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# NOMINAL INCOME TARGETING IN AN OPEN-ECONOMY OPTIMIZING MARKET

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## **ABSTRACT**

This paper presents stochastic simulation results pertaining to the performance of nominal income targeting, here represented as a monetary policy rule that sets quarterly values of an interest rate instrument in response to deviations on existing studies of nominal income growth from a specified target rate. It attempts to improve on existing studies by conducting analysis in a macroeconomic model that is designed to respect both neoclassical theory and empirical regularities. Accordingly, the basic theoretical framework is one in which individual economic agents are depicted as solving dynamic optimization problems with rational expectations, but in an environment such that prices respond only gradually to changes in conditions. The adjustment specification used is the P-bar model, which satisfies the strict natural rate hypothesis. Two improvements over previous work by the authors are that consumption choices reflect habit formation, which lends some inertia to the system, while the modeled economy is open to international flows of goods and securities. Both of these features have major effects on the system's properties. Quantitatively, the model is calibrated to post-Bretton Woods U.S. quarterly data. The results suggest that nominal income targeting deserves serious consideration as a monetary policy strategy.

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#### 1. Introduction

The purpose of this paper is to examine the merits of monetary policy rules that utilize as their principal target variable the level or growth rate of some aggregate measure of nominal spending, such as nominal GDP, rather than a monetary aggregate or an index of inflation (either alone or in combination with some measure of the output gap). Considerable academic support for nominal spending targets has existed since the early 1980s, and therefore predates the upsurge of interest in inflation targeting that began in the early 1990s with the adoption of inflation targeting by the central banks of New Zealand, Canada, the United Kingdom, and Sweden.<sup>2</sup> In our discussion we shall adopt the term "nominal income targeting" because of its widespread usage, although it does not most accurately reflect the logic of the approach, according to the discussion below. Also, we shall use the word "targeting" in the manner familiar from the existing literature, rather than in the more tightly defined sense suggested by Svensson (1997a) and Rudebusch and Svensson (1998). That is, we shall use the term "X-targeting" when the central bank sets its instrument in response to a rule that refers to deviations from a desired path for the variable X, whereas Rudebusch and Svensson would call this "responding to the variable X," and would reserve the term "target" for variables appearing in the central bank's objective function.

Because there is a large and rich literature on nominal income targeting (briefly, NIT), we begin in Section 2 with a short review of existing arguments in its favor. Then in Section 3 we present some evidence which suggests that NIT is in effect utilized in practice in the United States. Our paper's main objective, however, is to develop new results concerning the possible desirability of NIT in the context of a quantitative structural macroeconomic model that represents an improved and extended version of the semi-classical framework presented in McCallum and Nelson (1998). Toward that end, aggregate demand and aggregate supply specifications are developed in Sections 4 and 5. Both of these sections feature modifications designed to make the model one that depicts an economy open to trade and capital flows. In addition, our new demand specification incorporates habit-formation features that increase its ability to match aggregative U.S. data at

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For some discussion of these different terminological conventions, see Appendix A.

<sup>&</sup>lt;sup>1</sup> We do not thoroughly consider a policy of targeting some foreign exchange rate because the merits of such a regime seem to be largely based on either political or microeconomic (i.e., resource allocation) grounds, which are not amenable to study with macroeconomic models. Nevertheless, we report a small bit of evidence regarding the macroeconomic properties of an exchange rate target.

<sup>&</sup>lt;sup>2</sup> For discussions of inflation targeting see Mishkin and Posen (1997), McCallum (1997a), and individual papers in the volumes edited by Haldane (1995) and Leiderman and Svensson (1995).

the quarterly frequency. The model is summarized and log-linearized in Section 6. Calibration of the model, based on properties of quarterly data for the United States, is undertaken in Section 7. The main simulation exercises are finally reported in Section 8 and their implied messages are summarized in Section 9.

