NBER WORKING PAPER SERIES

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Working Paper 6689
http://www.nber.org/papers/w6689

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1998

Financial support from the Mellon Foundation (Grant 9608), FONDECYT (Grants 1980658 and 1981188), and the Dirección de Investigación, Fac. Cs. Físicas y Matemáticas, University of Chile (to Fischer) is gratefully acknowledged. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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JEL No. D81, H42, H54, L91

ABSTRACT

In recent years several countries have started massive highway franchising programs auctioned to private firms. In these auctions, the regulator typically sets the franchise term and firms bid on tolls, or, alternatively, the regulator sets tolls and the winner is the firm that asks for the shortest franchise term. In this paper we argue that many of the problems that highway franchises have encountered are due to the fact that the franchise term cannot adjust to demand realizations. We propose a new auction mechanism where the firm that bids the least present value of revenue from tolls (LPVR) wins the franchise. With this scheme, the franchise length adjusts endogenously to demand realizations.

Assuming that the regulator is not allowed to make transfers to the franchise holder, and that firms are unable to diversify risk completely due to agency problems, we show that LPVR auctions are optimal, even when the regulator does not know firms' construction costs. Furthermore, for demand uncertainty and risk aversion parameters typical of developing countries, welfare gains associated with substituting a LPVR auction for a fixed-term auction are large (e.g. one-third of the cost of the highway).

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1 Introduction and motivation

There is widespread agreement that most developing countries urgently need massive highway construction programs.[2] Highways have traditionally been viewed as public goods that should be financed and operated by the public sector. However, in recent decades chronic budgetary problems have led governments to neglect the upkeep of existing roads. Moreover, traffic has grown well ahead of their capacity. The task of rebuilding and making new roads is beyond the capabilities of most governments, so that it has become increasingly accepted that private firms should build, finance and operate highways, and that drivers should pay for the costs of building and operating them.[3]

In recent years many countries have started massive highway franchising programs via so-called build-operate-and-transfer (BOT) contracts.[4] Under such a contract a private firm builds and finances the highway and then collects tolls for a long period (usually between 10 and 30 years). When the franchise ends the road reverts to the state. The first franchises were usually awarded in bilateral negotiations, but increasingly, competitive auctions are being used to award them. Typically, the regulator fixes the franchise term, and the road is awarded to the firm that bids the lowest toll; alternatively, the regulator fixes the toll and the winner is the firm that bids the shortest franchise term. Many highways are natural monopolies and the premise that underlies the use of auctions is that they lead to efficient outcomes—competition *for* the field as a good substitute for competition *in* the field (Chadwick [1859], Demsetz [1968], Stigler [1968], Posner [1972]).

Highway franchises have several distinctive features. First, a large fraction of the costs of the franchise are sunk when the road is built and before demand becomes known; operating and maintenance costs are small by comparison. Second, highway franchises, specially in developing countries, usually award a natural monopoly, since there are no close substitutes for the highway.[5] Third, in order to alleviate strained budgets, roads have to be financed by tolls on users. This implies that tolls may have to be set above those that optimally regulate congestion. Fourth, medium- and long-term traffic forecasts are notoriously imprecise so

[2]See for example Irwin et al. (1997).

[3]For example, according to *The Economist*, (February 1, 1997, p. 63): "As many countries have neither the finances nor the managerial resources for the task [of raising infrastructure investment in East Asia], private companies will have to do much of the job."

[4]See Gómez-Ibáñez and Meyer (1993) for a thorough discussion and description of the international experience with road franchising.

[5]Mexico was an interesting exception, where the franchised highways were built parallel to free (but congested) public highways. Perhaps coincidentally, most of these projects had to be rescued by the government.

that there is considerable demand uncertainty, most of it beyond the control of the franchise holder. Moreover, in many cases firms are unable to diversify idiosyncratic risks.[6] Fifth, demand uncertainty implies that optimal tolls are state contingent. Last, while there are information asymmetries on construction costs, there are no significant differences between the quality of demand predictions made by the regulator and firms.

The purpose of this paper is to characterize the full-information socially optimal franchising contract in this framework and to show that it can be implemented with an auction where the winner is the firm that bids the least present value of toll revenue (LPVR). This optimal auction does not require that the planner know the cost of construction.

The planner's problem is to choose demand-contingent tolls and franchise lengths that maximize social welfare subject to making it attractive for risk averse firms to hold the franchise. The only source of incomes for the franchise holder are the toll revenues during the franchise; we call this the *self-financing constraint.* We show that the key feature of the planner's problem is the tradeoff between providing insurance to the franchise holder and setting optimal congestion tolls, where the latter refer to the optimal tolls in the absence of the self-financing constraint. By equalizing revenues across states of demand the planner provides insurance to franchise holders, thereby reducing the revenue they require. Yet this may lead to distortionary tolls in low demand states, thus forcing the planner to weigh the benefits of risk reduction against the costs imposed on toll users by distortionary tolling. This tradeoff is not relevant in one important case: if demand is high enough in all states of demand, optimal congestion tolls generate enough income (in present value) to cover construction costs. In that case the planner fixes non distortionary tolls and chooses the term of the franchise in each state of demand so that the present value of toll revenue equals construction costs.[7] By contrast, when demand is such that in some states the optimal congestion toll does not generate enough income to cover construction costs, it is not optimal to set tolls that fully insure the franchise holder, because distortions in some states are too costly. In that case, the optimal contract leads to a simple classification of states of demand: first, either tolls are higher than optimal congestion tolls and the franchise lasts forever. or the optimal congestion tolls are charged and the franchise lasts

[6] It is a well established fact that private firms and financiers usually refuse to participate unless governments pledge guarantees against commercial risks. If project specific risks could be diversified, there would be no demand for guarantees. See Irwin et al. (1997) for an extensive discussion of government guarantees in private infrastructure projects and Appendix D for a model where agency problems prevent the franchise holder from diversifying idiosyncratic risks.

[7] Our model ignores maintenance and operation costs.

until a predetermined present value of revenue (higher than construction costs) is collected. The latter value is the same across states of demand where non-distortionary tolls are optimal. Second, the revenues of the franchise holder are higher in those states where the optimal congestion toll is charged. Third, in some of the states where tolls are distorted the franchise holder loses money (in present value terms).

In order to implement the contract described above the planner must be able to commit to let the franchise holder lose money in some states of demand. For this reason we call this contract the *optimal commitment contract.* Experience suggests that this seldom happens in practice—contracts are usually renegotiated when demand turns out to be lower than expected.[8] Thus we also study the case where the planner must set tolls that guarantee that the franchise holder receives a normal return in every state of demand, i.e., full insurance. We show that unless optimal tolls are able to finance the road in each state of demand, the franchise holder is given too much insurance, at the cost of introducing more distortions than would be necessary in the commitment contract. We call this contract the *optimal no-commitment contract.*

The main result of the paper is that, even if the planner ignores firms' construction costs, in both cases (with and without commitment) the optimal contract can be implemented using an LPVR auction. In addition, we show that neither the planner nor firms need to know the probability distribution of the states of demand in two important cases: when demand is high in all states and in the case of no-commitment.

As we already mentioned, most highway franchises have been awarded using fixed-term contracts. Our results imply that a fixed-term auction is (almost) never optimal. Furthermore, we use data from Chile, which has embarked in a massive program of highway franchising, to estimate the efficiency gains that would be obtained by using an LPVR auction. These gains are substantial: approximately one-third of building costs.

Our paper is related to the literature on franchise bidding pioneered by Chadwick (1859)

[8]For example, in Spain, 12 concessions were awarded before 1973. In several of these, building costs were 4 to 5 times higher than projected, and traffic was about one-third of original projections. As a result, three firms went bankrupt, two were absorbed into stronger franchise holders, and toll increases and term extensions were granted to various firms by the government; see Gómez-Ibáñez and Meyer (1993, chs. 8, 9 and 10). As another example, Mexico franchised the construction and operation of more than 3,000 miles of highways in the late 1980's and early 1990's. Virtually all concessions were renegotiated after cost overruns and low revenues, with a (declared) cost to the government of US$6 billion. This amount does not include the cost to users due to term extensions, since in several cases the terms more than doubled (see *El Mercurio*, June 17, 1996, p. A8, "Apertura Vial Lleva a Desastre Económico," an article reproduced from the *Los Angeles Times*, and the article in the Mexican weekly *Proceso* of February 12, 1996).

and Demsetz (1968) (see also Stigler [1968] and Posner [1972]).[9] Following this literature, we show how competition for the franchise can be used to regulate a monopoly. Our contribution is to study how demand risk affects the optimal contract, explicitly considering the intertemporal nature of franchise contracts. Our paper is also related to the literature on the optimal regulation of natural monopolies (see, for example, Laffont and Tirole [1993]). We show that when costs are sunk prior to the revelation of the state of demand, a competitive auction can be used to reveal costs. Hence tolls and franchise length can be set to ensure firms the normal rate of return.

The rest of the paper is organized as follows. In section 2 we present the model and the planner's problem. In section 3 we solve the planner's problem. In section 4 we show that an LPVR auction implements the social optimum. Moreover, we show that a fixed-term auction generically cannot implement the optimum. In section 5 we make a quantitative comparison between LPVR and fixed-term auctions. Section 6 concludes and discusses extensions. Several appendices follow.

2 The model and the planner's problem

A benevolent social planner wants to hire a private firm to build a highway whose technical characteristics are exogenous.[10] The firm can only be compensated with toll revenues, as we assume that other sorts of compensation, such as monetary transfers from the planner to the firm, are not allowed. The planner's objective is to maximize the expected present value of driver welfare subject to finding a firm willing to build the road.[11] The road is franchised for a period during which the franchise holder collects tolls. When the franchise ends the road reverts to the state and any future tolls are returned to drivers lump-sum.

There are n possible states of demand. In state i, which occurs with probability $\pi_i > 0$, the marginal benefit of an additional trip when Q trips are made is $B_i(Q)$. We assume that the state of demand becomes known immediately after the road is built, so that demand remains constant through time. The toll charged for using the road in state i is $P_i \geq 0$, and the time-cost of using the road when Q vehicles are on it is $c(Q)$, which is independent of the state of demand. Then $P + c(Q)$ is the generalized travel cost, and, in equilibrium,

[9] But see Williamson (1976, 1985) for a critique.

[10] Thus, in this paper we do not study the problem of choosing the optimal scale and timing of the project.

[11] This objective function assumes that income of users is uncorrelated with the benefit of using the road, so that if users spend a small fraction of their incomes on tolls they will value the benefits produced by the road as if they were risk neutral. See Arrow and Lind (1970).

the number of cars on the road in state i is determined by

$$B_i(Q_i) = P_i + c(Q_i). \tag{1}$$

We impose some technical restrictions on the marginal benefit and cost functions:

$$B_i > 0,\ B_i' < 0 \text{ and } B_i'' \leq 0; \tag{2}$$

$$c,\ c',\ c'' \geq 0; \tag{3}$$

$$\lim_{Q\to\infty} B_i(Q) - c(Q) < 0,\ \lim_{Q\to 0^+} B_i(Q) - c(Q) = \infty. \tag{4}$$

That is, in all states the marginal benefit function is strictly positive, strictly decreasing and concave and the time-cost function is increasing and convex in the number of drivers on the road.[12]

It will be useful to work with a demand function $Q_i(P)$ that is determined from the equilibrium condition (1). In the appendix (see Proposition A.1) we show that this demand function is well defined, concave and strictly decreasing (that is, $Q_i'(P) < 0$, $Q_i''(P) \leq 0$). Moreover, the demand elasticity $\eta_i(P)$ is strictly decreasing with $\eta_i(0) = 0$ and $\eta_i(P_i^M) = -1$, where P_i^M is the monopoly toll in state i.[13]

In state i consumer surplus is given by

$$CS_i(P) \equiv \int_0^{Q_i(P)} B(q)dq - Q_i(P)\left[P + c(Q_i(P))\right], \tag{5}$$

which is assumed to be finite. Since tolls paid by drivers redistribute income between drivers and the franchise holder, the net instantaneous social surplus is

$$G_i(P) \equiv CS_i(P) + PQ_i(P). \tag{6}$$

The function G_i is strictly concave by conditions (2)–(4).[14] It follows that when congestion costs are unimportant $G_i(P)$ is decreasing for all P, and therefore attains its maximum at $P_i^* = 0$. On the other hand, when congestion costs are considerable, $G_i(P)$ has a unique interior maximum at $P_i^* > 0$. In the appendix (see Lemma A.3) we show that when $P_i = P_i^*$, drivers exactly internalize the congestion externality they create. Thus, henceforth we call

[12] Thus, we are assuming that there is no hypercongestion.
[13] See Proposition A.2 in the appendix.
[14] See Proposition A.4 in the appendix.

