

NBER WORKING PAPER SERIES

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Working Paper 7199
<http://www.nber.org/papers/w7199>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1999

We are grateful to Peter Diamond, Michael Hurd, Larry Kotlikoff, Olivia Mitchell, Antonio Rangel, Mark Warshawsky and participants in the NBER Summer Institute on Aging for helpful comments, and to TIAA-CREF, the National Institute on Aging, the Social Security Administration, and the National Science Foundation for research support. All opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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Joint Life Annuities and Annuity Demand by Married Couples

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NBER Working Paper No. 7199

June 1999

JEL No. H55, J14

ABSTRACT

This paper explores the value of purchasing joint life annuities for married couples. It describes the existing market for joint life annuities, and summarizes the range of annuity products that are currently available to couples. It then considers the value that married couples would place on access to an actuarially fair annuity market, and defines a measure of willingness-to-pay for annuities. This calculation differs from the analogous one for a single individual for two reasons. First, joint-and-survivor life tables differ from individual life tables. The life expectancy of the second-to-die in a married couple is substantially greater than that for a single individual. Second, joint life annuities provide time-varying payouts, because survivor benefit options permit the payout when both members of a couple are alive to differ from that when one member has died. The paper develops a new annuity valuation model and applies it to evaluate a married couple's utility gain from annuitization. The findings suggest that previous estimates of the utility gain from annuitization, which applied to individuals, overstate the benefits of annuitization for married couples. Since most potential annuity buyers are married, these findings may help to explain the limited size of the private market for single premium annuities.

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Annuities play an important role in the theory of consumer choice when there is uncertainty about length of life. Yaari (1965) showed that an individual with a fixed stock of resources and an uncertain lifetime should purchase an annuity contract to insure against the risk of outliving his resources. More recent work shows that the gains to annuitization for individual life-cycle consumers can be substantial. Mitchell, Poterba, Warshawsky, and Brown (1999) find that a typical 65-year old retired male life cycle consumer would be willing to give up roughly one third of his wealth to gain access to an actuarially fair nominal annuity market.

A number of studies have observed that despite the apparent benefits of annuitization, the market for individual annuity contracts in the United States is very small. This has resulted in several attempts to explain the limited flow of new annuity purchases. One group of empirical studies, including Friedman and Warshawsky (1988, 1990) and Mitchell, et al. (hereafter MPWB) (1999), have explored the extent of adverse selection in the individual annuity market. While annuitants are, on average, significantly longer-lived than individuals in the population at large, the degree of adverse selection does not seem sufficient to explain the absence of annuity purchases.

A second set of studies, surveyed in Laitner (1997) and Altonji, Hayashi, and Kotlikoff (1997), have focused on intergenerational altruism as a potential explanation for limited annuity demand. This work suggests that simple altruistic models do not provide a satisfactory explanation for observed patterns of intergenerational transfers.

Yet a third group of studies has shown that while individuals rarely purchase annuity contracts, a substantial fraction of their retirement resources is already annuitized. Auerbach, Kotlikoff, and Weil (1992) show that when Social Security benefits, private defined benefit pension plan payments, and Medicare benefits are added together, more than half of the resources of the current elderly in the United States take the form of life-contingent payouts (annuities). This suggests that one reason

individuals do not purchase additional annuities is because they are already substantially annuitized.

The functioning of private annuity markets has recently attracted attention as part of the policy debate on "individual accounts" alternatives to current pay-as-you-go, defined benefit Social Security systems. A central policy design issue concerns the way a retiree could spread the resources accumulated in an individual account over his, or his and his spouse's, remaining lifetime. While the treatment of married couples is an important issue in Social Security program design, virtually all of the previous research on annuities has focused on individuals rather than couples as decision-making units. This paper presents new evidence on the structure of joint-life annuity products that insurers currently offer, and it evaluates these annuity contracts from the standpoint of couples.

Two previous studies have investigated the post-retirement consumption behavior of married couples as opposed to individuals. The first, by Kotlikoff and Spivak (1981), focused on the demand for individual annuities by married rather than single persons. This paper showed that if married individuals, or more generally those in an extended family, were able to contract to share resources over their respective lifetimes, then the benefits of purchasing an individual annuity contract were smaller than those for independent individuals. This result arises from the fact that risk sharing within families is a partial substitute for risk sharing in an organized annuity market. Kotlikoff and Spivak (1981) did not consider the demand for joint life annuities by married couples.

A second related study is Hurd's (1999) investigation of optimal consumption by married individuals when both members of the couple face uncertainty about length of life. This paper presents new findings on how couples should structure their consumption in the absence of an annuity market. Hurd's study shows that optimal consumption depends on the structure of the household utility function, and it draws attention to the absence of an agreed-upon framework for modeling the joint decisions of married couples.

Modeling difficulties should not obscure the key role that couples rather than single individuals are likely to play in both public and private annuity markets. In the United States in 1995, 77 percent of men and 43 percent of women over age 65 were married. Most individuals are members of married couples at the beginning of their retirement years, the age at which annuity purchase is most likely. Not surprisingly, given these demographic facts, LIMRA (1997) reports that married persons buy 77 percent of all annuity contracts, and 85 percent of single-premium contracts. Single-life annuity contracts without any provisions for spouses or survivors are unusual. Period certain and joint-and-survivor annuity contracts, both of which provide some spousal protection, are much more common.

In this paper, we explore two issues related to married couples' demand for private annuities. First, we describe the range of joint life annuity products that is currently available. We show that existing annuity products are much more complex than textbook level-premium, single-life annuities, and that these products provide married couples with resource allocation options that cannot be achieved with single-life annuity products alone. Second, we extend previous work on the amount that individuals, or in our case married couples, would be prepared to pay to obtain access to an actuarially fair annuity market. We specify a household utility function and explore the increase in household utility that takes place when a couple is able to participate in an actuarially fair market for joint life annuities. We then translate this increase in utility into an "annuity equivalent wealth," which is the amount of additional wealth that a couple would need to be as well off without access to an annuity market as with such access. This segment of our research extends the dynamic programming analysis of optimal lifetime consumption in MPWB (1999) and Brown, Mitchell, and Poterba (1999) from an individual to a couple. By focusing on joint annuity products, we consider a wider menu of annuity payout options than previous studies.