## 2. Existing Arguments for NIT

To a considerable extent, the case for NIT as developed in the early 1980s grew naturally from a perception that the then-prominent strategy of monetary aggregate targeting could be improved upon. Large and unpredictable changes in payments industry technology and regulatory practices had led to well-publicized instability in the short-run relationships between monetary aggregates, such as M1, and nominal GDP (as well as other measures of aggregate nominal spending). In addition, several economists held the belief that a policy that smoothed out fluctuations in nominal GDP would be more effective in stabilizing real output and employment than a policy that smoothed the path of a monetary aggregate. These ideas were put forth in papers by Meade (1978), Tobin (1980), Hall (1984), Gordon (1985), Taylor (1985), and McCallum (1985).

At about the same time, papers by Bean (1983) and Aizenman and Frenkel (1986) developed analytical results suggesting that NIT would be superior to other targeting schemes in terms of the implied automatic policy responses to shocks of particular types. Root-mean-square (RMS) deviations of macroeconomic aggregates relative to desired values would be smaller, that is, according to small analytical models in which such magnitudes could be calculated precisely. More recently, this line of work has been extended by Frankel and Chinn (1995) and Ratti (1997). It has been shown by various authors, including West (1986) and Henderson and McKibbin (1993), however, that claims for the theoretical superiority of NIT have been overstated in some of these studies as a consequence of the non-robustness of results to details of model construction, including the failure to take account of some types of shocks. Other notable recent contributions include Feldstein and Stock (1994), Hall and Mankiw (1994), and Cecchetti (1995).

The criterion of robustness to model specification has been a prominent element in the NIT discussions of McCallum (1988, 1993, 1997b). The argument in this work has been that keeping nominal GDP, or some other measure of nominal spending, close to a target path that grows smoothly, at a rate equal to the long-run average rate of growth of real output plus a desired inflation rate, would result in an average inflation rate close to the desired value and perhaps in reduced fluctuations of real output and employment. The target inflation rate would be more accurately achieved than with monetary aggregate targets, the argument goes, because advance knowledge of average velocity growth would not be required. In addition, real output and employment fluctuations might be smaller on average than with

pure inflation targeting because of the implied response of the rule to unusually high or low output growth rates. The latter result can not be assured, because of the profession's ignorance concerning the mechanism by which nominal income growth is split between inflation and real output growth components, but the conjecture was that it would be achieved in a wide variety of models — this is the robustness idea — and in actuality. Also, in recent writings (including McCallum [1997b] and McCallum and Nelson [1998]) it has been stressed that NIT — especially in its growth-rate version — avoids dependence on unreliable measures of the "capacity" or "natural" output level that are required in several prominent rules, including the one proposed in Taylor's influential "Discretion versus policy rules in practice" (1993a).

Regarding the relative desirability of nominal income or inflation targeting rules, McCallum (1997a, p. 232) has suggested that "because the prices of goods and services... react more slowly than output in response to monetary actions, cycling and dynamic instability are more likely to occur with a price level or inflation target. In other words, the problem of 'instrument instability,' which would render the targeting attempt entirely unsuccessful, is intensified." Also, the same passage suggests that it may be "more difficult to devise a policy rule for hitting inflation targets than nominal GDP targets, because the former requires an understanding of the forces that determine the split of nominal GDP growth into its inflation and real growth components." These conjectures are not buttressed by any theoretical results, however, so it is at best an open question whether they are consistent or inconsistent with quantitative results in (a variety of) carefully-specified models.

A frequently expressed objection to NIT is that national income statistics are not produced often enough or quickly enough — and are often significantly revised after their first release. It can be argued, however, that the essence of the strategy is to use some reasonably comprehensive measure of aggregate nominal spending; it does not have to be GDP or GNP per se. Thus, other measures could readily be developed on the basis of price and quantity index measures that are reported more often and more promptly. It might even be possible to devise a monthly measure that is conceptually more attractive than GDP, by making the price index more closely tailored to public perceptions of inflation and using a quantity index that treats government purchases more appropriately. But in any event, if policy-

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<sup>&</sup>lt;sup>4</sup> The view that nominal spending growth is fundamental and relatively well understood is a characteristic of some monetarist writings, such as Friedman (1971) and Lucas (1972). McCallum (1997a, pp. 225-226; 1997b) contends that the determinants of the mentioned split are especially poorly understood by macroeconomic researchers, a view that does not presume that NI growth is causally prior.