P_i^* the *congestion toll* in state i.

For each possible state of demand the planner chooses two tolls, one that users pay to the franchise holder during the life of the franchise and a second toll that is collected by the planner after the franchise ends. The latter is returned to users as a lump-sum. The corresponding tolls in state i are denoted by P_i^F and P_i^A, where the superscripts "F" and "A" stand for *franchise* and *after*, respectively. The franchise term in state i is denoted by T_i.

Since we are not interested in construction cost uncertainty, we assume that there are many identical firms that can build the highway at cost $I > 0$. There are no maintenance costs and the road does not depreciate.[15] Firms are risk-averse expected-utility maximizers, with twice-continuously differentiable utility functions u defined over net revenue $\text{PVR}_i - I$, where

$$\text{PVR}_i \equiv \int_0^{T_i} P_i^F Q_i(P_i^F) e^{-rt} dt$$

is the present value of the franchise holder's income in demand state i, discounted at the risk-free interest rate, r. Each firm has an outside option that yields utility $u(0)$.[16]

We assume that a dollar in the pockets of drivers is socially more valuable than in the pocket of the franchise holder.[17] Given this assumption, it is easy to show that there is no loss of generality in assuming that the objective function of the planner does not include the rents accruing to the franchise holder (as in Laffont and Tirole [1993]).[18] Thus, the planner wants to extract all rents from the franchise holder and the firm's participation constraint holds with equality:[19]

$$Eu(\text{PVR}_i - I) = u(0). \tag{7}$$

Since the planner returns the revenue she receives after the franchise ends to users, as

[15] With a minor change in notation all results in this paper can be shown to hold when maintenance costs are proportional to the number of vehicles using the road.

[16] As mentioned in the Introduction, in Appendix D we derive the firm's risk aversion from the agency relation between investors and the franchise holder.

[17] One justification could be social preferences on the distribution of income; another that, particularly in developing countries, many foreign firms participate in the highway business.

[18] The optimality of LPVR does not depend on the assumption that franchise rents are less valuable.

[19] Even if tolls are set at the socially optimal level the franchise holder may earn a rent if the franchise period is too long.

a lump sum, her payoff in state i may be written as:

$$W_i(P_i^F, P_i^A, T_i) \equiv \int_0^{T_i} CS_i(P_i^F)e^{-rt}dt + \int_{T_i}^{\infty} CS_i(P_i^A)e^{-rt}dt + \int_{T_i}^{\infty} P_i^A Q_i(P_i^A)e^{-rt}dt,$$

which after some rewriting, and defining $L_i \equiv e^{-rT_i}$, is equal to

$$\frac{G_i(P_i^F)}{r}(1 - L_i) + \frac{G_i(P_i^A)}{r}L_i - \mathrm{PVR}_i. \tag{8}$$

The planner chooses a toll and franchise-period schedule $(P_i^F, P_i^A, L_i)_{i=1}^n$ to maximize the expected value of (8) subject to the firm's participation constraint (7).

If the planner could make monetary transfers to the franchise holder, she would choose P_i^F and P_i^A equal to the congestion toll P_i^*.[20] Since the participation constraint is no longer relevant after the franchise ends, we have that in all states the planner sets $P_i^A = P_i^*$.[21] Nevertheless, in order to raise revenue and satisfy the participation constraint, the planner may need to distort tolling during the franchise. The properties of the demand function Q_i imply that the optimal toll in state i during the franchise, which we denote by P_i^O,[22] satisfies

$$P_i^M \geq P_i^O \geq P_i^*$$

(see Proposition B.2 in the appendix for a proof). That is, the optimal toll lies between the congestion toll and the monopoly toll. In the remainder of the paper, the following definitions and notation will be useful. First, suppose

$$\frac{P_i Q_i(P_i)}{r} \geq I.$$

Then we say that *the road is self-financing in state i charging toll P_i*. Second

$$\mathrm{PVR}_i^* \equiv \frac{P_i^* Q_i(P_i^*)}{r} \tag{9}$$

is the present value of revenue collected by the franchise holder if the franchise lasts forever and the toll equals the congestion toll. Analogously, PVR_i^M is defined by substituting P_i^M

[20] As taxes are usually distortionary, the optimal toll should be slightly above the congestion toll.
[21] See Proposition B.1 in the appendix for a formal proof of this assertion.
[22] Henceforth the superscript "O" will denote the optimal value of a variable during the franchise period.

for P_i^* in (9). Finally,

$$\text{PVR}_i^O \equiv \frac{P_i^O Q_i(P_i^O)}{r}(1 - L_i^O)$$

is the present value of revenue collected by the franchise holder if both tolls and the franchise term are chosen optimally given the participation constraint.[23] We are now ready to study the planner's problem.

3 The planner's solution

In this section we find the contract that solves the planner's problem, and develop a simple classification of roads based on this contract.

3.1 The *commitment* case

Most highway franchises have been awarded under a contract that fixes a state-independent toll and franchise term before the road is built; that is for all i, j, $P_i^F = P_j^F = P$ and $T_i = T_j = T$. In such fixed-term contracts the government has committed in principle (though often not in practice) to change neither tolls nor the franchise period. This is a special case of a more general contract where the planner commits to a toll and franchise-term schedule $(P_i^F, P_i^A, L_i)_{i=1}^n$ before the realization of demand.[24] In this subsection we characterize the optimal contract within this class.

From (8) we have that the planner solves

$$\text{(10)} \qquad \max_{(P_i^F, P_i^A, L_i)_{i=1}^n} \sum_i \pi_i \left[\frac{G_i(P_i^F)}{r}(1 - L_i) + \frac{G_i(P_i^A)}{r} L_i - \text{PVR}_i\right]$$

subject to the firm's participation constraint (7). Suppose that $\sum_i \pi_i u\left(\text{PVR}_i^M - I\right) \geq u(0)$, that is, that the road is self-financing under monopoly tolls. Then there exists a solution for this problem.[25] The key implication of commitment is that the planner can compel the franchise holder to accept losses in some states, and compensate him with profits in other states; that is, $u(\text{PVR}_i - I) = u(0)$ need only hold on average, not in every state of demand. Commitment gives the planner the possibility of distorting less in low demand states and

[23] Recall that $L_i^O = e^{-rT_i^O}$, where T_i^O is the optimal franchise term in state i.

[24] This assumes that, as mentioned before, the planner can observe and verify the different states of demand.

[25] See Proposition B.3 in the appendix.

compensating the franchise holder with a longer franchise in high demand states, thereby trading off user distortions against the risk borne by the franchise holder. We start with an important lemma that characterizes this trade off and will be used repeatedly below.

Lemma 3.1 *(a) For all states i, $P_i^O > 0$, and $T_i^O > 0$ (i.e., $L_i^O < 1$).*

(b) The following term is independent of the state i:

$$\frac{Q_i(P_i^O)[1+\eta_i(P_i^O)]}{Q_i(P_i^O)[1+\eta_i(P_i^O)] - G_i'(P_i^O)}\, u_i'. \tag{11}$$

With $u_i' \equiv u'(\mathrm{PVR}_i^O - I)$.

Proof See Theorem B.1 in the appendix. ∎

Part (a) of the lemma says that the franchise holder receives positive revenues in all states. Part (b) summarizes the insurance-distortion tradeoff. In the planner's solution, the term in (11) is smaller when firms' revenue is larger (since it is increasing in u') and when tolls are higher (as reflected both by $\eta_i(P_i^O)$ and $G_i'(P_i^O)$, both of which have an absolute value that increases with P_i^O).

We are now ready to characterize the solution when the planner can commit to the toll schedule $(P_i^F, P_i^A, L_i)_{i=1}^n$. The first proposition shows that when $\mathrm{PVR}_i^* \geq I$ for all states i (that is, when in all states of demand the road is self-financing if the congestion toll is charged) then the congestion toll will be charged in all states, the participation constraint will hold in every state of demand and the franchise holder will receive full insurance.[26]

Proposition 3.1 (Full insurance) *Let $\mathrm{PVR}_i^* \geq I$ for all i. Then the optimal franchise contract is such that for all states i, $P_i^F = P_i^*$ and $\mathrm{PVR}_i^O = I$.*

Proof Since $\mathrm{PVR}_i^* \geq I$, the solution is feasible and meets the participation constraint. If $P_i^F = P_i^*$ then $G_i'(P_i^*) = 0$, and from Lemma 3.1 we have that $u_i' = u_j'$ for all i, j, so that $\mathrm{PVR}_i^O = \mathrm{PVR}_j^O$. Finally, $\mathrm{PVR}_i^O = I$ minimizes the transfer to the franchise holder. ∎

The intuition behind this proposition is quite straightforward. First, when in all states of demand the road is self-financing if the congestion toll is charged, there is no need to

[26] In Engel, Fischer and Galetovic (1997a) we prove this result assuming perfectly inelastic demands and no congestion.

distort to satisfy the participation constraint. Second, since the franchise holder is risk-averse, the transfer is minimized when he is given full insurance. Last, since in general $\text{PVR}_i^* \neq \text{PVR}_j^*$, the franchise term is variable; the franchise lasts longer when demand is low.

Proposition 3.1 is not general, because nothing ensures that $\text{PVR}_i^* \geq I$ for all i. For roads such that $\text{PVR}_i^* < I$ in at least some state i, the planner must trade off the benefit of insuring the franchise holder (i.e., that reduced risk implies a smaller transfer to the franchise holder) against the costs of raising tolls and creating a distortion. In what follows we characterize this tradeoff.

When $\text{PVR}_i^* < I$ in at least some state i, states of demand can be classified in two categories: those where the planner sets congestion tolls and those where the planner, optimally, chooses to distort tolls by setting $P_i^F > P_i^*$. We start by studying tolling in a state i where the planner optimally sets $P_i^F > P_i^*$. Suppose that, for the optimal contract, the franchise holder's revenue in state i is PVR_i^O. In principle the planner faces the following tradeoff: given PVR_i^O, a lower toll means a smaller instantaneous distortion, but for a longer term. The next proposition shows that the concavity of G_i implies that the planner has a preference for *toll smoothing*, so that her optimum is to charge forever the lowest possible toll consistent with $\text{PVR}_i = \text{PVR}_i^O$.

Proposition 3.2 (Toll smoothing) *For all states i such that $P_i^O > P^*$, $T_i^O = \infty$.*

Proof To ease notation we drop the subscript i and the superscript F. Consider $P > P^*$ and $L > 0$ (i.e., $T < \infty$) that generate PVR^O. Since we know from Proposition B.3 in the appendix that a solution exists to problem (10), it suffices to show that there exist small reductions in P and L such that the revenue collected does not change and the value function W is larger. So consider the pair (P', L') also generating PVR^O but with $P' = P - dP$ and $L' = L - dL$ with $dP, dL > 0$. In order to ensure that both pairs generate the same revenue we impose $d[PQ(P)(1-L)] = 0$ or

$$(1-L)\frac{Q + PQ'}{PQ}dP = dL. \tag{12}$$

Since $P^A = P^*$, it follows from (8) that

$$\begin{aligned} dW(P, P^*, L) &\propto -G'(P)(1-L)dP - [G(P^*) - G(P)]dL \\ &= -G'(P)(1-L)dP - [G(P^*) - G(P)](1-L)\frac{Q + PQ'}{PQ}dP \end{aligned}$$

$$
\begin{aligned}
&= (1-L)\frac{dP}{P}\left\{-G'(P)P - \frac{Q+PQ'}{Q}[G(P^*)-G(P)]\right\} \\
&> (1-L)\frac{dP}{P}\left\{-G'(P)P - [G(P^*)-G(P)]\right\}.
\end{aligned}
$$

Where the first equality follows from (12) and the inequality from the fact that $G(P^*) - G(P) > 0$ and $PQ' < 0$. Now by concavity of G, $G(P) + (P^* - P)G'(P) > G(P^*)$ so that $-G'(P)P - [G(P^*) - G(P)] > -P^*G'(P) > 0$, where the second inequality follows from the fact that $G'(P) < 0$ since $P > P^*$. Since $dP > 0$, it follows that $dW > 0$. Hence $L^O = 0$. ∎

Next we characterize revenues in those states where congestion tolls are charged.