The paper is divided into four sections. The first describes the structure of currently-available

joint annuity products. Section two presents our algorithm for evaluating the utility gains associated with participating in an actuarially fair joint annuity market. Section three reports our basic results on the “annuity equivalent wealth,” along with sensitivity tests for the impact of changing assumptions about mortality rates and the degree of pre-existing annuitization. The fourth section concludes and suggests several further research issues.

1. The Marketplace for Joint and Survivor Annuity Contracts

To analyze the role of joint life annuity contracts, it is important to recognize that a married couple is concerned with four distinct states of the world. The first state is that in which both members of the couple are alive, and income is used to support the consumption of both spouses. The three other states are one in which the wife is a widow, one in which the husband is a widower, and one in which both spouses are deceased. A couple that makes rational financial decisions will seek to optimally allocate wealth across these four states. In the presence of a bequest motive, the couple may choose to allocate some resources to the state in which both members of the couple are dead. Joint annuities allow couples to make their income stream contingent on these survival states.

There are two primary types of joint annuity contracts. The first is a joint life annuity with a last survivor payout rule. This rule specifies a periodic payment, typically monthly or quarterly, that the annuitants will receive provided both of them are still alive. In addition, it specifies a fraction of this payment, ϕ , that will be paid to the survivor after the death of one member of the couple. The fraction ϕ is usually set at 1, $2/3$, or $1/2$ although LIMRA (1997) reports that insurance companies will provide virtually any fractional survivor benefit at the request of the annuity buyer. In the special case of $\phi = 1$, the annuity provides a level payout stream from the time it is purchased until the death of the last surviving spouse. This is sometimes referred to simply as a “joint and survivor annuity.”

To define an actuarially fair joint and survivor annuity with a last survivor provision, let A denote the fixed nominal benefit that is paid as long as both members of the couple are alive, and let $S_{m,j}$ denote the probability that the husband in the annuity-purchasing couple survives for at least j months after purchasing the annuity. In an analogous fashion, define $S_{f,j}$ as the j -period survival probability for his wife. The equation for actuarial fairness of the premium (P) associated with a joint and survivor annuity contract is:

$$P = \sum_{j=1}^{\infty} \{A * S_{m,j} * S_{f,j} + \mathbf{f} * A * (S_{f,j} * (1 - S_{m,j}) + S_{m,j} * (1 - S_{f,j}))\} / (1 + i)^j. \quad (1)$$

We use i to denote the nominal, after-tax interest rate at which the insurance company discounts future payouts.

The second type of joint life annuity policy is a joint life policy with a contingent survivor benefit. The key distinction between this type of policy and a last survivor policy is that the contingent benefit policy specifies one member of the couple as a primary annuitant. Provided the primary annuitant is alive, the annuity payout is A per period. If the primary annuitant predeceases the secondary annuitant, however, the payout declines to a fraction θ of the primary annuitant's payment. If $\theta = 1$ then the contingent survivor annuity is equivalent to a last survivor annuity with $\phi = 1$, but when $\theta < 1$, the policy differs from a last survivor policy with $\phi < 1$. With a contingent survivor annuity, the order in which the two annuitants die matters for the time profile of benefits, and thus the spouses are treated asymmetrically with regard to the survivor benefit.

The condition that defines an actuarially fair joint life annuity with a contingent survivor benefit, assuming for purposes of illustration that the husband is the primary annuitant, is:

$$P = \sum_{j=1}^{\infty} \{A * S_{m,j} + \mathbf{q} * A * (1 - S_{m,j}) * S_{f,j}\} / (1 + i)^j. \quad (2)$$

Contingent payout annuities are likely to be most attractive to individuals in couples with clear ideas

about their relative consumption needs. Because we do not have a solid basis for specifying such consumption needs, we focus our analysis below on last survivor joint annuities.

Joint life annuities play a potentially important role in completing the market for life-contingent claims. While a joint life annuity can be structured to perfectly replicate any combination of single life annuities by adjusting the survivorship ratios, the reverse is not true. Standard single life annuities limit the extent to which couples can allocate consumption across the four states of survival discussed above.

This can be illustrated by considering two potential allocations across survivorship states. First, consider a couple whose optimal state-contingent allocation has either surviving spouse receive 50% of the income that the couple has while both were alive. This is easily achieved through the use of single life annuities by dividing the wealth in such a way that the annuity income generated for each spouse is equal. Purchasing a joint and 50% survivor annuity can also achieve this.

Now consider a second couple that chooses for the surviving spouse to receive the same income, after the death of the first-to-die-spouse, as the couple received when both were alive. A joint and full (100%) survivor annuity can generate this stream. A portfolio of individual annuities cannot replicate this income flow, however, since income contingent upon one life will cease upon the death of the first-to-die spouse. A couple could use the payments from a pair of single life annuity payments to purchase life insurance, and then use the proceeds of the life insurance on the first-to-die spouse to purchase an additional annuity for the survivor after the death of the first spouse. However, in the presence of transaction costs or any deviation from actuarial fairness in the life insurance market, this may be an expensive way to replicate a joint life annuity.

Joint life annuities represent a relatively small share of the single premium immediate annuity (SPIA) market, although they account for a very large fraction of the annuities written in conjunction with defined benefit pension plans. In 1996, LIMRA (1997) reports that joint annuities accounted for

11 percent of SPIA premium payments. Of this 11 percent, 7 percent (64 percent of all joint annuity premiums) went to purchase joint life annuities with a “period certain” payout. Joint life annuities were only one third as important as single life annuities in the private market: 33 percent of SPIA premiums were devoted to single life annuities, with the majority (24 percent) going to single-life contracts with period certain provisions. Data from LIMRA (1997) also show that roughly half of the annuity purchases in the individual annuity market are not for life-contingent payout streams, but for period certain annuities. This under-recognized fact leads to frequent overstatement of the size of the private annuity market in the United States.

