+1} = Y_t - C_t - G_t + (1 - \delta)K_t - F_{t+1} + \overline{R}F_t \qquad (5.1)$$

To convert the closed economy to a small open economy, simply replace equation (3.6) with (5.1) and impose that R equals the world rate $\overline{R}$. Similarly, to modify the steady state simply replace the steady state resource constraint in equation (4.13) with

$$(x + n + \delta)k = 1 - \pi\{(1 - \lambda)R(k + \overline{b}) + h + s^w + \epsilon[\lambda R(k + \overline{b}) + s]\} - \overline{g} - [(1 + x + n) - R]f$$

where $f = \frac{F}{Y}$.

Figure 4.6 reports the steady state impact on f of government debt and of social security.
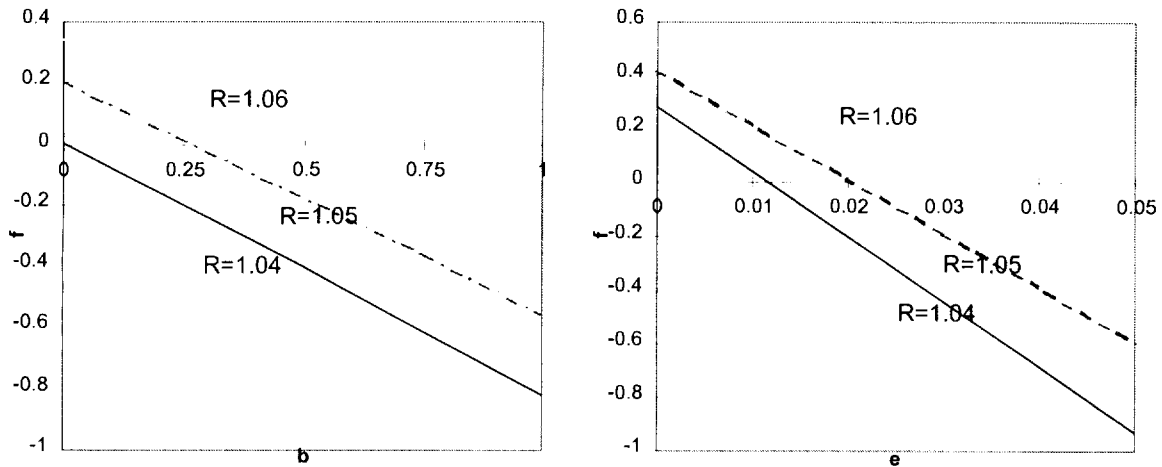
32

Figure 5.1: Steady-State Impact of Debt and Social Security in the Open Economy

A dollar rise in government debt leads to a decline in net foreign assets of between fifty and sixty cents. The impact of a rise in social security on net foreign assets is similarly substantial.

6. Concluding Remarks

The framework of this paper embeds life-cycle behavior within a dynamic general equilibrium economy. The model is very tractable, yet permits individuals to have realistic average lengths of life, work, and retirement. Government debt and social security have significant and plausible effects on capital intensity and real interest rates.

Many extensions are possible. Perhaps the most interesting would be to allow for a non-stationary demographic structure. It is possible, for example, to capture the baby boom experience by allowing for a temporary period of high fertility, and then letting the effect on the demographic structure work its way out through time.By doing so, it would be possible to more accurately capture the current dilemma over old age entitlements. Another possibility would be to model spending on health in order to more explicitly capture the effects of medicare. It would also be interesting to consider the welfare effects

of distortionary taxation, particularly social security taxes. Finally, it might interesting to allow for endogenous growth. Within an endogenous growth setting, the effects of policy on saving arising from the life-cycle setup would translate into effects on the growth rate.

Appendix

Retiree Decision Problem: Maximize

$$V_t^r = [(C_t^r)^\rho + \beta\gamma(V_{t+1}^r)^\rho]^{\frac{1}{\rho}}$$

subject to the budget constraint given by equation (2.4). The F.O.N.C. are given by

$$(C_t^r)^{\rho-1} = \beta\gamma\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r}(V_{t+1}^r)^{\rho-1}$$