Proposition 3.3 *For all states i, j such that $P_i^O = P_i^*$ and $P_j^O = P_j^*$,* $\mathrm{PVR}_i^O = \mathrm{PVR}_j^O$.

Proof Note that $G_i'(P_i^*) = G_j'(P_j^*) = 0$. From Lemma 3.1 we have that $u_i' = u_j'$ and hence that $\mathrm{PVR}_i^O = \mathrm{PVR}_j^O$. ∎

The intuition behind this result is quite simple, at least in the case where the optimal franchise length in both states is finite. Consider two states i, j where $P_i^O = P_i^*$ and $P_j^O = P_j^*$, but where $\mathrm{PVR}_i^O < \mathrm{PVR}_j^O$. Then if we extend the franchise a bit in state i and shorten it in state j in such a way that expected revenue does not change, the planner's objective function does not change and the firm's participation constraint becomes slack. It follows that the original franchise terms in states i and j were suboptimal.[27]

The next proposition shows that the franchise holder will collect more revenue in those states where congestion tolls are charged.

Proposition 3.4 *For all states i, j such that $P_i^O > P_i^*$ and $P_j^O = P_j^*$,* $\mathrm{PVR}_i^O < \mathrm{PVR}_j^O$.

Proof Suppose that $P_i^O > P_i^*$ and $P_j^O = P_j^*$. Since $G_i'(P_i^*) = 0$, by Lemma 3.1 we have that

$$
\frac{Q_i(P_i^O)[1+\eta_i(P_i^O)]}{Q_i(P_i^O)[1+\eta_i(P_i^O)] - G_i'(P_i^O)} u_i' = u_j'.
$$

[27]The argument also holds when the franchise term is infinite in the state where revenue is higher. Finally we note that the case where the franchise term is infinite in the state with lower revenue cannot be optimal, since increasing the toll in that state and increasing the franchise length in the other state implies a first order welfare improvement due to risk reduction and, due to the optimality of tolls, only a second order welfare reduction because of an increase in toll distortion.

Since $G_i'(P_i^O) < 0$ and $\eta_i(P_i^O) \geq -1$, the fraction on the LHS is smaller than one. Thus $u_i' > u_j'$ and hence, by concavity of u, $\text{PVR}_i^O < \text{PVR}_j^O$. ∎

Note that Propositions 3.3 and 3.4 imply that if there exists at least one state where optimal tolls are distortionary, then in those states where congestion tolls are charged we have $\text{PVR}^O > I$, that is, the franchise holder makes a profit. It follows that if $\text{PVR}_i^* < I$ then $P_i^O > P_i^*$.[28] Moreover, since the participation constraint must hold with equality, the franchise holder must lose money in some states.

To conclude we show that if in a given state it is optimal to charge the congestion toll, then in all states with higher PVR* it is also optimal to charge the corresponding congestion toll.

Proposition 3.5 *If* $\text{PVR}_i^* \leq \text{PVR}_j^*$ *and* $P_i^O = P_i^*$, *then* $P_j^O = P_j^*$.

Proof See Corollary B.1 in the appendix. ∎

Proposition 3.5 allows us to order states of demand in a simple way. Without loss of generality, assume that $\text{PVR}_1^* \leq \text{PVR}_2^* \leq \ldots \leq \text{PVR}_n^*$ (we will keep this convention in the rest of the paper). It follows that if $P_i^O = P_i^*$, then $P_{i+1}^O = P_{i+1}^*, \ldots, P_n^O = P_n^*$. Conversely, if $P_i^O > P_i^*$, then $P_{i-1}^O > P_{i-1}^*, \ldots, P_1^O > P_1^*$.

To summarize, the preceding results show that when the planner can commit, the structure of the optimal contract $(P_i^O, L_i)_{i=1}^n$ is quite simple. First, either tolls are distorted and the franchise lasts forever, or congestion tolls are charged and the franchise lasts until a given PVR is collected (Propositions 3.2 and 3.3). Second, the revenues of the franchise holder are higher in those states where the road is optimally tolled (Proposition 3.4). Finally, if it is optimal to charge the congestion tolls in some state, they should also be used in all other states that generate more income if that toll rate is set forever. (Proposition 3.5).

3.2 The *no-commitment* case

As mentioned in the introduction, in the real world it is common for franchise contracts to be renegotiated in those states of demand where the franchise holder loses money under the original contract.[29] For political economy reasons, once it becomes apparent that the

[28] The converse is not true: if $\text{PVR}_i^* \geq I$, it does not follow that $P_i^O = P_i^*$.

[29] See footnote 8 for evidence.

franchise holder will suffer losses, governments seem unable to resist pressures to renegotiate. Since the franchise holder will necessarily lose money in some states of demand for any road such that $\text{PVR}_i^* < I$ for some i, it follows that in many cases it may be unrealistic to expect governments to implement the optimal contract. However, as in the case of utilities, the government may be able to precommit to grant the franchise holder at least a normal rate of return *in every state of demand*: that is, after the road is built, for all states i she will set tolls such that $P_i Q_i(P_i) = rI$. In that case, for all i the planner solves

$$\max_{P_i^F, P_i^A, L_i} \frac{G_i(P_i^F)}{r}(1 - L_i) + \frac{G_i(P_i^A)}{r} L_i - \text{PVR}_i \tag{13}$$

subject to

$$\text{PVR}_i = I. \tag{14}$$

The following proposition characterizes the optimum.

Proposition 3.6 *Assume that for all states i: $\text{PVR}_i^M \geq I$. Then:*

(a) if $\text{PVR}_i^* \geq I$, then $P_i^F = P_i^A = P_i^*$, and T_i is set so as to satisfy (14);

(b) if $\text{PVR}_i^* < I$, then the franchise lasts forever ($T_i = \infty$) and the optimal toll is determined by

$$\frac{P_i^O Q_i(P_i^O)}{r} = I.$$

Proof In case (a), the maximum is attained at $P_i^F = P_i^A = P_i^*$ and the self-financing constraint determines the franchise length T_i. The proof of part (b) is similar to that of Proposition 3.2. ∎

Just like in the case when the planner can commit, states of demand can be ordered in a simple way: if $P_i^O = P_i^*$, then $P_{i+1}^O = P_{i+1}^*, \ldots, P_n^O = P_n^*$. Conversely, if $P_i^O > P_i^*$, then $P_{i-1}^O > P_{i-1}^*, \ldots, P_1^O > P_1^*$. Contrary to the case of commitment, however, the optimal no-commitment contract always gives full insurance to the franchise holder. Consequently, when $\text{PVR}_i^* \geq I$ in all states, the solution to problem (13) is identical to the commitment contract: in all states the franchise ends when $\text{PVR}_i = I$. But when $\text{PVR}_i^* < I$ in at least one state of demand, the optimal contract is inferior to the commitment contract. First, the participation constraint must not only hold on average, but in every state of demand. Thus, insurance and distorted tolls cannot be traded off and this contract gives too much insurance and distorts tolls too much. Second, roads for which $\text{PVR}^M < I$ in at least one state of demand will never be built, independently of their profitability in other states, whereas they might have been built under the optimal commitment contract.

Note that the optimal no-commitment contract is analogous in spirit to traditional rate of return regulation, which seeks to set the price of the service so that the utility earns a normal rate of return contingent on the particular realization of demand and cost parameters.[30] The main difference is that the franchise period is limited, a consequence of the assumption that all investments are sunk and need to be made only once.

3.3 A classification of roads

Since in both cases states of demand can be ordered in a simple way, we can introduce an elementary classification of roads. Before doing so it is helpful to define, for given demand schedules and probability distribution across states of nature, the highest construction cost for a given road consistent with firms' participation constraint, both in the case with and without commitment. We denote these quantities by $I^c_{\max}$ and $I^{nc}_{\max}$, respectively. Then $I^c_{\max}$ is the unique I satisfying

$$\sum_i \pi_i u\left(\mathrm{PVR}_i^M - I\right) = u(0),$$

while

$$I^{nc}_{\max} = \min_i \mathrm{PVR}_i^M.$$

It is obvious that $I^c_{\max} \geq I^{nc}_{\max}$, with equality only in exceptional cases.[31] Then one of the following holds:

1. In all states of demand the optimal toll is equal to the congestion toll, that is $P_i^O = P_i^*$ for all i. We call such a road a *high-demand* road.

2. In all states of demand the optimal toll is above the congestion toll, that is $P_i^O > P_i^*$ for all i. We call such a road a *low-demand* road.

3. There exists an index k between 2 and n, such that $P_i^O > P_i^*$ for all $i < k$ and $P_i^O = P_i^*$ for all $i \geq k$. We call such a road an *intermediate-demand* road.

In Appendix B we show that whether the road is low, intermediate or high demand depends only on the values of PVR_1^*, PVR_n^* and I. This classification does not depend on

[30] Since demand is exogenous in the present model, there is no tradeoff between rent extraction and incentives. Moreover, in this section the regulator acts with full information about the relevant parameters. Thus, rent extraction is the sole aim of the regulator and rate of return regulation is appropriate.

[31] A sufficient condition for the inequality to be strict is that, for at least two states of demand, the demand schedules be different at all prices.

whether the planner can commit or not, nor on the probability distribution of the states of demand. When $I \leq \text{PVR}_1^*$ the road is a high-demand road. For $\text{PVR}_1^* < I \leq \text{PVR}_n^*$ the road is an intermediate demand road. As I increases from PVR_1^* to PVR_n^* the number of states of demand with (optimally) distorted tolls increases monotonically. Finally, when $I > \text{PVR}_n^*$ the road is a low-demand road.[32] It follows that there exists a pair of toll functions

$$\begin{aligned} \mathbf{P}_c^O : (0, I_{\max}^c] &\rightarrow \mathbb{R}_{++}^n, \\ \mathbf{P}_{nc}^O : (0, I_{\max}^{nc}] &\rightarrow \mathbb{R}_{++}^n \end{aligned}$$

which completely characterize tolling as a function of I under the optimal franchise contract if, respectively, the planner can or cannot commit.[33] In the case without commitment we have that the ith coordinate of $\mathbf{P}_{nc}^O$ is equal to P_i^* if $\text{PVR}_i^* \geq I$, otherwise it is equal to the unique P satisfying $PQ_i(P) = rI$. No explicit expressions exist for $\mathbf{P}_c^O$ but Appendix B describes how to construct this function.

The function that determines the present value of revenue that the franchise holder receives in each state of demand, in the commitment case, for each feasible value of I, is denoted by

$$\mathbf{PVR}_c^O : (0, I_{\max}^c] \rightarrow \mathbb{R}_{++}^n, \tag{15}$$

and will be useful in the following section. Every coordinate of this function is strictly increasing in I.[34] Since in the no-commitment case the franchise holder receives $\text{PVR}_i = I$ for all states of demand i, the function $\mathbf{PVR}_{nc}^O : (0, I_{\max}^{nc}] \rightarrow \mathbb{R}_{++}^n$ is constant and equal to I. And, as T_i^O can be directly inferred from P_i^O and PVR_i^O, the pairs $(\mathbf{P}_c^O, \mathbf{PVR}_c^O)$ and $(\mathbf{P}_{nc}^O, I)$ completely characterize, respectively, the optimal commitment and no-commitment contract.