Although joint annuities represent a small fraction of the single premium individual annuity market, they represent a substantial fraction of the group annuity policies that are associated with private defined benefit pension plans. This is partly a result of legislation. ERISA, enacted in 1974, includes “Joint-and Survivor Annuity Requirements” which specify that pension plans must offer a default annuitization option that provides at least a joint and one half survivor annuity. The expected present discounted value of this option must be equal to that of a single life, individual worker annuity. ERISA apparently increased the use of joint and survivor annuities. Holden (1997) reports that while 48.1% of married men with pensions initiated prior to ERISA chose a survivor benefit, 63.9% of married men initiating pensions after 1974 did so. The ERISA joint-and-survivor rule was amended in the 1984 Retirement Equity Act to require a spouse’s notarized signature when the survivor option is not selected. Prior to this amendment, the worker could select a single life annuity without the spouse’s consent or notification. We are not aware of any research that has evaluated how this legislative change affected annuitization decisions.

2. “Annuity Equivalent Wealth”

To obtain an estimate of married couples' willingness to pay for actuarially fair joint annuities, we extend the analytical approach of Brown, Mitchell, and Poterba (1999). We consider a couple with an initial stock of wealth that they can fully annuitize in an actuarially fair joint annuity market. We compute their expected joint utility from complete annuitization. Then, we close the annuity market, and calculate the incremental amount of wealth that the couple would need to have, in the no-annuity case, to achieve the same level of joint expected utility that they would have had with their actual wealth holdings and access to the actuarially fair annuity market.

We note in passing that the "annuity equivalent wealth" is related to, but different from, the "wealth equivalent" calculations reported in MPWB (1999). Those calculations ask how much wealth an individual who does not initially have access to an annuity market would be prepared to give up in order to obtain such access for his remaining wealth. When a household or individual does not have any pre-existing annuity wealth, "annuity equivalent wealth" is simply the reciprocal of the "wealth equivalent." The calculations in MPWB (1999) suggest that a 65 year old single man with no previously annuitized income, i.e., no Social Security or pension annuities, would be willing to give up approximately one-third of his wealth in return for the opportunity to buy an annuity. This "wealth equivalent" of 0.67 would translate into an "annuity equivalent wealth" of 1.50. With pre-existing annuities, however, the simple relationship between these two concepts breaks down, and we cannot relate the various findings.

Two factors create a presumption that the annuity equivalent wealth for married couples will be different from that for single individuals. First, the joint and survivor mortality curve facing a married couple differs from individual life table facing a single individual. Because the mortality experience of spouses is not perfectly correlated, the probability that at least one spouse will be alive after any number of years is always higher than the survival probability for a single individual of the same age as either

member of the couple. The couple's "life expectancy" is longer than that of either individual in the couple.

Second, individuals and couples may have different time paths for their consumption needs. The consumption needs of the couple may change when one member of the couple dies. The direction of such a change is not clear a priori. While direct outlays on food, medical care, and other items may decline when one spouse dies, if the surviving spouse needs to replace the other spouse's unpriced contributions to home production, consumption outlays might actually increase.

Yagi and Nishigaki (1993) show that an individual will find a constant real consumption stream to be optimal only under very specialized conditions. The conditions for such optimality on the part of a couple are even more restrictive. Thus an annuity product that offers a constant payment, regardless of which members of the couple are still alive, may be less attractive to a couple than a level-payout annuity would be to an individual.

2.1 The Household Problem without Annuities

To evaluate the annuity equivalent wealth associated with joint and survivor annuities, we need to model the optimal consumption behavior of married couples. We consider a setting in which the utility of a married couple depends on the consumption of the husband (C^m) and the wife (C^f) according to an additively separable utility function given by:

$$U(C_t^m, C_t^f) = U_m(C_t^m + \mathbf{I}C_t^f) + \mathbf{j}U_f(C_t^f + \mathbf{I}C_t^m). \quad (3)$$

The parameter φ determines the relative weights of the husband's and wife's utility in the household utility aggregate. Kotlikoff and Spivak (1981) used a similar specification in their analysis of the gains from annuitization for married individuals. One could imagine using other utility functions to model household behavior, or allowing for within-household bargaining by husbands and wives. Our analysis

focuses on the case of $\varphi=1$, which implies that $C^m=C^f$ at all dates when both members of the couple are alive.

We extend Kotlikoff and Spivak's (1981) analysis to consider complementarities in consumption, or “consumption externalities,” between the two members of a couple. In particular we allow the utility of the husband to depend on $C^m + \lambda C^f$, and we make a symmetric assumption for the utility of the wife. When $\lambda = 0$, there is no jointness in consumption and only the husband's (wife's) consumption enters his (her) sub-utility function. When $\lambda = 1$, all consumption is joint, and the consumption needs of a surviving spouse are the same as those of the couple.

Another way to model joint consumption is through the budget constraint rather than through the utility function. Specifically, one can set $\lambda=0$ in (3) and allow $(C^m+C^f)/(1+\sigma) = C$, where C is total consumption in the couple's budget constraint and σ is the parameter controlling the degree of joint consumption. In the special case we are considering, with $\varphi=1$ and $U^m = U^f$, these approaches are equivalent and $\sigma = \lambda$.

Varying the degree of jointness (λ) clearly affects the utility level of the couple, both with and without annuities. In general, it also affects the marginal value of additional annuitization. In one important special case, however, that of log utility with equal division of consumption within the couple ($\varphi=1$), λ can be factored out of the objective function and it has no effect on behavior.

We focus in our analytical presentation on the case of $\lambda = 0$, though our numerical results also consider varying degrees of joint consumption as well. We assume that the household utility function is a weighted sum of the sub-utility functions for the husband and the wife, and we further assume that each of these sub-utility functions exhibits constant relative risk aversion. Thus,

$$U_m(C_t^m, C_t^f) = \frac{(C_t^m + \mathbf{1}C_t^f)^{1-b}}{1-b} \text{ and } U_f(C_t^f, C_t^m) = \frac{(C_t^f + \mathbf{1}C_t^m)^{1-b}}{1-b}. \text{ One important simplification}$$

in our analysis is the assumption that within each couple, the risk aversion parameter γ is the same for the husband and the wife. In this case it is straightforward to show that the wife's share of total household consumption will be $1/(1+\phi^{-1/\beta})$. The assumption that the husband and wife have the same sub-utility functions could be relaxed in future work, although there is limited empirical work that can be used to calibrate these functions.

We assume that married couples attempt to maximize the expected present discounted value of their joint utility over their remaining lifetimes. When couples cannot purchase annuities, they will balance the marginal utility of current period consumption against the expected marginal utility of holding additional resources at the end of the period. Formally, this gives rise to a stochastic dynamic programming problem, in which the couple's value function depends on its current wealth. Each spouse faces some risk of dying before the next period, so the couple's value function is more complex than that of a single individual.