<sup>&</sup>lt;sup>5</sup> In the United States, for instance, one could in principle use the product of the CPI and the Fed's Industrial Production Index, both of which are produced monthly and infrequently revised. Monthly availability is also a feature of the Bureau of Economic Analysis nominal series entitled "Personal Income".

instrument adjustments are based on expected future target discrepancies, rather than past misses, then this issue is not directly relevant.

Quite recently a novel and rather extreme argument against NIT has been put forth by Ball (1997) and seconded by Svensson (1997a). This argument is that successful stabilization of the growth rate (or the growing level) of nominal income would result in non-stationarity of the output and inflation components considered separately — an outcome that Ball (1997) labels "disastrous." This result is shown to hold, however, only in a particular theoretical model, with no attempt being made to consider its robustness or fragility. The model in question is, moreover, a non-optimizing model of an entirely backward-looking type, with no expectational terms included in either of its behavioral equations. (These are an IS function and a price-adjustment or Phillips-curve relation.) The nature of the demand portion of the model — i.e., the IS function — turns out to be unimportant for the non-stationarity result in question, but McCallum (1997c) shows that the price-adjustment specification is crucial. In particular, replacement of the Ball-Svensson price-adjustment relation with any of seven other popular specifications eliminates the non-stationarity result. Also see Dennis (1998).

The upshot of the foregoing discussion is that arguments pro and con regarding NIT rules relative to ones with inflation and output gap target variables depend upon details of the dynamic relationships between nominal and real variables, about which prevailing theory is not particularly helpful. In Sections 4-8 of this paper, consequently, we shall attempt to explore some quantitative aspects of the issues by simulations conducted in a model that has been carefully specified to respect both neoclassical theory and also the quantitative time-series data for the United States economy. Here we examine only one model in several variants so further investigation of the robustness of our findings is needed, but the present study represents a start.<sup>7</sup>

Before proceeding to our theoretical model, however, we provide evidence on the *empirical* relevance of nominal income targeting, by comparing the fit on U.S. data of estimated Taylor rules with that of interest rate rules that respond to nominal income growth.

## 3. Nominal Income Targeting and Historical Policy

In his now-famous paper, Taylor (1993a) demonstrated that U.S. monetary policy in the period since 1987, a period considered very successful in terms of delivering low inflation alongside relatively stable and satisfactory output growth, can be

<sup>&</sup>lt;sup>6</sup> This problem is recognized by Svensson (1997a).

<sup>&</sup>lt;sup>7</sup> Other studies have examined these issues, of course, but are open to objections on the grounds of non-operationality, as discussed in McCallum and Nelson (1998).

well characterized as a regime in which the federal funds rate responds with positive, fixed coefficients to (proxies for) expected inflation and the output gap. In a recent paper, Clarida, Gali, and Gertler (CGG) (1997), using instrumental variables estimation over the period 1979:3-1996:4 (a sample period beginning with the onset of Chairman Volcker's incumbency), provide formal econometric corroboration of Taylor's finding. Their regression for the federal funds rate, which employs a partial adjustment specification to allow for interest rate smoothing, indicates that expected inflation enters with a long-run coefficient of approximately 1.5, in keeping with Taylor (1993a); the estimated coefficient on the output gap is, however, smaller than the 0.5 value that Taylor used.