4 Least-Present-Value-of-Revenue auctions

The informational requirements needed to implement the optimum are quite formidable, because the planner needs to know construction costs I. In this section we assume that

[32]Theorems B.2–B.4 in the appendix provides the proofs for the commitment case. The no-commitment case is trivial.

[33]The set of strictly positive real numbers is denoted by $\mathbb{R}_{++}$; the set of n-tuples of elements in $\mathbb{R}_{++}$ by $\mathbb{R}_{++}^n$.

[34]A formal proof is provided in Corollary B.3 in the appendix.

the planner does not know I (but knows $(\pi_i, Q_i)_{i=1}^n$ and u) and describe a new auction mechanism that implements the planner's optimum.[35] The mechanism, which we call a Least-Present-Value-of-Revenue (LPVR) auction, is as follows:

- The planner announces a function $\mathbf{P} : \mathbb{R}_{++} \to \mathbb{R}^n_{++}$ which assigns a toll schedule in every possible state of demand to the winning bid.
- Each one of the $l \geq 2$ identical firms participates in the auction if there exists a bid that satisfies its participation constraint.[36] If firm k participates, its bid is denoted by $\beta_k \in \mathbb{R}_{++}$, $k = 1, \ldots, l$.
- The franchise is won by the firm that submits the lowest bid, $\beta_W \equiv \min_k \beta_k$; if there is a tie between s firms, each firm wins the franchise with probability $1/s$. If no firm participates, we say the auction is *non contested.* The following points apply if there is at least one bidder.
- The planner observes the state of demand (after the road is built). If state i attains she fixes the toll equal to the ith coordinate of $\mathbf{P}(\beta_W)$.
- The franchise ends when $\text{PVR}_i = \beta_W$ or lasts forever if it never attains a present value of revenue equal to β_W.

We assume that $I \leq I^c_{\max}$ in the commitment case and that $I \leq I^{nc}_{\max}$ in the case without commitment, for otherwise there exists no toll schedule that will attract bidders.

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[35]The planner needs to know u, otherwise she could not design a mechanism that meets the franchise holder's participation constraint. Moreover, it is reasonable to assume that firms do not have an informational advantage over the planner about traffic demand. First, even when roads are franchised, in most cases governments decide which roads are built. Second, franchising has been introduced only recently, so that up to now governments have built most roads.

[36]In the case with commitment, and denoting the firm's bid by β, this means that

$$Eu(\text{PVR}_i(\beta) - I) \geq u(0).$$

Where, as argued below,

$$\text{PVR}_i(\beta) = \min\left(\frac{P_i(\beta_W)Q_i[P_i(\beta_W)]}{r}, \beta_W\right).$$

And in the case without commitment it means that for all states i

$$\text{PVR}_i(\beta) \geq I.$$

Consider an LPVR auction. If the planner announces a toll schedule $\mathbf{P}$ and the winning bid is β_W, the present value of the revenue that the franchise holder receives in state i is

$$(16) \qquad \min\left(\frac{P_i(\beta_W)Q_i[P_i(\beta_W)]}{r}, \beta_W\right).$$

That is, either β_W is collected in finite time and revenue is equal to β_W, or the franchise lasts forever in which case the revenue collected equals $P_iQ_i(P_i)/r$. Typically, the length of the franchise period is variable, unless $(P_i(\beta_W)Q_i[P_i(\beta_W)]) \leq r\beta_W$ for all i, in which case the franchise lasts indefinitely in all states of demand.

The next proposition is the main result of this paper: even when the planner does not know I, she can use an LPVR auction to implement the optimum, in both the commitment and no-commitment cases. The following definitions will be needed in what follows. As mentioned at the end of Section 3, every coordinate of $\mathbf{PVR}_c^O$, hence the largest among them (which we denote $\max_j \mathbf{PVR}_c^O$), is strictly increasing. It follows that the inverse function $\mathcal{I} \equiv [\max_j \mathbf{PVR}_c^O]^{-1}$ is well defined. It takes the largest revenue, over all possible states, in the commitment case, and associates to it the level of investment defined by (15), given that the toll function $\mathbf{P}_c^O$ is used. Consider the function $\overline{\mathbf{P}}_c : \mathbb{R}_{++} \to \mathbb{R}_{++}^n$, which is equal to $\mathbf{P}_c^O \circ \mathcal{I}$ over $(0, \max_j \mathrm{PVR}_j^M]$, and equal to $\mathbf{P}^M \equiv (P_1^M, \ldots, P_n^M)$ otherwise. This is a well defined function that associates the optimal toll to each state of the demand, for all feasible toll revenue levels, and the monopoly toll in the rest of the domain. Similarly, when the planner cannot commit, we define the ith coordinate of $\overline{\mathbf{P}}_{nc}$ evaluated at β as P_i^* if $\mathrm{PVR}_i^* \geq \beta$. Otherwise it is equal to the unique P satisfying $PQ_i(P) = r\beta$ or, should such a P not exist, equal to P_i^M.

Proposition 4.1 *(a) Assume that the planner can commit and $I \in (0, I_{\max}^c]$. Then an LPVR auction implements the social optimum if the planner announces $\overline{\mathbf{P}}_c : \mathbb{R}_{++} \to \mathbb{R}_{++}^n$. (b) Assume that the planner cannot commit and $I \in (0, I_{\max}^{nc}]$. Then an LPVR auction implements the social optimum if the planner announces $\overline{\mathbf{P}}_{nc} : \mathbb{R}_{++} \to \mathbb{R}_{++}^n$.*

Proof (a) If $I = I_{\max}^c$ then for any $\beta \geq \max_j \mathrm{PVR}_j^M$ the franchise holder receives PVR_i^M in all states i, and his expected utility is

$$\sum_i \pi_i u(\mathrm{PVR}_i^M - \overline{I}) = u(0).$$

Moreover, no firm will bid $\beta < \max_j \mathrm{PVR}_j^M$ because it would violate the participation constraint. It follows that the winning bid, β_W, is equal to $\max_j \mathrm{PVR}_j^M$, thereby achieving the planner's solution for this case.

Next assume $I \in (0, I^c_{\max})$. Since $\overline{\mathbf{P}}_c \equiv \mathbf{P}^O_c \circ \mathcal{I}$ over $(0, \max_j \mathrm{PVR}^M_j]$, it is sufficient to show that the winning bid, β_W, will be equal to $\max_j \mathrm{PVR}^O_j(I)$.

Consider first the case when $\beta_W > \max_j \mathrm{PVR}^M_j$. From (16) and the definition of PVR^M_i it follows that in each state the winner of the auction receives PVR^M_i, so that his expected utility is

$$\sum_i \pi_i u(\mathrm{PVR}^M_i - I) > u(0).$$

But this cannot be a Nash equilibrium, since a firm could bid slightly below β_W and still make a profit.

Now assume $\max_j \mathrm{PVR}_j(I) < \beta_W < \max_j \mathrm{PVR}^M_j$. Then the winner's expected utility is

$$\sum_i \pi_i u\left(\mathrm{PVR}^O_i\left(\mathcal{I}(\beta_W)\right) - I\right) > \sum_i \pi_i u(\mathrm{PVR}^O_i(I) - I) = u(0). \tag{17}$$

Where the inequality follows from the fact that $\mathcal{I}(\beta_W) > I$ since $\beta_W > \max_j \mathrm{PVR}_j(I)$ and $\mathcal{I}$ is strictly increasing, and $\mathbf{PVR}^O_c$ is also strictly increasing in every coordinate. But this cannot be a Nash equilibrium either, because a firm could bid slightly below β_W and still make a profit.

Last, an expression similar to (17) shows that $\beta_W < \max_j \mathrm{PVR}^O_j(I)$ cannot be a Nash equilibrium since it would violate the participation constraint.

Thus in any Nash equilibrium when the planner can commit, the winner must bid $\beta_W = \max_j \mathrm{PVR}^O_j(I)$. Moreover, at least two firms must make this bid for otherwise it would pay for the winner to unilaterally deviate and bid slightly above this value. Last, it follows from the argument above that in such a candidate equilibrium no firm can gain by unilaterally deviating.

(b) Since $\mathrm{PVR}^O_i = I$ for all i, for $\beta = I$ we have that $\mathrm{PVR}_i(\beta)$ defined via

$$\mathrm{PVR}_i(\beta) \equiv \min\left(\frac{P_i Q_i(P_i)}{r}, \beta_W\right),$$

is equal to I, and therefore the participation constraint holds for the winning firm with equality:

$$\sum_i \pi_i u(\mathrm{PVR}_i(\beta) - I) = u(0). \tag{18}$$

Furthermore, for any $\beta < I$ the left hand side of (18) is smaller than $u(0)$ therefore violating the participation constraint. It follows that the winning bid is $\beta_W = I$.∎

There are various features of Proposition 4.1 that are worth mentioning. First, with an LPVR auction the planner does not need to know the construction cost I to implement the optimum, because competition will force firms to reveal it in the auction. The planner can then use the winning bid to set tolls that are optimal subject to the self-financing constraint. The endogeneity of the franchise period, which ends when $\text{PVR}_i = \beta_W$, takes care of rent extraction. Thus, an LPVR auction enables the planner to optimally regulate the rate of return of the franchise.

Second, when the road is self-financing in all states of demand charging the congestion toll, or when the planner cannot commit, neither firms nor the planner need to estimate the state probabilities π_i. Firms only need to know that, $\min_j \text{PVR}_j^* \geq I$ (case with commitment) or $\min_j \text{PVR}_j^M \geq I$ (case without commitment) in order to bid. In both cases the winning bid is equal to the construction cost I. Moreover, to implement the optimum the planner does not need to know u either, since the franchise holder is granted full insurance. Thus LPVR auctions greatly reduce the chances of a winner's curse due to overly optimistic traffic projections.[37]

Finally, fixed-term auctions, which are the standard highway auction mechanisms throughout the world, are optimal only if PVR_i^* is the same across all states of demand and the common value is larger than I. Thus generically fixed-term auctions are suboptimal.[38] Furthermore, as we show in the next section, not only are LPVR auctions better than their fixed-term counterpart, also the welfare differences involved are important.

5 LPVR and fixed-term franchises compared

As we mentioned before, most highways that have been franchised around the world have been awarded under a fixed-term contract. In this section we develop a procedure to quantitatively compare LPVR auctions with fixed-term auctions and apply it to data from Chilean highways to obtain estimates of the savings involved in using an LPVR auction (a massive highway franchising program is currently underway in Chile, see Engel, Fischer and Galetovic [1996]). Since we do not have data to estimate demand elasticities we work with a simplified version of the model where demand in each state is perfectly inelastic.

[37]It should be stressed that LPVR auctions do not prevent a winner's curse due to an overly optimistic estimate of construction costs.

[38]For a formal proof see Propositions C1 in the appendix.

Uncertainty comes from the fact that demand depends on user income, whose growth is stochastic. Given that tolls play no allocational role in this setting, we also assume that the toll is the same in all states. Finally, we assume that tolls are high enough to finance the road in each state of demand.

5.1 Model

In a fixed term auction the planner can either set the franchise term T and the auction is won by the firm who bids the lowest toll; or it can fix a toll P and the auction is won by the firm who bids the shortest franchise term. In both cases Bertrand-Nash competition implies that the following identity must hold in equilibrium:

$$\sum_i \pi u(P \cdot \text{PVQ}_i(T) - I) = u(0). \tag{19}$$

Where $\text{PVQ}_i(T)$ denotes the present value of traffic flow in state of demand i,[39] and $\text{PVR}_i = P \cdot \text{PVQ}_i$. Note that if the term of the franchise is fixed, PVQ_i varies with the state of demand. Thus with a fixed-term franchise the franchise holder cannot be offered full insurance. By contrast, an LPVR auction gives full insurance to the franchise holder.[40]

Let $\zeta(T) \equiv E[\text{PVQ}_i(T)]$ be the expected present value of traffic flows if the term of the franchise is T, and let $\sigma^2(T) \equiv E\left[[\text{PVQ}_i(T) - \zeta(T)]^2\right]$ denote the corresponding variance. Proposition 5.1 calculates the risk premium charged by the franchise holder in a fixed-term auction.