We use $V(W_t)$ to denote the value function for a couple with wealth stock W_t at time t , and ρ to denote the time preference rate for each member of the couple. We assume that both members of the couple have the same discount rate. We let q^m (or q^f) denote the one-period mortality rate for the husband (or wife). With probability $(1-q^m)(1-q^f)$ both members of the couple survive until the next period; in this case, the value function at t depends on the discounted value of the same value function at $t+1$. If one member of the couple does not survive, however, then the appropriate value function for the next period is either that associated with the husband, or that associated with the wife. We denote these value functions by $M()$ and $F()$ respectively, and define them as:

$$M(W_t) = \max U(C_t^m) + \frac{(1-q_t^m)*M(W_{t+1})}{(1+r)} \quad (4)$$

and

$$F(W_t) = \max_{\mathbf{j}} U(C_t^f) + \frac{(1-q_t^f) * F(W_{t+1})}{(1+r)} \quad (5)$$

The probability that the husband's value function applies is $(1-q^m)q^f$, while the probability that the wife's value function applies is $q^m(1-q^f)$. There is also a probability q^mq^f that neither member of the household survives until the next period. We assume that the couple derives zero utility of wealth in this state; there is no bequest motive. Generalizing our analysis to allow for intentional bequests would alter this conclusion.

The foregoing considerations yield the couple's value function in period t:

$$\begin{aligned} V(W_t) = \max & U_m(C_t^m + \lambda C_t^f) + \varphi U_f(C_t^f + \lambda C_t^m) + (1+\rho)^{-1}(1-q^m)(1-q^f)V(W_{t+1}) \\ & + (1+\rho)^{-1}(1-q^m)q^fM(W_{t+1}) + (1+\rho)^{-1}q^m(1-q^f)F(W_{t+1}). \end{aligned} \quad (6)$$

In this expression and the similar ones that follow, we suppress age subscripts on all of the mortality rates. In fact, the value function is age-dependent.

When the couple does not have any pre-existing annuity income from a pension or Social Security, its budget constraint when there is no private annuity market has two parts. The first is a non-negativity constraint on wealth at all dates, $W_t \geq 0$ at all t. The second is a recursive relationship that describes the evolution of the couple's wealth:

$$W_{t+1} = (W_t - C_t^m - C_t^f)(1+r). \quad (7)$$

The parameter r is the after-tax return that the couple can earn on its investments.

Finding the consumption rule that determines consumption as a function of wealth in each period is straightforward. In some special cases, the optimal consumption path can be solved for analytically. In more general cases, however, it is not possible to obtain closed form solutions. We therefore rely on

a numerical stochastic dynamic programming algorithm to find the optimal consumption path. We use “years” as the time period for the analysis, and we consider the optimal consumption problem for households in which the husband is age 65. We further assume that no one lives beyond age 115, so there is a fixed last period for our analysis.

To compute annuity equivalent wealth, we need to combine this approach to calculating the couple's lifetime expected utility when it is not possible to purchase private annuities, with an analysis of lifetime utility when such annuity purchases are possible. In the latter case, we assume that the couple annuitizes all of its wealth, even though such a policy may not yield the highest possible level of lifetime utility among all annuitization strategies.

The first step in our annuity equivalent wealth calculation is evaluating the value function when the couple uses all of its wealth to purchase an annuity. We denote the private market annuity flows by A' . Given our assumptions, when a couple uses all of its retirement wealth to purchase a private annuity, it has no remaining non-annuitized wealth at the beginning of the retirement period. The couple may, nevertheless, choose to save some of the income from annuity payouts early in the retirement period, and accumulate a wealth stock in periods following the initial annuitization decision.

When the couple has an annuity policy that offers benefits $\{A'_b, A'_m, A'_f\}$ when both members of the couple, the husband, or the wife are alive, respectively, the value function in (6) is replaced by

$$V(W_t; A'_b, A'_m, A'_f) = \max U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) + (1+\rho)^{-1} \{ (1-q^m)(1-q^f)V(W_{t+1}; A'_b, A'_m, A'_f) + (1-q^m)q^f M(W_{t+1}; A'_m) + q^f(1-q^f)F(W_{t+1}; A'_f) \}. \quad (8)$$

The couple maximizes this value function subject to:

$$W_{t+1} = (W_t + A'_t - C_t^m - C_t^f)(1+r). \quad (9)$$

Once again, the non-negativity constraints on wealth at all dates apply to this problem. In addition, at the beginning of retirement, when the couple has just purchased an annuity, $W_{65} = 0$.

We assume that joint and survivor annuities are priced in an actuarially fair fashion. The couple's age-65 wealth stock before purchasing the annuity, which we denote as W^*_{65} , is related to the private sector annuity payouts, $\{A'_b, A'_m, A'_f\}$, by the equation:

$$W^*_{65} = \sum_{j=65}^{115} \{A'_b * S_{m,j} * S_{f,j} + A'_f * S_{f,j} * (1 - S_{m,j}) + A'_m * S_{m,j} * (1 - S_{f,j})\} / (1 + i)^{j+1-65}. \quad (10)$$

We consider several possible structures on annuity payouts, including a level nominal payout stream while either member of the couple is still alive, and various options with survivor payouts that are less than the payouts while both members of the couple are alive. The value function that results from this calculation, assuming that the husband is 65 years old and that the couple chooses to fully annuitize any retirement wealth, is $V(0; A'_b, A'_m, A'_f)$. The private sector annuity payouts are determined by (10) and the wealth level that the couple had at the beginning of retirement.

The opening of an annuity market, which provides access to insurance that was not otherwise available, will typically raise the couple's utility level. For a couple with a wealth stock at retirement of W^*_{65} , this typically implies that $V(0; A'_b, A'_m, A'_f) > V(W^*_{65}; 0, 0, 0)$ where the feasible private annuity choices are given by (10) above. However, there may be cases in which the structure of payouts in the private annuity market is unattractive relative to the couple's prospective consumption needs. In such cases, our requirement that the couple fully annuitize retirement assets if they participate in the annuity market could make the couple worse off. Our calculations below, however, indicate that this does not occur for most plausible parameter values.