From the envelope theorem: $\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = \frac{R_{t+1}}{\gamma}(C_{t+1}^r)^{\rho-1}(V_{t+1}^r)^{1-\rho}$. Then $(C_t^r)^{\rho-1} = R_{t+1}\beta(C_{t+1}^r)^{\rho-1}$ or, equivalently,

$$C_{t+1}^r = (R_{t+1}\beta)^\sigma C_t^r$$

with $\sigma = \frac{1}{1-\rho}$. To find a solution for the value function, conjecture that $V_t^r = \Delta_t^r C_t^r$. Then, to obtain an expression for Δ_t^r, substitute the conjectured solution for V_t^r into the objective to obtain

$$\Delta_t^r C_t^r = [(C_t^r)^\rho + \beta\gamma(\Delta_{t+1}^r C_{t+1}^r)^\rho]^{\frac{1}{\rho}}$$

Then substitute the first order conditions for consumption in for C_{t+1}^r to obtain $\Delta_t^r C_t^r = [(C_t^r)^\rho + \beta\gamma[\Delta_{t+1}^r(R_{t+1}\beta)^\sigma C_t^r]^\rho]^{\frac{1}{\rho}}$, which implies that Δ_t^r is given by

$$(\Delta_t^r)^\rho = 1 + \beta^\sigma R_{t+1}^{\sigma-1}\gamma(\Delta_{t+1}^r)^\rho$$

From this equation and equation (2.7), it is straightforward to show that

$$\Delta_t^r = (\epsilon_t\pi_t)^{\frac{-1}{\rho}}$$

The difference equation for $\epsilon_t \pi_t$ in equation (2.7) is obtained by substituting the conjectured solution for retiree consumption $C_t^{rjk} = \epsilon_t \pi_t (R_t/\gamma) A_t^{rjk}$ into the first order condition given by equation (2.6).

Worker Decision Problem. Maximize

$$V_t^w = \{(C_t^w)^\rho + \beta[\omega V_{t+1}^w + (1 - \omega)V_{t+1}^r]^\rho\}^{\frac{1}{\rho}}$$

subject to the budget constraint given by (2.8). The F.O.N.C are given by

$$(C_t^w)^{\rho-1} = \beta[\omega \frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + (1 - \omega)\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r}][\omega V_{t+1}^w + (1 - \omega)V_{t+1}^r]^{\rho-1}$$

From the envelope theorem $\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} = R_{t+1}(C_{t+1}^w)^{\rho-1}(V_{t+1}^w)^{1-\rho}$ and $\frac{\partial V_{t+1}^r}{\partial A_{t+1}^r} = R_{t+1}(C_{t+1}^r)^{\rho-1}(V_{t+1}^r)^{1-\rho}$ (for an individual that is a worker at t and a retiree at t+1). Conjecture that $V_t^w = \Delta_t^w C_t^w$ and that $\Delta_t^w = \pi_t^{\frac{-1}{\rho}}$ Given this conjecture and given that $V_t^r = \Delta_t^r C_t^r$ (see above), $(C_t^w)^{\rho-1} = R_{t+1}\beta[\omega + (1 - \omega)(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w})^{1-\rho}][\omega C_{t+1}^w + (1 - \omega)\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}C_{t+1}^r]^{\rho-1}$, implying

$$\omega C_{t+1}^w + (1 - \omega)\Lambda_{t+1}C_{t+1}^r = \{R_{t+1}\Omega_{t+1}\beta\}^\sigma C_t^w$$

where $\Lambda_{t+1} = \frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}$ and $\Omega_{t+1} = [\omega + (1 - \omega)(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w})^{1-\rho}] = [\omega + (1 - \omega)(\epsilon_{t+1})^{\frac{1}{1-\sigma}}]$. To verify the conjectured solution for V_t^w, substitute the implied solution for V_{t+1}^w and the solution for V_{t+1}^r into the expression for the value function to obtain

$$\Delta_t^w C_t^w = \{(C_t^w)^\rho + \beta[\omega \Delta_{t+1}^w C_{t+1}^w + (1 - \omega)\Delta_{t+1}^r C_{t+1}^r]^\rho\}^{\frac{1}{\rho}}$$

Then substitute in the first order condition for consumption to obtain $\Delta_t^w C_t^w = \{(C_t^w)^\rho + \beta[\Delta_{t+1}^w (R_{t+1}\Omega_{t+1}\beta)^\sigma C_t^w]^\rho\}^{\frac{1}{\rho}}$, or equivalently,

$$(\Delta_t^w)^\rho = 1 + \beta^\sigma (R_{t+1}\Omega_{t+1})^{\sigma-1}(\Delta_{t+1}^w)^\rho$$

36

From this equation and equation (2.12) in the text it is straightforward to show that $\Delta_t^w = \pi_t^{\frac{-1}{\rho}}$, as conjectured. To obtain the difference equation for π_t described in equation (2.12) substitute the conjectured solution for worker consumption $C_t^{wj} = \pi_t(R_t A^{wj} + H_t^j)$ into the first order condition given by equation (2.11) and make use of the fact that $\Lambda_{t+1} = \frac{\Delta_{t+1}^r}{\Delta_{t+1}^w} = (\epsilon_{t+1})^{\frac{\sigma}{1-\sigma}}$.