Proposition 5.1 *To a first order approximation, the risk premium charged by the franchise holder in a fixed-term franchise is*

$$\left(\frac{CV\sqrt{A/2}}{1 - CV\sqrt{A/2}}\right) I. \tag{20}$$

Where A denotes the coefficient of relative risk aversion (evaluated at $P\zeta - I$) and $CV \equiv \sigma/\zeta$ denotes the coefficient of variation of the present value of traffic flows.

Proof Given T or P, equilibrium tolls or franchise terms are determined by condition (19). A first-order Taylor expansion of the RHS of (19) and a second-order Taylor expansion of

[39] Note that Q_i is no longer a function of P_i. Also note that, in contrast with the preceding sections, we do not assume that uncertainty is resolved in the first period.

[40] Where we assume that the road is self-financing under the monopoly toll.

the LHS, both around the risk premium $P\zeta(T) - I$, lead to:

$$\sum_i \pi_i \left[\bar{u} + P(\text{PVQ}_i - \zeta)\bar{u}' + \frac{1}{2}P^2(\text{PVQ}_i - \zeta)^2\bar{u}''\right] \cong \bar{u} - (P\zeta - I)u'.$$

Where $\bar{u} \equiv u(P\zeta(T) - I)$, $\bar{u}' \equiv u'(P\zeta(T) - I)$ and $\bar{u}'' \equiv u''(P\zeta(T) - I)$. It follows that $-\frac{1}{2}P^2\sigma^2\frac{\bar{u}''}{\bar{u}'} \cong P\zeta - I$, and hence, multiplying both sided by $P\zeta - I$

(21)
$$\frac{1}{2}P^2\sigma^2 A \cong (P\zeta - I)^2,$$

which leads to

$$P \cong \frac{I}{\zeta[1 - CV\sqrt{A/2}]}.$$

Substituting P back into (21) and taking the square root yields (20) which completes the proof.∎

Now consider an LPVR auction. If tolls are set high enough to make the road self-financing in all states, then the following corollary follows trivially:

Corollary 5.1 *If the toll P is fixed so that the road is self-financing in all states, then expression (20) is also the expected value of the reduction in toll income for the franchise holder in a competitive auction.*

5.2 Empirical implementation

We calculate risk premia for values of A between 1.0 and 3.0 (see Table 1).[41] We obtain CV as follows. We assume that traffic flows increase according to

$$Q_{t+1} = e^{g_t}Q_t$$

and define

(22)
$$\text{PVQ} \equiv \sum_{t=0}^{T-1} e^{-rt}Q_t.$$

There are two sources of uncertainty: the annual growth rates of the traffic flow, g_t, and the initial traffic flow, Q_0. We assume that annual growth rates are independently distributed and satisfy

$$g_t \equiv (\eta_y + \varepsilon_t^{\eta})(g_y + \varepsilon_t^M + \varepsilon_t^m).$$

[41]These values are representative of those estimated in the literature.

Where η_y denotes the average income elasticity of traffic flows, ε_t^η are random shocks that affect this elasticity, g_y is the average growth rate of GDP, and ε_t^M and ε_t^m are, respectively, the variations in this rate due to macro- and micro-economic factors. The parameter η_y is taken as 1.6, the estimated income elasticity of traffic flows in Chile in the period 1985–1995; g_y is set equal to 0.06, the average rate of growth of Chile's GDP over the same period; ε_t^η, ε_t^M and ε_t^m are assumed to be mutually independent and uncorrelated over time, following a normal distribution with zero mean and standard deviations of, respectively, 0.2, 0.02 and 0.04. The standard deviations assumed for macro- and micro-economic risk are consistent with the growth rates of national and regional GDP in Chile over the 1985–1995 period.[42] The variation of Q_0 cannot be estimated from actual data. Thus, as in the case of the coefficient of relative risk aversion, we calculate risk-premia for values of the coefficient of variation of initial traffic flows within a certain range, in this case between 0.05 and 0.25.

If the length of the franchise (T), the discount rate (r), the relative risk aversion coefficient (A) and the coefficient of variation of Q_0 are all given, the coefficient of variation of the sum (22) can be estimated by simulating paths for g_t. We assume that $T = 20$ years (several highways in Chile were franchised with that term) and $r = 0.06$ (this has been close to the average real rate paid by a 20-year bond issued by the Central Bank during the nineties). CV can be calculated assuming that vehicle flow growth rates are independent from their initial levels and holding constant the coefficient of variation of Q_0.[43]

Table 1 shows the savings to users as a percentage of the initial investment in the highway, for alternative combinations of the coefficient of variation of Q_0 and the relative risk aversion coefficient, A.[44]

It can be read from Table 1 that if the coefficient of risk aversion of firms is 2 and the coefficient of variation of Q_0 is 0.15, then the risk premium charged by the franchise holder if the term is fixed is approximately one-third (32.9%) of the initial investment. The median of the values in the table is 32.6%—the mean is even higher. With a discount rate of 8% instead of 6%, the median is 31.1%.

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[42] The standard deviations for ε_t^M and ε_t^m are obtained decomposing yearly regional GDP growth rates into the sum of a common component (equal to the average growth rate across regions) and an idiosyncratic component (the residual). The standard deviation of the common component is 1.82%, the standard deviation of idiosyncratic shocks varies between 2.79% (1989–1990) and 5.75% (1993–1994) with an average of 4.21% over the period considered. We thank Raimundo Soto for providing the regional GDP data.

[43] Here we use the result that relates the coefficient of variation of the product of two independent variables, X and Y, to the coefficient of variation of the individual variables: $CV_{X \cdot Y}^2 = CV_X^2 + CV_Y^2 + CV_X^2 \cdot CV_Y^2$.

[44] Each value in this table is based upon a coefficient of variation of the sum (22) obtained from 25,000 simulations. This leads to a relative approximation error smaller than 0.4%.

Table 1: SAVINGS AS A PERCENTAGE OF ORIGINAL INVESTMENT

		Coef. Rel. Risk Aversion				
		1.0	1.5	2.0	2.5	3.0
	0.05	16.6	21.1	25.2	29.0	32.7
CV	0.10	18.4	23.5	28.2	32.6	36.8
of	0.15	21.2	27.3	32.9	38.3	43.5
Q_0	0.20	24.8	32.2	39.1	45.8	52.5
	0.25	29.3	38.4	47.2	55.9	64.6

6 Conclusion

In this paper we have shown that fixed-term contracts, which are commonly used to franchise highways, do not assign demand risk optimally. We characterized the optimal risk sharing contract and showed that it can be implemented with a fairly straightforward mechanism—an LPVR auction—even when the planner ignores construction costs. Finally, we showed that the welfare gains that can be attained by replacing fixed-term auctions with LVPR auctions are substantial.

Throughout the paper we focused on the risk sharing properties of alternative highway franchising contracts. Worldwide evidence with highway franchising suggests that there are additional characteristics of these contracts that should be considered. We comment on them briefly in what follows.[45]

The actual experience of countries that have franchised highways to the private sector has often been unhappy. Two problems that have been prominent: private firms and financiers usually refuse to participate unless governments pledge guarantees against commercial risks;[46] and franchise holders are usually able to renegotiate and shift losses to taxpayers and users whenever they get into financial trouble.[47] As we have argued elsewhere (see Engel, Fischer and Galetovic [1997b]), government guarantees and renegotiations are undesirable, because they are not accounted for in the budget, blunt the incentives to be efficient, encourage firms with experience in lobbying to lowball in the expectation of a

[45]The presentation is at an intuitive level since we are currently working on formalizing the insights we describe.

[46]For example, for nine out of ten highways franchised in recent years in Chile, the government provided a guarantee that the revenue would equal 70% of construction and maintenance costs. See Irwin et et al. (1997) for more examples.

[47]For evidence, see footnote 8.

future renegotiation, and make white elephants more likely.[48] We believe LPVR franchises moderate these pitfalls.[49] By reducing demand risks, they reduce the demand for guarantees. Moreover, the fact that each firm's bid reveals the income required to earn a normal profit reduces the scope for post-contract opportunistic renegotiations, since any wealth transfer by the government must take the form of a cash transfer whose amount can be readily understood by the public and compared with the initial winning bid. For the same reason, it should be politically more difficult for the government to exploit the franchise holder by changing the original contract, because the winning bid is a clear and observable benchmark that makes it easy to compute any wealth loss sustained by the franchise holder.

LPVR auctions should also be more flexible than their fixed term counterparts. For example, if the regulator wishes to change tolls during the franchise in order to reduce congestion due to unexpected demand growth, she can change tolls (within a reasonable range) without affecting the franchise holder's revenue.[50] A second example illustrating the flexibility of LPVR auctions occurs if, for some reason, the franchise needs to be terminated ahead of time. In this case there is a simple and fair compensation for the franchise owner, namely the difference between the winning bid and revenue collected so far (minus expected maintenance costs). This should be contrasted with the case of a fixed term franchise, where any estimate of the expected profits during the remainder of the franchise is subject to dispute.[51]

LPVR auctions do not offer a solution to all the problems that arise in franchising. This is particularly relevant when there exists a tradeoff between risk and incentives, which we have not studied here. An LPVR contract reduces much of the undesirable demand risk borne by the franchise holder, but, at the same time, it provides insufficient incentives to exert effort in demand- and quality-enhancing activities (e.g. building a road of the right standard, maintaining it adequately, or providing expedite service at toll booths). In the case of monopoly highways, there appear to be few demand enhancing activities, so omitting the effects of incentives appears reasonable. Nevertheless, as Tirole (1997) has

[48]Where by white elephant we mean a road with negative net present social value.

[49]And are therefore more robust to Williamson's (1976, 1985) critique of franchise bidding.

[50]Profits do change, since the franchise holder saves on operational and maintenance costs, yet this effect is considerably smaller than under a fixed term franchise and should be outweighed by the benefit of not having to commit to a toll schedule ahead of time.

[51]In early 1997 the government of Argentina announced it wanted to end airport franchises in order to reauction them under new terms. These were fixed-term franchises. Estimates of adequate compensation for franchise holders varied between US$400 million (government estimates) and US$40 million (former Economics Minister Domingo Cavallo's estimates). See *El Mercurio*, February 6, 1997.

stressed, this suggests that LPVR contracts should be complemented with other regulatory innovations, such as third parties who verify minimum quality standards, and appropriate fines for non-compliance. In the case of highway franchises this should not be a major problem since objective measures for road and service quality can be defined and verified at a low cost.

Finally it is interesting to mention that LPVR auctions are not only a theoretical construct. An LPVR auction was used in March of 1998 in Chile to franchise the Santiago-Valparaíso highway, one of the main roads in the country (estimated cost of approximately US$300 million). The toll schedule was fixed in advance (in real terms) as was the discount rate. Five firms participated in the auction and the winning bid was *below* estimated construction and maintenance costs, probably reflecting the fact that the discount rate set by the regulator—equal to 4% above the risk free rate—was above firms' true discount rates for the relatively low risk level associated with LPVR auctions. Also, firms were given the option to buy government insurance against demand risks, but the winner declined the offer.

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Appendices

A Concavity of G_i

In this appendix we prove concavity of the net instantaneous social welfare function, G_i. The notation and definitions are those given in the main text.

Proposition A.1 (Existence of a demand function) *Given a price P the demand schedule in state i, $Q_i(P)$, is determined implicitly by*

$$B_i(Q_i(P)) - c(Q_i(P)) = P. \tag{23}$$

We also have:

$$\begin{aligned} Q'(P) &= [B'(Q(P)) - c'(Q(P))]^{-1} < 0, \\ Q''(P) &= -\frac{[B''(Q(P)) - c''(Q(P))]}{[B'(Q(P)) - c'(Q(P))]^3} \leq 0. \end{aligned} \tag{24}$$

Where in the last two expressions we dropped the subscript i for ease of presentation.[52]

Proof Since the generalized travel cost is $P + c(Q)$, in equilibrium the number of cars on the road is determined by (23), as long as there exists a Q satisfying this condition. That such a Q exists follows from the fact that the left hand side of (23) is decreasing in Q and that $B_i(Q) - c(Q)$ covers the positive real line as Q varies (due to assumption (4)).