The second step in the annuity equivalent wealth calculation involves finding how much more retirement wealth the couple would need, if it did not have access to an annuity market, to obtain the same expected utility that it would receive if it could fully annuitize its actual retirement wealth (W^*_{65}). We express the "annuity equivalent wealth" as a fraction of the couple's initial retirement wealth by

finding the value α for which

$$V(0; A'_b, A'_m, A'_f) = V(\alpha W^*_{65}; 0, 0, 0). \quad (11)$$

We numerically search for the value α that satisfies this equality. Thus in the first stage of the annuity equivalent wealth calculation, we evaluate a value function once, given a set of parameters, determine the value on the left hand side of (11). In the second step of the calculation, we evaluate the value function corresponding to the no-private-annuity case many times, until we find the wealth level, or alternatively the value of α , that satisfies (11).

2.2 The Household Problem with Pre-Existing Annuities

When the couple receives some income from a pre-existing annuity, the recursive equation for wealth evolution differs from the one specified above, and the value functions also differ. The wealth evolution equation without the purchase of a private annuity now depends on the couple's pre-existing annuity income, which can in turn depend on which members of the couple are alive. Recall that there are potentially three different annuity payouts, depending on whether both spouses are alive (A_b), only the husband survives (A_m), or only the wife survives (A_f). (Note that we use A' to denote privately purchased annuities, and A to denote pre-existing annuities. It would be straightforward to generalize the analysis to period-certain annuities, but we leave that for further work.)

The modified wealth accumulation equation with pre-existing annuities, but no privately purchased annuities, is

$$W_{t+1} = (W_t + A_t - C_t^m - C_t^f)(1+r) \quad (12)$$

where A_t denotes the couple's survivor-contingent annuity flow.

The value functions defined above now also depend on the pre-existing annuity income flows. The new value function for the couple in the absence of privately-purchased annuities, the analogue of (6), is:

$$V(W_t; A_b, A_m, A_f) = \max U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) + (1+\rho)^{-1} \{ (1-q^m)(1-q^f)V(W_{t+1}; A_b, A_m, A_f) + (1-q^m)q^f M(W_{t+1}; A_m) + q^m(1-q^f) F(W_{t+1}; A_f) \}. \quad (13)$$

While finding the values of C^m and C^f that maximize this value function is more difficult than finding the consumption choices that maximize equation (6), both problems can be solved using the same stochastic dynamic programming techniques.

In choosing plausible parameters for pre-existing annuity payouts, we consider the expected present discounted value (EPDV) of this annuity income as a share of the couple's wealth. For a couple that consists of two 65-year-olds, this EPDV is:

$$EPDV = \sum_{j=65}^{115} \{ A_b * S_{m,j} * S_{f,j} + A_f * S_{f,j} * (1 - S_{m,j}) + A_m * S_{m,j} * (1 - S_{f,j}) \} / (1+r)^{j+1-65}. \quad (14)$$

(Note that in our analysis we assume that the return available to an insurance company equals the return available to a couple ($r = i$.)

In our calculations below, we assume that half of the couple's wealth takes the form of pre-existing annuities. We consider two cases with respect to survivor payouts, one in which a surviving spouse receives half as much as the couple received when both members were alive ($A_f = A_m = .5 * A_b$), and one in which the survivor receives two thirds of the couple's benefit. We believe these cases roughly span the current structure of Social Security and private pensions.

Computing annuity equivalent wealth in the presence of pre-existing annuities proceeds in the same way as the calculation without pre-existing annuities. The couple's value function when it is possible to purchase private annuities, and when there are also pre-existing annuities, is

$$V(W_t; A_b, A_m, A_f, A'_b, A'_m, A'_f) = \max U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) + (1+\rho)^{-1} \{ (1-q^m)(1-q^f)V(W_{t+1}; A_b, A_m, A_f, A'_b, A'_m, A'_f) \}$$

$$+ (1-q^m)q^f M(W_{t+1}; A_m, A'_m) + q^f(1-q^f) F(W_{t+1}; A_f, A'_f)\}. \quad (15)$$

The couple maximizes this value function subject to:

$$W_{t+1} = (W_t + A_t + A'_t - C_t^m - C_t^f)(1+r), \quad (16)$$

the non-negativity constraints on wealth at all dates, and the constraint that if the couple purchases a private annuity, $W_{65} = 0$ at the beginning of retirement. The value function that results from this calculation, assuming that the husband is 65 years old and that the couple chooses to fully annuitize any retirement wealth, is $V(0; A_b, A_m, A_f, A'_b, A'_m, A'_f)$.

If the couple cannot purchase an annuity, the value function corresponding to joint utility maximization is given by $V(W_{65}^*; A_b, A_m, A_f)$. The annuity equivalent wealth parameter, α , is defined implicitly in this case by:

$$V(0; A_b, A_m, A_f, A'_b, A'_m, A'_f) = V(\alpha W_{65}^*; A_b, A_m, A_f). \quad (17)$$

As in the case without pre-existing annuities, we numerically search for the value α that satisfies this equality.

Brown, Mitchell, and Poterba (1999) used the annuity equivalent wealth concept to describe the relative value of different annuity schemes, such as level-payment nominal annuities, inflation-indexed annuities, and escalating nominal annuities, for single individuals. The results of the calculation in (14) can be compared with those findings to obtain some insight on the value of annuities to couples rather than single individuals.

2.3 Calibration

To evaluate the annuity equivalent wealth from access to a joint annuity market, we need to parameterize the value functions described above. This section discusses the choice of mortality rates and risk aversion coefficients for our stochastic dynamic programs. Some parameters that are likely to

have an important effect on the value of an annuity, such as the degree of jointness in household consumption, are difficult to calibrate based on the existing literature. For these parameters we will rely on sensitivity analysis in reporting our results.

2.3.1 Mortality Rates

Specifying the mortality rates facing potential annuitants is an essential component of the annuity equivalent wealth calculation. MPWB (1999) explain that there are two mortality tables that could be used to value annuity payouts. The first is the population life table, which is compiled by the Social Security Administration Office of the Actuary and describes the mortality experience of randomly selected individuals in the population. The second is an “annuitant” mortality table, which more accurately captures the mortality experience of individuals who have historically purchased annuity contracts. In both cases it is essential to use a cohort mortality table which describes the mortality experience at different ages for individuals who were born in a given year. (This is distinct from a “period” mortality table that describes the mortality risk facing individuals of different ages in a given year.) Since we are primarily interested in the annuity equivalent wealth for representative couples, we use population cohort mortality tables in our calculations.