References

[1] Aiyagari, S. Rao and Mark Gertler (1985). "The Backing of Government Bonds and Monetarism." **Journal of Monetary Economics** 16.(July): 19-44.

[2] Auerbach, Alan J. and Laurence Kotlikoff (1987). **Dynamic Fiscal Policy** (Cambridge University Press, Cambridge).

[3] Barro, Robert J.(1974). "Are Government Bonds Net Wealth?" **Journal of Political Economy** 82 (November/December): 1095-1117.Hill

[4] Barro, Robert J. and Xavier Salai-I-Martin (1995). **Economic Growth.** (McGraw-, New York.)

[5] Blanchard, Olivier J. (1985). "Debt, Deficits and Finite Horizons." **Journal of Political Economy** 93 (April): 223-247.

[6] Blanchard, Olivier J. and Charles Kahn (1981). "The Solution to Linear Difference Equations Under Rational Expectations." **Econometrica** 48 (July): 1305-1311.

[7] Bernheim. Douglas B. (1987). "Ricardian Equivalence: An Evaluation of Theory and Evidence." **NBER Macroeconomics Annual** 2: 263-304.

[8] Campbell, John Y. and Robert J. Shiller (1988). "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." **Review of Financial Studies** 1 (Fall): 195-228.

[9] Cass, David (1965). "Optimum Growth in an Aggregative Model of Capital Accumulation.." **Review of Economic Studies** 32 (July): 233-40.

[10] Clarida, Richard H. (1991). "Aggregate Stochastic Implications of the Life-Cycle Hypothesis." **Quarterly Journal of Economics** CVI. (August):851-868

[11] Diamond, Peter A. (1965). "National Debt in a Neoclassical Growth Model" **American Economic Review**55 (December): 1126-50.

[12] Farmer, Roger (1990). "Rince Preferences." **Quarterly Journal of Economics** 105 (February): 43-60.

[13] Gali, Jordi (1990). "Finite Horizons, Life-Cycle Saving and Time Series Evidence on Consumption." **Journal of Monetary Economics** 90: 433-452.

[14] Gokhale, Jagadeesh, Laurence J. Kotlikoff, and John Sabelhaus. (1996). "Understanding the Postwar Decline in U.S. Saving: A Cohort Analysis," **Brookings Paper on Economic Activity** (Spring): 319-390.

[15] Hayashi, Fumio. (1982) "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables." **Journal of Political Economy** 90 (October): 895-916.

[16] Hubbard, R. Glenn and Kenneth L. Judd (1987). "Social Security and Welfare." **American Economic Review 77** (September): 630-646.

[17] King, Robert G., Charles I. Plosser and Sergio Rebelo (1988). "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." **Journal of Monetary Economics** (March/May): 195-232.

[18] Koopmans, Tjalling C. (1965). "On the Concept of Optimal Economic Growth." In **The Econometric Approach to Development Planning.** (North Holland, Amsterdam.)

[19] Modigliani, Franco (1966). "The Life-Cycle Hypothesis of Saving, the Demand for Wealth and the Supply of Capital." **Soc. Res.** 33 (June): 160-217.

[20] Romer, David. (1989)."What Are the Costs of Excessive Deficits?" **NBER Macroeconomics Annual:** 63-98.

[21] Tobin, James. (1967), "Life-Cycle Saving and Balanced Growth. In **Ten Economic Studies in the Tradition of Irving Fisher.** (Wiley, New York).

[22] Weil, Philippe. (1989). "Overlapping Families of Infinitely-Lived Agents." **Journal of Public Economics** 38 (March): 410-21.

[23] Weil, Philippe. (1990). "Nonexpected Utility in Macroeconomics." **Quarterly Journal of Economics** 105 (February):29-42.

[24] Yaari, Menahem E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." **Review of Economic Studies** 32: (April): 137-150.