The expressions for $Q'(P)$ and $Q''(P)$ follow from implicitly differentiating (23) with respect to P. ∎

Definition A.1 (Net Social Surplus, Elasticity) *It will be helpful to define net social surplus as a function of Q:*

$$\tilde{G}_i(Q) \equiv \int_0^Q B_i(q)dq - Qc(Q).$$

We define the price-elasticity of demand and (instantaneous) revenue in state i at price P as

$$\eta_i(P) \equiv \frac{PQ_i'(P)}{Q_i(P)}$$

[52] We will do this often throughout this appendix.

and

$$\mathcal{R}_i(P) = PQ_i(P).$$

Lemma A.1 *The net social surplus, as a function of P, satisfies:*

$$G'(P) = Q(P)[\eta(P) - c'(Q(P))Q'(P)], \tag{25}$$

$$G''(P) = \frac{-(B'' - c'')(B - c - Qc') + (B' - c')(B' - 2c' - Qc'')}{(B' - c')^3}. \tag{26}$$

Where we have omitted the subscript i and all functions on the right hand side of (26) are evaluated at $Q(P)$.

Proof From (5) and (6) it follows that

$$G_i(P) = \int_0^{Q_i(P)} B_i(q)dq \; - \; Q_i(P)c(Q_i(P)).$$

Differentiating both sides of this identity with respect to P and rearranging terms leads to (25). Differentiating (25) with respect to P leads to:

$$G''(P) = Q''(P)[P - c'(Q(P))Q'(P)] + Q'(P)[1 - Q'(P)c'(Q(P)) - Q(P)Q'(P)c''(Q(P))].$$

Substituting $Q'(P)$ and $Q''(P)$ by the expressions in Proposition A.1 leads to (26). ∎

Lemma A.2 *The social surplus, as a function of Q, satisfies:*

$$\tilde{G}'(Q) = B(Q) - c(Q) - Qc'(Q), \tag{27}$$

$$\tilde{G}''(Q) = B'(Q) - 2c'(Q) - Qc''(Q) < 0. \tag{28}$$

It follows that $\tilde{G}(Q)$ is strictly concave.

We also have the following relations between net social surplus as a function of P and as a function of Q:

$$G(P) = \tilde{G}(Q(P)), \tag{29}$$

$$G'(P) = \tilde{G}'(Q(P))Q'(P), \tag{30}$$

$$G''(P) = \tilde{G}''(Q(P))[Q'(P)]^2 + \tilde{G}'(Q(P))Q''(P). \tag{31}$$

Proof Expressions (27) and (28) follow directly from the definition of $\tilde{G}(Q)$. Expression (29) is by definition; expressions (30) and (31) follow directly from (29). ∎

Definition A.2 (Monopolist's toll) *We denote the toll a monopolist charges in state* i, *that is the toll that maximizes* $PQ_i(P)$, *by* P_i^M. *We also denote* $Q(P_i^M)$ *by* Q_i^M.

The following proposition shows that our assumptions—the non-trivial one being the concavity of the B_i's—ensure that demand is inelastic for all tolls below the monopolist's tolls, becoming more elastic as tolls increase.

Proposition A.2 (Properties of η and $\mathcal{R}$) *The price-elasticity* η_i *satisfies:*

$$\eta_i(0) = 0, \tag{32}$$

$$\eta_i'(P) < 0, \tag{33}$$

$$\eta_i(P_i^M) = -1. \tag{34}$$

Also, the (instantaneous) revenue function, $\mathcal{R}_i$, *satisfies:*

$$\begin{aligned} \mathcal{R}_i'(P_i^M) &= 0, \\ \mathcal{R}_i'(P) &> 0 \quad \textit{for } P < P_i^M, \\ \mathcal{R}_i'(P) &< 0 \quad \textit{for } P > P_i^M. \end{aligned} \tag{35}$$

Proof From (24) we have that:

$$\eta(P) = \frac{P}{Q(P)[B'(Q(P)) - c'(Q(P))]}. \tag{36}$$

Evaluating this expression at $P = 0$ proves (32).

Differentiating (36) with respect to P we have:

$$\eta'(P) = \frac{Q[B' - c'] - PQ'[B' - c'] + PQ'Q[B'' - c'']}{[Q(B' - c')]^2}.$$

Where the functions Q, B, c and their derivatives are evaluated at $Q(P)$. Since our assumptions ensure that the three terms in the numerator have negative signs (with the first one being strictly negative), we conclude that $\eta'(P) < 0$.

Finally, (34) follows from the monopolist's first order condition:

$$Q(P) + PQ'(P) = 0.$$

The properties for $\mathcal{R}$ follow from (33) and (34). ∎

Proposition A.3 (Unimodality of G_i) *The net social surplus function, $G_i(P)$, is unimodal for $P \geq 0$.*

Proof We have that $\tilde{G}'(Q=0) = B(0) - c(0) > 0$ (due to Assumption (4)). Since $\tilde{G}$ is concave (Lemma A.2), there are two possibilities:

1. $\tilde{G}'(Q) > 0$ for all Q. In this case it follows from (30) and (24) that $G'(P) < 0$ for all $P > 0$. Hence $G(P)$ is unimodal attaining its maximum at $P = 0$.

2. There exists a $Q^* > 0$ such that $\tilde{G}$ is strictly increasing for $Q < Q^*$ and strictly decreasing for $Q > Q^*$, with $\tilde{G}'(Q^*) = 0$.[53]

   Let P^* denote the unique P such that $Q(P) = Q^*$. It then follows from (30) and (24) that $G(P)$ is strictly increasing for $P < P^*$ and strictly decreasing for $P > P^*$, attaining its maximum value at P^*. ∎

Lemma A.3 *The congestion toll, P_i^*, satisfies:*

$$P_i^* < P_i^M, \tag{37}$$

$$G_i'(P_i^*) = 0, \tag{38}$$

$$P_i^* = Q(P_i^*)c'(Q(P_i^*)). \tag{39}$$

Proof To prove (37) we show that the number of trips chosen by the social planner, Q^*, is larger than the number of trips chosen by the monopolist, Q^M.[54] It then follows from (24) that $P^* < P^M$.

The monopolist chooses Q^M as to maximize $[B(Q) - c(Q)]Q$, which leads to the first order condition:

$$[B(Q) - c(Q)] - Qc'(Q) + QB'(Q) = 0.$$

Thus it follows from (27) that $\tilde{G}'(Q^M) > 0$, and since $\tilde{G}'' < 0$, we have that $Q^M < Q^*$.

To prove (38) we consider the two situations into which we broke up the proof of the preceding proposition. Equation (38) obviously holds in the second case. In the first case, where $P_i^* = 0$,[55] we have, from (25) and (32), that:

$$G'(P^*) = -Q(0)c'(Q(0))Q'(0) \geq 0.$$

[53] We convene throughout this appendix that a function $f(x)$ is increasing when $f'(x) \geq 0$ and strictly increasing when $f'(x) > 0$. A similar convention holds for what we call decreasing and strictly decreasing.

[54] As usual, we drop the subscript i.

[55] Strictly speaking in this case we have $G'(0^+) = 0$.

Yet, from what we saw in the proof of Proposition A.3, in this case $G'(P) < 0$ for all $P > 0$. From continuity of $G'(P)$ it follows that $G'(0) = G'(P_i^*) = 0$. ∎

Corollary A.1 *If $P_i^* = 0$ then $c(Q)$ is constant.*

Proof From Lemma A.3 we have that $G'(P^*) = 0$ implies that either $Q'(0) = 0$ or $c'(Q(0)) = 0$. The former cannot hold due to Proposition A.1. Thus $c'(Q(0)) = 0$, and since $c(Q)$ is convex and $Q(0) \geq Q(P)$, it follows that $c'(Q) \leq 0$. Since we assumed that $c'(Q) \geq 0$, it follows that $c'(Q) \equiv 0$ and thus that $c(Q)$ is constant. ∎

Lemma A.4 *Define*

$$J(P) \equiv B(Q(P)) - c(Q(P)) - Q(P)c'(Q(P)).$$

Then:

$$J(P^*) = 0, \tag{40}$$

$$J'(P) \geq 1 \quad \textit{for all } P > 0, \tag{41}$$

$$J(P) > 0 \quad \textit{for all } P > P^*. \tag{42}$$

Proof Expression (40) follows from (23) and (39).

It follows from (40) that:

$$J'(P) = 1 - Q'(P)c'(Q(P)) - Q(P)Q'(P)c''(Q(P)).$$

Then (41) follows from the fact that both $Q'c'$ and $QQ'c''$ are negative.

Finally, (42) is a direct consequence of (40) and (41). ∎

Proposition A.4 (Concavity of the net social surplus) *The function $G_i(P)$ is strictly concave for $P \geq P_i^*$.*

Proof Since $B' - c' < 0$, $B' - 2c' - Qc'' < 0$, and $B'' - c'' \leq 0$, it follows from (26) that a sufficient condition for $G''(P) < 0$ is that $B - c - Qc' \geq 0$, which holds due to Lemma A.4. ∎

Example A.1 Consider $B(Q) = B_0 - B_2Q^2$, for $Q \leq (B_0/B_2)^{1/2}$ and $B(Q) = 0$ elsewhere. Also consider $c(Q) = C_0 + C_2Q^2$ and assume that the constants B_0, B_2, C_0 and C_2 are positive with $B_0 > C_0$. It then follows from (4) that the demand function is given by:

$$Q(P) = \begin{cases} \sqrt{\frac{B_0 - C_0 - P}{B_2 + C_2}} & \text{if } P \leq B_0 - C_0, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for $P \leq B_0 - C_0$:

$$Q'(P) = -\frac{1}{\sqrt{(B_2 + C_2)(B_0 - C_0 - P)}} < 0, \tag{43}$$

$$\eta(P) = -\frac{P}{2(B_0 - C_0 - P)}. \tag{44}$$

Consistent with Proposition A.2, $\eta(0) = 0$ and $\eta'(P) < 0$. We also have that $\eta(P^M) = -1$ leads to:

$$P^M = \frac{2}{3}(B_0 - C_0).$$

From (25), (43) and (44) we have that:

$$G'(P) = \frac{2C_2(B_0 - C_0) - (B_2 + 3C_2)P}{2(B_2 + C_2)^{3/2}\sqrt{B_0 - C_0 - P}}.$$

From the preceding expression and (38) it follows that:

$$P^* = \frac{2C_2(B_0 - C_0)}{3C_2 + B_2} = \frac{2(B_0 - C_0)}{3 + \frac{B_2}{C_2}},$$

which implies that, as implied by (37), $P^* < P^M$. Finally we have that from (26):

$$G''(P) = -\frac{B_0 - C_0 + (B_2 + 3C_2)Q^2}{4(B_2 + C_2)^2 Q^3} < 0.$$

B Planner's solution

In this appendix we first present two results that are useful both in the cases with and without commitment. Then we characterize the planner's solution in the commitment case.

Proposition B.1 (Optimal toll after the franchise ends) *Both in the case with and without commitment, the optimal toll after the franchise ends is* $P_i^A = P_i^*$.

Proof In both cases the proof follows directly from the fact that the toll charged after the franchise ends plays no role in the firms' participation constraint. ▮

Proposition B.2 (Bounds for optimal toll) *For both problems, the optimal toll during the franchise in state i, P_i^O, satisfies:*

$$P_i^* \leq P_i^O \leq P_i^M.$$

Proof Having $P_i^O < P_i^*$ is not possible, since a small increase in the toll improves the objective function (due to Proposition A.4) and increases revenue (due to Proposition A.2). A similar argument rules out the possibility that $P_i^O > P_i^M$. ▮

Proposition B.3 *There exists a solution for the social planner's problem with commitment.*

Proof It follows from Proposition B.2 that the set of feasible values of the P_i^F's and L_i's is compact. Thus we are maximizing a continuous function over a compact set and there exists a solution. ▮

As will become clear shortly, the following functions are closely related to the degree to which the self-financing constraint leads to distortions in a particular state of demand.