We noted above that a key difference between the annuitization choice for a couple and that for an individual is that the joint-and-survivor mortality table differs from that for an individual. Figure 1 illustrates this point. It shows the probability that a male aged 65 will survive to various ages, that a female aged 65 will survive to various ages, and the probability that at least one member of a married couple, both of whom are 65, will survive to various ages. The survivor curve for the couple lies above the individual survival curves, showing that the probability of at least one member of the couple surviving to a given age is larger than that of either individual. For example, the probability of at least one member of a 65-year-old couple living to age 80 is .86, compared with .54 for a single man. The

corresponding life expectancies are 15.7 years for the male, 19.4 years for the female, and 23.1 years for the second-to-die spouse in a married couple.

The married couple survivor curve in Figure 1 lies above the individual survivor curves, and it also has a different shape. There is less probability mass in the tails of the joint length-of-life distribution than in the individual distributions. The standard deviation of the couple's longest life expectancy is 7.8 years, which is lower than that for a man (9 years) or for a woman (9.5 years). This is potentially important because an annuity is usually more valuable when there is greater uncertainty about remaining life length.

2.3.2 Risk Aversion

The second important parameter in our analysis is the coefficient of relative risk aversion, which determines the shape of the sub-utility functions for husbands and wives. A substantial literature in macroeconomics has found levels of risk aversion near unity, which correspond to log utility. Laibson, Repetto, and Tobacman (1998) survey much of this work. Mehra and Prescott (1985) note, however, that much higher levels of risk aversion are needed to explain the large historical return premium of U.S. equities over riskless bonds. In addition, recent survey work, such as that in Barsky, *et al.* (1997), also suggests that household risk aversion levels are higher than unity. Therefore, we will consider a range of values for risk aversion, including 1, 2, 5, and 10. Hurd (1989) shows that if consumers are highly risk averse they will want to guard against having to consume at a low level if they were to live longer than they expected. Similarly, couples will value annuitization more highly when the risk aversion parameter (β) is larger.

3. Results of Annuity Equivalent Wealth Calculations

We compute the utility level that a married couple achieves when it uses all its wealth to

purchase an actuarially fair joint and survivor annuity, as well as the utility level associated with no purchase of such an annuity. We then place these utility levels in perspective using the concept of annuity equivalent wealth. We begin our analysis with a base case of a 65-year old husband with a 62-year old wife, and with no consumption complementarities ($\lambda=0$). The three-year difference in the age of the husband and wife is consistent with U.S. experience according to Social Security actuaries.

Table 1 presents our annuity equivalent wealth results for this “base case couple.” There are two central findings. First, the value of annuitization is lower for the married couple than for a “comparable” single individual. Second, the value of annuitization rises with risk aversion, and with the age of the couple. These results parallel findings for single individuals.

Table 1 reports findings for various sets of parameter choices. The columns of the table show alternative assumptions about ϕ , the ratio of survivor annuity income from the privately-purchased annuity to the income received by the couple when both are alive. We consider the three most common survivor benefit ratios of 1/2, 2/3, and 1. We also consider four different levels of risk aversion, including a relative risk aversion coefficient of 1 (log utility), 2, 5, and 10.

The table also shows three different scenarios with regard to pre-existing annuity income. The top panel corresponds to the case in which there is no pre-existing annuity. The middle panel assumes the couple begins with half its wealth in a pre-existing real annuity that pays a survivor benefit equal to one-half of the couple benefit. This is a stylized representation of the case in which both members of a couple are entitled to equal Social Security benefits, and have half their wealth in Social Security. Upon the death of the first spouse, the survivor will continue to receive only his or her own worker benefit. In the bottom panel, we again assume that half of the couple's wealth is pre-annuitized, but now offer a survivor payout equal to 67 percent of the couple's benefit. This corresponds to a stylized case in which the couple's Social Security benefit consists of a primary worker benefit plus an additional 50

percent in dependent benefits. Upon the death of the first spouse, the Social Security benefit drops to 67 percent of the couple's benefit.

The results in the first panel of Table 1 provide an illustration of our findings. When the actuarially fair private annuity offers a joint life contract with a 50 percent survivor's benefit to our base case couple with log utility, access to annuitization is equivalent to a 17.5 percent increment to the couple's non-annuitized wealth. The annuity equivalent wealth, parameterized by α above, is 1.175. Increasing risk aversion from 1 to 2 raises the annuity equivalent wealth from 1.175 to 1.244, and further increases in risk aversion result in further increases in annuity equivalent wealth.

The results in the upper panel of Table 1 also suggest that the ratio of survivor benefits to annuity payouts when both members of the couple are alive has a modest effect on annuity equivalent wealth, at least at low levels of risk aversion. When the relative risk aversion coefficient is two, for example, varying the survivor payout relative to the couple payout from 0.5 to 1.0 changes the annuity equivalent wealth by only 0.044, i.e. 4.4 percent of initial non-annuitized wealth. All of the annuities that we consider provide a constant nominal payout stream, so different survivor payout structures partly affect the degree to which real benefits decline over time. Without consumption complementarities, the survivor ratio of 0.67 provides the highest annuity valuation for most levels of risk aversion, though 0.5 is preferred for a risk aversion coefficient of 10. As can be seen, full survivor benefits are not optimal for any of the levels of relative risk aversion that we consider. This is because upon the death of the first spouse, the income required to provide the survivor with a given level of consumption declines. Providing full survivor benefits gives too much income to the survivor at the expense of too low a consumption level when both spouses are alive.

The final column of Table 1 provides a comparison to the case of a single male, age 65, who maximizes an individual utility function and who does not have opportunities to pool risk within a

marriage. For all levels of risk aversion, the annuity equivalent wealth for couples is significantly lower than that for single individuals. The difference between individual annuity valuation, and annuity valuation by a married couple, is partly explained by the fact that risk sharing takes place within couples. If one member of the couple lives an unexpectedly long life, there is some probability of inheriting resources from an earlier-to-die spouse. This provides some mortality insurance, even without a formal annuity contract.