In this section we estimate a specification closely related to CGG's, and compare it to a specification in which policy responds to expected nominal income growth instead of expected inflation. Our version of CGG's (1997) specification is as follows:

$$4R_{t} = c + (1 - \rho_{R}) \cdot \phi_{P} (4E_{t-1} \Delta p_{t+1}) - (1 - \rho_{R}) \cdot \phi_{GAP} E_{t-1} u_{t} + (1 - \rho_{R}) \cdot d_{1}D7982_{t} + \rho_{R} (4R_{t-1}) + \varepsilon_{Rt}$$
(3.1)

with  $\rho_R \in (0,1)$  and  $\varepsilon_{Rt}$  white noise. Here  $4R_t$  and  $4\Delta p_t$  are the annualized values of the federal funds rate and the log-change in the GDP deflator, and  $u_t$  is the unemployment rate (expressed as a fraction). There are two differences in specification (3.1) from CGG's baseline model. First, we use  $u_t$  to measure the output gap, whereas CGG's unemployment-based measure of the output gap was the detrended unemployment rate. (We also obtained estimates using detrended  $u_t$ ). Secondly, we add  $D7982_t$ , a dummy variable taking the value 1.0 during the "new operating procedures" period 1979:4-1982:2, to the equation. We include this variable because we found in our earlier estimates of the Fed's policy rule (McCallum and Nelson [1998]) that it entered significantly.

Apart from our inclusion of D7982<sub>t</sub>, our instrument list is the same as in CGG: a constant term, and lags 1-4 of inflation, the federal funds rate, the 10 year bond / funds rate spread, the unemployment rate, commodity price inflation (from the Producer Price Index), and M2 growth. Our data were downloaded from the Federal Reserve Bank of St. Louis' FRED database, and our estimation period is 1979:3-1997:3.

The first column of coefficients in Table 1 reports our estimates of equation (3.1), which is closely related to CGG's specification. In the second column, we replace expected next-period inflation in (3.1) with expected next-period nominal income growth,  $E_{t-1} \Delta x_{t+1}$ . The residual standard error declines slightly relative to the fit of (3.1). Moreover, the New Operating Procedures dummy becomes significant (its t-

Table 1: Instrumental Variables Estimates of (3.1) and Variants, Dependent Variable: Annualized Federal Funds Rate ( $4R_t$ )							
Parameter	Equation	Equation (3.1) with	Equation (3.1) with	Equation (3.1) with			
	(3.1)	$\mathbf{E}_{t-1} \Delta x_{t+1}$	$\mathbf{E}_{t-1} \Delta x_t$	both			
		replacing	replacing	$E_{t-1} \Delta x_t$ and			
		$\mathbf{E}_{t-1}\Delta p_{t+1}$	$\mathbf{E}_{t-1}\Delta p_{t+1}$	$\mathbf{E}_{t-1}\Delta p_{t+1}$			
Long-run response	1.526			-0.034			
to inflation	(0.534)	_	_ '	(0.752)			
$(\phi_P)$							
Long-run response	0.3116	1.677	1.078	1.084			
to output gap	(0.4813)	(1.122)	(0.636)	(0.713)			
$(\phi_{GAP})$							
Interest rate							
smoothing	0.746	0.844	0.810	0.811			
coefficient	(0.065)	(0.058)	(0.049)	(0.056)			
$(\rho_R)$							
Long-run response							
to nominal income	_	1.037	1.203	1.214			
growth		(0.5422)	(0.3697)	(0.4713)			
$(\phi_X)$							
Long-run							
coefficient on New	0.0233	0.1051	0.0860	0.0875			
Operating	(0.0288)	(0.0228)	(0.0179)	(0.0407)			
Procedures							
$\frac{\text{dummy } (d_1)}{C}$	0.000	2016					
Constant term $(c)$	0.0093	0.0168	0.0114	0.0606			
C4 1 - 1 - C	(0.0083)	(0.0070)	(0.0061)	(0.0423)			
Standard error of	0.0114	0.0111	0.0004	0.0005			
estimate	0.0114	0.0111	0.0094	0.0095			
D.W.	1.77	1.99	2.51	2.51			

Notes to Table 1: Standard errors in parentheses. Estimation is on quarterly data for 1979:3-1997:3. For all three regressions in the table, the instrument set is: a constant; the dummy variable  $D7982_t$  defined in the text; and lags 1-4 of: inflation, the federal funds rate, the 10 year bond rate / funds rate spread, the unemployment rate, commodity price inflation (the log-difference of a commodity price index), and the log-difference of M2.

value rises from 0.81 to 3.79) and the estimate of the output gap response coefficient,  $\phi_{GAP}$ , is five times larger and is much more precisely estimated. CGG's result that the policy response to the output gap is insignificant after 1979 thus appears sensitive to the specification of the nominal aggregate to which policy reacts —inflation or nominal income growth.