Definition B.1 (Distortion functions) *We define:*

$$H_i(P) \equiv \frac{Q_i(P)(1+\eta_i(P))}{Q_i(P)(1+\eta_i(P)) - G_i'(P)},$$

$$v_i(P, L) \equiv H_i(P)u'(PVR_i(P, L) - I), \tag{45}$$

where:

$$PVR_i(P, L) \equiv \frac{PQ_i(P)}{r}(1 - L).$$

Lemma B.1 *The functions $H_i(P)$ and $v_i(P, L)$ satisfy:*

$$H_i(P_i^*) = 1, \tag{46}$$

$$H_i(P_i^M) = 0, \tag{47}$$

$$H_i(P) < 1, \quad \textit{for all } P > P_i^*, \tag{48}$$

$$H_i'(P) < 0, \quad \textit{for } P_i^* \leq P < P_i^M, \tag{49}$$

$$H_i(P) = \frac{1+\eta_i(P)}{1+c'(Q_i(P))Q_i'(P)}, \tag{50}$$

$$
\begin{aligned}
(51) \qquad \frac{\partial v_i}{\partial P}(P, L) &< 0, \qquad \text{for } P_i^* \leq P \leq P_i^M, \\
\frac{\partial v_i}{\partial L}(P, L) &> 0, \qquad \text{for } 0 \leq L \leq 1.
\end{aligned}
$$

Proof Identity (46) follows from (38); identity (47) from (34).

Expression (48) follows from Proposition A.3 and the fact that $P > P_i^*$. Expression (49) follows from the fact that $Q(P)(1+\eta(P))$ is positive and strictly decreasing in P for $P \leq P^M$ (see Proposition A.2) and $G'(P)$ is negative and strictly decreasing in P for $P > P^*$ (see Proposition A.4). Expression (50) follows from (25).

That $v(P, L)$ is strictly decreasing in the first argument follows from the fact that it is the product of two positive, strictly decreasing functions of P.[56] Finally, $v(P, L)$ is strictly increasing in its second argument because $\text{PVR}_i(L)$ is decreasing in L and u concave. ∎

Theorem B.1 (Optimality conditions) *The planner's solution to the problem with commitment satisfies $P_i^O > 0$ and $T_i^O > 0$ for all states i. Also, for any pair of states k and l we have:*

$$(52) \qquad v_k(P_k^O, L_k^O) = v_l(P_l^O, L_l^O),$$

or equivalently:

$$H_k(P_k^O)u'_k = H_l(P_l^O)u'_l.$$

Where $v_i(P, L)$ is defined in (45) and $u'_i = u'(\text{PVR}_i^O - I)$.

Proof We divide the states of demand in two groups. The first group includes those states where $L_k^O < 1$ (or equivalently $T_k^O > 0$) and $P_k^O > 0$. The second group includes all the remaining states, that is those where either $L_k^O = 1$ or $P_k^O = 0$. Note that P_k^O can take any value when $L_k^O = 1$, since $T_k^O = 0$ in this case. Thus we may assume, without loss of generality, that $P_k^O = 0$ and $L_k^O < 1$ for all states in the second category.

The first group of states has to be non-empty, since otherwise the firm's participation constraint cannot be satisfied (all states in the second group provide no revenue for the firm). The initial statement of the proposition is that all states belong to the first group.

The remainder of the proof proceeds as follows. We first prove (52) for any pair of states in the first category. Next we show that no state can belong to the second group.

[56] That $u'\left(\frac{PQ_i(P)}{r}(1-L) - I\right)$ is strictly decreasing in P follows from Proposition A.2

The Lagrangian corresponding to the social planner's problem is:

$$\mathcal{L} = \frac{1}{r}\sum_{i=1}^{n}\pi_i\left\{[G_i(P_i^F) - P_i^F Q_i(P_i^F)](1-L_i) + G_i(P_i^A)L_i\right\} + \lambda\sum_{i=1}^{n}\pi_i u(\mathrm{PVR}_i - I).$$

The first order condition in P_k^F for a state in the first category implies:

$$v_k(P_k^O, L_k^O) = \frac{1}{\lambda}, \tag{53}$$

so that (52) holds for any pair of states i and k in this category.

If state k belonged to the second category, we would have:

$$v_k(P_k^O, L_k^O) \leq \frac{1}{\lambda}. \tag{54}$$

From Proposition B.2 and Corollary A.1 this implies that $P_k^* = 0$ and $c(Q)$ is constant. Thus (25) implies that $G'(P_k^O) = 0$, and hence $H_k(P_k^O) = 1$. It follows from (53), (54) and (48) that:

$$\begin{aligned} u'_k &\leq \frac{1}{\lambda}, \\ u'_l &\geq \frac{1}{\lambda}. \end{aligned}$$

Where l is a state in the first category. Concavity of u and the two preceding inequalities imply that the revenue obtained by the firm in state k is larger or equal than that obtained in state l. Since the former is zero, the latter is also zero. This contradicts the firms' participation constraint, thus showing that there exist no states in the second category. ∎

Corollary B.1 *If $PVR_i^* \leq PVR_j^*$ and $P_i^O = P_i^*$ then $P_j^O = P_j^*$.*

Proof We assume $P_j^O > P_j^*$ and arrive at a contradiction.

If $P_j^O > P_j^*$, then $T_j^O = \infty$ (Proposition 3.2). Since $H_j(P_j^O) < 1$ and $H_i(P_i^O) = 1$ (Lemma B.1), from Theorem B.1 it follows that $u'_i < u'_j$ and therefore

$$\mathrm{PVR}_i^O > \mathrm{PVR}_j^O. \tag{55}$$

On the other hand, from Propositions A.2 and B.2 it follows that:

$$\mathrm{PVR}_j^O > \mathrm{PVR}_j^*. \tag{56}$$

Also, trivially (since the optimal toll is P_i^*) we have:

$$\text{PVR}_i^O \leq \text{PVR}_i^*. \tag{57}$$

From (57), (55) and (56):

$$\text{PVR}_i^* \geq \text{PVR}_i^O > \text{PVR}_j^O > \text{PVR}_j^*,$$

and therefore $\text{PVR}_i^* > \text{PVR}_j^*$, contradicting one of our assumptions. ∎

Lemma B.2 *Fix $s \in \{1, 2, \ldots, n\}$ and define:*

$$\begin{aligned} P_k(s) &= P_k^*, \qquad \textit{for } k = s, s+1, \ldots, n; \\ L_k(s) &= 1 - \frac{PVR_s^*}{PVR_k^*}, \qquad \textit{for } k = s, s+1, \ldots, n; \\ L_k(s) &= 0, \qquad \textit{for } k = 1, 2, \ldots, s-1. \end{aligned}$$

Denote by $P_k(s)$, $k = 1, 2, \ldots, s-1$, the unique P that satisfies:[57]

$$v_k(P, 0) = v_s(P_s^*, 0).$$

Then there exists a unique value of I, which we denote by $I(s)$, for which the tolls and franchise lengths defined above correspond to the social planner's choice when commitment is possible. Furthermore, $I(s)$ is increasing in s.

Proof The tolls and franchise lengths satisfy the first order conditions specified in (52) by construction. Denote the present value of the revenue the franchise holder receives in state i by $\text{PVR}_i(s)$. To complete the proof we must show that there exists $I = I(s)$ such that:

$$\sum_{i=1}^{n} \pi_i u(\text{PVR}_i(s) - I) = u(0). \tag{58}$$

This follows from the fact that the left hand side of the preceding equation is (a) strictly decreasing in I; (b) larger than $u(0)$ when evaluated at $I = 0$; and (c) smaller than $u(0)$ when evaluated at I larger than $\max_i \text{PVR}_i(s)$.

[57]The existence of a unique solution follows from the fact that $v_k(P, 0)$ is strictly decreasing in P (see (51)), with $v_k(P_k^*, 0) \geq v_k(P_s^*, 0)$ (since $\text{PVR}_k^* \leq \text{PVR}_s^*$) and $v_k(P_k^M, 0) = 0 < v_k(P_s^*, 0)$ (due to (47)).

To show that $I(s)$ increases with s, we note that by construction the $P_i(s)$'s are increasing in s, which implies that the $\text{PVR}_i(s)$'s also increase with s (from (35)) and therefore, due to (58), so do the $I(s)$. ∎

Definition B.2 *Lemma B.2 shows that there exists a unique value of I for which the social planner's solution sets an indefinite franchise in state s with toll equal to P_s^*. The corresponding tolls, franchise lengths and franchise revenues are denoted by $P_i(s)$, $T_i(s)$ and $PVR_i(s)$, respectively. The corresponding value of I is denoted by $I(s)$.*

Theorem B.2 (Characterization when $I \leq I(1)$) *Assume $I \leq I(1)$.*[58] *Then the unique solution to the planner's problem is obtained setting $P_i = P_i^*$ and $L_i(I) = 1 - (I/\text{PVR}_i^*)$.*

Proof It follows from the definition of P_i and L_i that the franchise holder's revenue in all states of demand is I. Since we also have non distortionary tolls, the P_i's and L_i's satisfy the first order conditions (52). Finally, the firm's participation constraint is (trivially) satisfied. ∎

Theorem B.3 (Characterization when $I > I(n)$) *Assume $I > I(n)$,*[59] *For $0 \leq \alpha \leq u'(PVR_n^* - I)$ define $P_i(\alpha)$ as the unique (due to (51)) solution to:*

$$v_i(P_i(\alpha), 0) = \alpha, \quad \textit{for } i = 1, \ldots, n, \tag{59}$$

and set the franchise lengths at infinity for all states of demand:

$$L_i(\alpha) = 0, \quad \textit{for } i = 1, \ldots, n.$$

Then there exists a unique value of α such that the tolls $P_i(\alpha)$ and franchise lengths $L_i(\alpha)$; $i = 1, \ldots, n$, fully characterize the planner's solution. The corresponding value of α is the unique solution to:

$$\sum_i \pi_i u(PVR_i(\alpha) - I) = u(0). \tag{60}$$

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[58] This is the case where the road is relatively cheap to build compared with expected revenues, so that it can be financed charging the congestion toll in every state of demand.

[59] This means that the road is relatively expensive compared with the revenue it can generate. As usual, the firm's participation constraint holds, that is:

$$\sum_i \pi_i u(\text{PVR}_i^M - I) \geq u(0).$$

Where PVR_i^M denotes the firm's revenue if it charge's the monopoly toll in an indefinite franchise in state i.

Where $PVR_i(\alpha) = P_i(\alpha)Q_i(P_i(\alpha))/r$.

Proof By construction the $(P_i(\alpha), L_i(\alpha))$'s satisfy the first order conditions (52). From (59) and (51) it follows that P_i, and therefore PVR_i is strictly decreasing in α. Denoting the left hand side of (60) by $S(\alpha)$ it then follows that S is continuous and strictly decreasing in α. We also have, due to (47):

$$S(0) = \sum_i \pi_i u(\text{PVR}_i^M - I) \geq u(0). \tag{61}$$

And, with the definition of $I(n)$ and $\text{PVR}_i(n)$ given in Definition B.2:

$$\begin{aligned} S(u'(\text{PVR}_n^* - I)) &= \sum_i \pi_i u(\text{PVR}_i(n) - I) \\ &< \sum_i \pi_i u(\text{PVR}_i(n) - I(n)) \\ &= u(0). \end{aligned}$$

Where the strict inequality follows from the assumption that $I > I(n)$. Existence and uniqueness of α satisfying (60) now follow. ∎

Theorem B.4 (Characterization when $I(1) < I \leq I(n)$) *Assume that* $I(s) \leq I \leq I(s+1)$ *for* $s \in \{1, 2, ..., n-1\}$.[60] *Given* $\gamma \in [0,1]$ *define:*

$$\begin{aligned} P_i(\gamma) &= P_i^*, \quad \text{for } i = s+1, \dots, n; \\ L_i(\gamma) &= 1 - \frac{PVR_{s+1}^*}{PVR_i^*}(1-\gamma), \quad \text{for } i = s+1, \dots, n. \end{aligned}$$

For $i \leq s$ *set* $L_i(\gamma) = 0$ *and define* $P_i(\gamma)$ *as the unique* P *satisfying:*[61]

$$v_i(P, 0) = u'(PVR_i(\gamma) - I).$$

Where

$$PVR_i(\gamma) = \frac{P_i(\gamma)Q_i(P_i(\gamma)))}{r}(1 - L_i(\gamma)).$$

[60] That a unique integer s between 1 and $n-1$ satisfying these inequalities exists follows from the fact that $I(s)$ is increasing in s, see Lemma B.2.