The results in the two lower panels of Table 1 indicate how our findings change when the couple has access to a pre-existing annuitized income stream such as Social Security or a defined benefit pension plan. We assume that the pre-existing annuity benefit is indexed to inflation. Comparing the results to the top panel, we see that the couple's annuity equivalent wealth (now computed as a multiple of initially non-annuitized wealth) declines when half of the couple's wealth is already annuitized. The annuity equivalent wealth when the couple has some pre-existing annuity income is the ratio of the couple's non-annuitized wealth that would be required to make them as well off as if they were fully annuitized. For all levels of risk aversion, and for all combinations of survivor benefits, the annuity equivalent wealth is lower when the couple has some pre-annuitized wealth. This suggests that pre-existing annuities reduce the demand for additional private annuities.

Table 2 explores the sensitivity of the results to alternative assumptions about the degree of "jointness" in consumption. This table continues with our base case couple that consists of a 65-year old man married to a 62-year old woman, but it now allows λ , the consumption complementarity parameter, to vary. The first column again reports results for the case of no jointness ($\lambda=0$). The second column assumes that half of all consumption is joint ($\lambda=.5$) and the third column assumes complete jointness in consumption ($\lambda=1$). The jointness parameter λ does not affect the annuity equivalent wealth in the case of log utility, so Table 2 reports results for risk aversion values of 2, 5, and

10. We restrict our attention to the case without pre-existing annuities, the case considered in the first panel of Table 1, to focus on the effect of the private annuity survivor ratio. We vary the survivor ratio on the privately purchased nominal annuity from 0.4 to 1.0 in increments of 0.1.

The jointness parameter clearly affects the desired survivor ratio. Without consumption externalities, a survivor benefit ratio of 0.6 is preferred to the other ratios considered for all levels of risk aversion that we consider. The preferred survivor ratio rises with the jointness parameter, with full survivor benefits being preferred in the case of full consumption externalities ($\lambda=1$). Full jointness means that a surviving widow, for example, needs just as much total income as she and her husband needed when he was alive to sustain a given level of consumption. With full jointness in consumption, two can consume for the price of one, so the death of a spouse does not reduce a household's expenses. Survivor benefits are more attractive when there are positive consumption externalities within a couple.

Table 3 explores the effect of the age of the husband and wife on the annuity equivalent wealth. The table considers two levels of risk aversion, 1 and 5, but it confines attention to $\lambda=0$ and the case in which the privately purchased annuity has a survivor benefit ratio of 0.67. We present results both for the case of no pre-existing annuity wealth, and for the case in which half of wealth is pre-annuitized with a survivor ratio of 0.5. The table shows that the annuity equivalent wealth increases with the age of either spouse. Gaining access to annuities for two 70 year olds with log utility and no pre-existing annuities is equivalent to a 24.2% increase in their wealth, compared to only 11.7% for two 55 year olds. These age effects arise due to differences in mortality risk faced by the individuals. The rate of return on an actuarially fair annuity consists of a mortality premium that is a function of the mortality hazard. Older couples face higher mortality probabilities, and thus have more to gain from annuitization than do younger couples.

4. Conclusions and Future Directions

This paper presents new results on the market for joint annuities and on the utility gains available to married couples who are able to participate in actuarially fair annuity markets. A couple consisting of a 65 year old man and a 62 year old woman find access to actuarially fair joint and survivor annuities roughly equivalent to an 18% increase in non-annuitized wealth, assuming log utility. Their valuation of annuities is even higher if risk aversion is higher or if the spouses are older at the date of annuitization. We confirm previous findings for single individuals that suggest that pre-existing annuity wealth reduces the demand for additional annuitization.

The results suggest that married couples value the opportunity to purchase joint and survivor annuity products less than single individuals value the opportunity to purchase single life annuities. Moreover, at high levels of risk aversion, annuity design features such as the relationship between the annuity payouts for surviving spouses and the benefits paid to the couple when both members were alive can have an important impact on the couple's valuation of the annuity.

Our estimates of the amount that couples would be prepared to pay to obtain joint and survivor annuity products can be contrasted with the estimates of the expected present discounted value (EPDV) of joint and survivor annuity payouts in Mitchell, Poterba, Warshawsky, and Brown (1999). Those estimates suggested that for a 65 year old couple, the EPDV of the average annuity in the marketplace is 84 percent of its initial premium. Because couples find annuities less valuable than single individuals, couples may find that these “load factors” in the private marketplace are significant enough to deter them from annuitizing their resources. Given the importance of married couples in the population age groups that are most likely annuity buyers, this may help to explain the rather limited size of the annuity market.

One of the important issues that we have not considered is the possible relationship between mortality risk of married individuals. Frees, Carriere, and Valdez (1996) document a “broken heart effect” in the mortality of married couples: conditional on the death of one spouse, the mortality risk of the surviving spouse rises. Their calculations suggest that this effect reduces the expected discounted value of joint and survivor annuity payouts by about five percent relative to what they would be without this effect.

A second issue that we have not considered is the impact of bequest motives on the demand for joint and survivor annuities. To the extent that couples value wealth that is left behind to their heirs, this may lessen the value of annuitization. Jousten (1998) discusses in detail how one can model the utility of gifts and bequests for the case of an individual life-cycle consumer. This analysis can be extended to the couples’ context. We have not done this because there is remarkably little empirical guidance regarding the parameterization of the utility of bequest function. One study that estimated the necessary utility parameters for a bequest motive, Hurd (1987), estimated it for individuals and found the marginal utility of bequests to be statistically indistinguishable from zero. Further work exploring the impact of bequests in a couple’s context is left for future research.

A related issue that could explain the limited demand for private annuities is the potentially important role of medical expense uncertainty. Both individuals and couples may be concerned about the risk of future uninsured medical expenses (e.g., long-term care needs), and they may correspondingly be reluctant to annuitize their resources. This explanation for limited annuity demand has not yet been quantified, however, or evaluated using a well-calibrated model for health needs and consumption demands.