[61] The argument explaining why such a P is uniquely determined is the same as that in footnote 57.

Then the unique solution to the planner's problem with commitment is the set of $P_i(\gamma)$'s and $L_i(\gamma)$'s corresponding to the unique value of γ (in $[0, 1 - (\mathrm{PVR}^*_s/\mathrm{PVR}^*_{s+1})]$) that satisfies:

$$S(\gamma) \equiv \sum_i \pi_i u(PVR_i(\gamma) - I) = u(0).$$

Proof We first note that the assumption $I(s) \leq I \leq I(s+1)$ implies that:

$$\sum_i \pi_i u(\mathrm{PVR}_i(s) - I) \leq u(0), \tag{62}$$

$$\sum_i \pi_i u(\mathrm{PVR}_i(s+1) - I) \geq u(0). \tag{63}$$

By construction the $(P_i(\gamma), L_i(\gamma))$'s satisfy (52). Thus all that remains to be shown is that there exists a unique γ that satisfies $S(\gamma) = u(0)$.

We have that $\mathrm{PVR}_i(\gamma)$ is strictly decreasing in γ,[62] and therefore $S'(\gamma) < 0$. Furthermore, from the definition of the PVR(s)'s in Definition B.2, and (62) and (63) it follows that:

$$\begin{aligned} S(0) &= \sum_i \pi_i u(\mathrm{PVR}_i(s+1) - I) \geq u(0), \\ S\left(1 - \frac{\mathrm{PVR}^*_s}{\mathrm{PVR}^*_{s+1}}\right) &= \sum_i \pi_i u(\mathrm{PVR}_i(s) - I) \leq u(0). \end{aligned}$$

Thus the characterization of the planner's solution holds. ∎

Definition B.3 *For every i define $\widetilde{P}_i$ as the unique toll that satisfies $PQ_i(P) = rI$.*

Corollary B.2 (Comparison of solutions with and without commitment)

(a) *When $I \leq I(1)$ the planner's solution with and without commitment are the same.*

(b) *When $I > I(1)$, the planner's solutions with and without commitment are the same if and only if $H_i(\widetilde{P}_i)$ does not vary with i.*

Proof Since in the case without commitment the franchise holder's revenue is the same in all states of demand, (52) implies that the $H_i(\widetilde{P}_i)$'s do not vary with i when both solutions coincide. Statements (a) and (b) now follow directly. ∎

[62] The argument for the case where $i \geq s+1$ may be found in the proof of Theorem B.2; the one for the case where $i \leq s$ in the proof of Theorem B.3.

In most cases (generically) we have that the planner's solutions without and with commitment are the same only when $I \leq I(1)$, that is, when the road is sufficiently cheap to build (relative to expected demand) so that in all states of demand it can be financed setting the congestion toll. The only interesting exception we can think of is when there is no congestion ($c'(Q) \equiv 0$) and demand in different states of nature only differs by a multiplicative constant:

$$Q_i(P) = z_i B^{-1}(P).$$

Corollary B.3 *Denote by* $\mathrm{PVR}_k^O(I)$ *the present value of revenue received in state of demand* k *under the optimal contract with commitment when construction costs are equal to* I*. Then* PVR_k^O *is strictly increasing in* I*.*

Proof The intuition behind this result is the following. The franchise holder's revenue in state k increases with I either because the franchise length increases ($I < I(k)$) or because the optimal toll increases ($I > I(k)$).

The formal proof considers three ranges for I:

1. $I \leq I(1)$.

   It follows from Theorem B.2 that in this case PVR_k^O is equal to I (and therefore strictly increasing in I).

2. $I \geq I(n)$.

   With the notation introduced in the proof of Theorem B.3 we have that (59) and (51) imply that $\mathrm{PVR}_k(\alpha)$ is strictly decreasing in α. It then follows from the firm's participation constraint

   $$\sum_i \pi_i u(\mathrm{PVR}_i(\alpha(I)) - I) = u(0),$$

   that α is strictly decreasing in I. Thus PVR_k^O is strictly increasing in I.

3. $I(s) \leq I \leq I(s+1)$.

   With the notation introduced in the proof of Theorem B.4 we have that $\mathrm{PVR}_k(\gamma)$ is strictly decreasing in γ. It then follows from

   $$\sum_i \pi_i u(\mathrm{PVR}_i(\gamma(I)) - I) = u(0)$$

   that γ is strictly decreasing in I. Thus PVR_k^O is strictly increasing in I. ∎

C Sub-optimality of fixed-term auctions

Proposition C.1 (Suboptimality of fixed-term auctions) *A fixed term auction is optimal if and only if* PVR_i^* *is the same in all states of demand, and this common value is larger than* I. *Thus generically a fixed term franchise is suboptimal.*

Proof We present the proof for the case with commitment. The case without commitment is analogous.

A necessary condition for a fixed term franchise to be optimal is that, in the planner's solution, the franchise length be the same in all states of demand. From Proposition 3.6 it follows that this holds in two situations. First, when PVR_i^* is the same in all states and this common value is larger than I. In this case the optimal franchise length is the same across states of demand and finite. The planner sets $P_i = P_i^*$ and the winning bid attains the planner's solution.

The second case where the franchise length is the same in all states of demand is when it extends indefinitely. Yet in this case the planner cannot infer from the winning bid which is the value of I and therefore is unable to set the optimal tolls after the winning bid is revealed. It follows that a fixed term franchise is optimal only in the first case. ∎

D A model of a risk averse firm

In this appendix we present a model that rationalizes a paradoxical feature of the financing of highway franchises, namely that entrepreneurs seem to be unable to diversify risks.

Consider the case of the owner of a construction firm whose profits are s, a random variable with cumulative distribution function $F(\cdot\,; \bar{s}, \sigma_s^2)$, where $\bar{s}$ and σ_s^2 denote the corresponding mean and variance. The entrepreneur is the sole owner of the firm and is risk averse, so that his expected utility is

$$W = \int u(y)\, dG(y).$$

Where u is strictly increasing and strictly concave, y denotes the entrepreneur's net income and $G(y)$ the corresponding cumulative distribution function.

In general the entrepreneur will be willing to shed some risk. Consider a risk neutral investor who is considering whether to invest in this project. She knows that s is a private signal and that the entrepreneur, either as a member of a partnership or as the manager of a company fully owned by the investor, always declares that company profits are zero if not monitored.[63,64]

The investor can always verify, i.e., make claims to a fraction $\alpha(e)$ of total profits at a cost e of effort in monetary terms. As usual, $\alpha(e) \in [0,1]$, $\alpha' > 0$, $\alpha'' < 0$. For simplicity, we assume that the price p of the company set by the entrepreneur does not depend on the share that the investor buys.[65] The expected profits for the investor of buying a share β of the company is

$$\Pi(e,\beta) = \beta\alpha(e)\bar{s} - e - \beta p.$$

Since the investor is risk neutral, she maximizes expected utility as a function of effort and the share of the company she buys. As we assumed that p does not depend on β, the maximization problem leads to $\beta = 1$ if the investor buys at all, and hence effort satisfies

$$\alpha'(e)\bar{s} = 1. \tag{64}$$

It follows that the maximum price p at which the investor is willing to buy the company is the price that solves

$$\bar{s}\alpha(e^*) - e^* - p = 0. \tag{65}$$

Where e^* is the profit maximizing level of effort (characterized by (64)).

Consider now the utility of the entrepreneur in the two polar cases. If he asks for a price such that the investor does not buy into the firm (he holds the firm) his welfare is given by

$$W_h = \int u(s)\, dF(s). \tag{66}$$

Whereas if the investor buys out the firm and leaves the entrepreneur as the manager, the manager's welfare from selling is

$$\begin{aligned} W_s &= \int u((1-\alpha^*)s + p)\, dF(s) \\ &= \int u(s + \alpha^*(\bar{s} - s) - e^*) dF(s). \end{aligned} \tag{67}$$

Where we used (65) in the last step, $\alpha^* \equiv \alpha(e^*)$, and we have assumed that the manager ap-

[63] The investor leaves the entrepreneur as a manager due to his superior specialized knowledge.

[64] This strategy is weakly dominant in the subgame after the investment is committed.

[65] Even though the derivation is more complex, the result that follows continues holding if this assumption is relaxed.

propriates the non-verifiable profits. Next, consider two extreme cases for the entrepreneur's attitude toward risk. If he is risk neutral, he has a cost advantage over the investor, since he does not have to incur the verification cost and he gets the full profits from the project. In this case he does not sell the firm. On the other hand, if he is infinitely risk averse, he maximizes the lowest expected utility, which occurs when he sells the firm. It follows that as the entrepreneur's risk aversion grows, there is a positive degree of risk aversion at which he switches from keeping to selling the project. In what follows, we prove this intuition.

A second order Taylor approximation for u around $\bar{s}$ leads from (67) to

$$\begin{aligned} W_s &\simeq \int [u(\bar{s}) + \{(1-\alpha^*)(s-\bar{s}) - e^*\}u'(\bar{s}) + \tfrac{1}{2}\{(1-\alpha^*)(s-\bar{s}) - e^*\}^2 u''(\bar{s})]dF(s) \\ &= u(\bar{s}) - e^* u'(\bar{s}) + \tfrac{1}{2}(1-\alpha^*)^2\sigma_s^2 u''(\bar{s}) + \tfrac{1}{2}e^{*2}u''(\bar{s}), \end{aligned}$$

and from (66) to

$$W_h \simeq u(\bar{s}) + \tfrac{1}{2}\sigma_s^2 u''(\bar{s}).$$

Taking the difference, we have

$$W_s - W_h \simeq \tfrac{1}{2}[(\alpha^{*2} - 2\alpha^*)\sigma_s^2 + e^{*2}]u''(\bar{s}) - e^* u'(\bar{s}). \tag{68}$$

Denoting by $\rho = -u''(\bar{s})\bar{s}/u'(\bar{s})$ the entrepreneur's coefficient of relative risk aversion evaluated at $\bar{s}$, we have that (68) is equivalent to

$$W_s - W_h \simeq \frac{u'(\bar{s})}{\bar{s}}\left[\tfrac{1}{2}\left\{\alpha^*(2-\alpha^*)\sigma_s^2 - e^{*2}\right\}\rho - e^*\bar{s}\right].$$

Hence, given that the investor's optimal choice of effort e^* is independent of the entrepreneur's degree of relative risk aversion (see (64)), we have the following proposition:

Proposition D.1 *Suppose $\alpha^*(2-\alpha^*)\sigma_s^2 > e^{*2}$,*[66] *then there exists $\rho^* > 0$ such that for $\rho < \rho^*$, the entrepreneur does not sell the firm. When $\rho > \rho^*$, the firm is sold.* ∎

The relevant part of the proposition is that for all $\rho \in (0, \rho^*)$ the entrepreneur prefers not to sell and must, therefore, assume all the risk of the company. For these values of ρ the behavior of the firm is that of the risk averse entrepreneur.

[66] Other things equal, this holds if σ_s^2 is sufficiently large.