A final issue for further study concerns the nature of the utility functions for men and women in married couples. There is some evidence, for example from asset allocation patterns in defined benefit

plans, that women are more conservative investors than men are. If this reflects higher risk aversion, then it may be appropriate to modify our assumption that men and women have the same preferences, and hence sub-utility functions, in our analysis. More generally, it is possible that the couple's joint utility function, which results from bargaining between the husband and the wife, is more complex than our analysis suggests. Further progress in modeling the behavior of couples is likely to await clear empirical evidence that bears on these issues.

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TABLE 1
Annuity Equivalent Wealth For Married Couples and Single Persons

| | Married Couple, Joint Life Annuity with Survivor Ratio | | | 65 Year Old Single Man |
|--|--|-------|-------|---------------------------|
| | 0.5 | 0.67 | 1.0 | |
| No Pre-Existing Annuities | | | | |
| CRRA = 1 | 1.175 | 1.179 | 1.158 | 1.465 |
| CRRA = 2 | 1.244 | 1.247 | 1.205 | 1.576 |
| CRRA = 5 | 1.339 | 1.340 | 1.250 | 1.679 |
| CRRA = 10 | 1.407 | 1.402 | 1.266 | 1.703 |
| Half of Wealth Pre-Annuitized with Survivor Ratio = 0.5 | | | | |
| CRRA = 1 | 1.107 | 1.110 | 1.099 | 1.311 |
| CRRA = 2 | 1.153 | 1.159 | 1.138 | 1.405 |
| CRRA = 5 | 1.229 | 1.237 | 1.189 | 1.530 |
| CRRA = 10 | 1.289 | 1.297 | 1.223 | 1.625 |
| Half of Wealth Pre-Annuitized with Survivor Ratio = 0.67 | | | | |
| CRRA = 1 | 1.111 | 1.106 | 1.082 | 1.311 |
| CRRA = 2 | 1.164 | 1.153 | 1.109 | 1.405 |
| CRRA = 5 | 1.279 | 1.252 | 1.154 | 1.530 |
| CRRA = 10 | 1.368 | 1.301 | 1.170 | 1.625 |

Note: Calculations for the married couple assume that the husband is 65, the wife is 62, and that there are no complementaries in consumption.

TABLE 2
Effect of Consumption Complementary, Measured by λ , on Annuity Equivalent Wealth

| CRRA = 2 | $\lambda=0$ | $\lambda=.5$ | $\lambda=1$ |
|------------------|--------------|--------------|--------------|
| Survivor Ratio = | | | |
| .4 | 1.201 | 1.153 | 1.120 |
| .5 | 1.230 | 1.190 | 1.165 |
| .6 | 1.244 | 1.213 | 1.194 |
| .7 | 1.248 | 1.225 | 1.213 |
| .8 | 1.236 | 1.231 | 1.228 |
| .9 | 1.222 | 1.226 | 1.229 |
| 1.0 | 1.205 | 1.217 | 1.230 |
| | | | |
| CRRA = 5 | $\lambda=0$ | $\lambda=.5$ | $\lambda=1$ |
| Survivor Ratio = | | | |
| .4 | 1.312 | 1.152 | 1.057 |
| .5 | 1.339 | 1.216 | 1.142 |
| .6 | 1.345 | 1.256 | 1.197 |
| .7 | 1.335 | 1.274 | 1.233 |
| .8 | 1.313 | 1.280 | 1.256 |
| .9 | 1.284 | 1.277 | 1.268 |
| 1.0 | 1.250 | 1.264 | 1.272 |
| | | | |
| CRRA = 10 | $\lambda=0$ | $\lambda=.5$ | $\lambda=1$ |
| Survivor Ratio = | | | |
| .4 | 1.388 | 1.035 | < 1 |
| .5 | 1.407 | 1.175 | 1.021 |
| .6 | 1.412 | 1.255 | 1.131 |
| .7 | 1.394 | 1.293 | 1.204 |
| .8 | 1.354 | 1.298 | 1.249 |
| .9 | 1.308 | 1.293 | 1.272 |
| 1.0 | 1.266 | 1.280 | 1.278 |

Notes: All calculations assume that the couple does not have any pre-existing annuity wealth. The parameter λ indicates the degree of consumption complementarity; see text for further discussion. Calculations are for a married couple in which the husband is 65, and the wife is 62.

TABLE 3
Effect of Age Differentials on a Couple's Annuity Equivalent Wealth

| <u>Age of Wife</u> | <u>Age of Husband</u> | | | |
|---|-----------------------|-------|-------|-------|
| | 55 | 60 | 65 | 70 |
| CRRA = 1, Privately Purchased Annuity with Survivor Ratio = 0.67, No Pre-Existing Annuities | | | | |
| 55 | 1.117 | 1.131 | 1.146 | 1.160 |
| 60 | 1.133 | 1.151 | 1.170 | 1.188 |
| 65 | 1.150 | 1.171 | 1.194 | 1.215 |
| 70 | 1.168 | 1.193 | 1.217 | 1.242 |
| CRRA=1, Privately Purchased Annuity with Survivor Ratio 0.67, Half of Wealth Pre-Annuitized with Survivor Ratio 0.5 | | | | |
| 55 | 1.070 | 1.082 | 1.094 | 1.103 |
| 60 | 1.082 | 1.095 | 1.105 | 1.118 |
| 65 | 1.094 | 1.104 | 1.118 | 1.132 |
| 70 | 1.103 | 1.117 | 1.131 | 1.146 |
| CRRA = 5, Privately Purchased Annuity with Survivor Ratio = 0.67, No Pre-Existing Annuities | | | | |
| 55 | 1.216 | 1.243 | 1.261 | 1.283 |
| 60 | 1.253 | 1.281 | 1.320 | 1.339 |
| 65 | 1.286 | 1.334 | 1.366 | 1.411 |
| 70 | 1.322 | 1.374 | 1.433 | 1.475 |
| CRRA=5, Privately Purchased Annuity with Survivor Ratio 0.67, Half of Wealth Pre-Annuitized with Survivor Ratio 0.5 | | | | |
| 55 | 1.161 | 1.178 | 1.184 | 1.193 |
| 60 | 1.181 | 1.193 | 1.216 | 1.252 |
| 65 | 1.193 | 1.221 | 1.265 | 1.291 |
| 70 | 1.215 | 1.265 | 1.294 | 1.326 |

Notes: All calculations assume that there are no consumption complementarities between the husband and the wife.

**Figure 1:
Individual and Joint Survival**