In the third column of Table 1, expected next-period nominal income growth is replaced by expected *current* nominal income growth,  $E_{t-1} \Delta x_t$ . The improvement in fit over CGG's specification becomes more noticeable, with the residual standard deviation about a fifth lower than that of (3.1) (0.94% vs. 1.14%). Compared to the results with  $E_{t-1} \Delta x_{t+1}$ , the 1979-1982 dummy increases further in significance, although the estimated magnitude of the output gap term declines.

In the final column of Table 1, we add  $E_{t-1} \Delta p_{t+1}$  to the preceding regression, and find that it contributes no explanatory power, has a wrongly-signed coefficient, and leaves the remaining coefficient estimates virtually unaltered.

The regressions in Table 1 using nominal income growth agree with CGG's result that, at least from 1979, the long-run response coefficient of the funds rate to the nominal aggregate exceeded unity: the estimates of (3.1) indicate a value of  $\phi_P = 1.53$ , while the response coefficient when expected  $\Delta x_{t+j}$  is the nominal aggregate is  $\phi_X = 1.20$  (j = 0) or 1.04 (j = 1). All regressions in Table 1 are also uniform in implying a high degree of interest rate smoothing by the Fed, as indicated by the estimates of  $\rho_R$  in the 0.75-0.85 range.

Our results suggest therefore that actual U.S. monetary policy since 1979 is well approximated empirically as a rule that reacts to expected nominal income growth. The success of U.S. monetary policy in recent years can then perhaps be regarded as evidence in favor of monetary policies directed towards the stabilization of nominal income growth.

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<sup>&</sup>lt;sup>8</sup> CGG report that the output gap does enter their funds rate regression significantly if the sample period is truncated to Alan Greenspan's period as Federal Reserve Board Chairman (1987 onwards). Similarly, Judd and Rudebusch (1998) find that the level of the output gap enters significantly in the Greenspan period but not the Volcker period.

<sup>&</sup>lt;sup>9</sup> Results not reported here show that when the detrended unemployment rate is the output gap proxy, the estimated response to expected  $\Delta x_{t+j}$  is below unity for j = 1, but the difference from values exceeding unity is not statistically significant. Other findings are similar to those in Table 1.

## 4. Aggregate Demand Specification

We now turn to specification of the model to be used in our simulations. The aggregate demand side is an open-economy extension of the optimizing IS-LM specification used in our earlier work (McCallum and Nelson [1997, 1998]);<sup>10</sup> our incorporation of a foreign sector draws on recent work on international versions of the Sidrauski-Brock model by Obstfeld and Rogoff (1996) and Kollmann (1996).

The model depicts an open economy that is small in a sense that will be spelled out below. This economy is populated by a continuum of households over (0,1). A typical household maximizes  $E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}, C_{t+j-1}, M_{t+j}/P^A_{t+j})$ , where  $C_t$  is an index of household consumption in period t,  $M_t/P^A_t$  denotes its end-of-period real money holdings (which facilitate period t transactions),  $P^A_t$  is the general price level, and  $u(C_t, C_{t-1}, M_t/P^A_t)$  is the instantaneous utility function. The appearance of  $C_{t-1}$  in  $u(\bullet)$  implies that preferences are not time-separable with respect to consumption; however, we assume that  $u(\bullet)$  is separable across consumption and money balances, taking the specific functional form

$$u(C_t, C_{t-1}, M_t/P^{A_t}) = \exp(v_t)(\sigma/(\sigma-1))(C_t/C_{t-1}^{h})^{(\sigma-1)/\sigma} + (1-\gamma)^{-1}(M_t/P^{A_t})^{(1-\gamma)}$$
(4.1)

with  $\sigma > 0$ ,  $\gamma > 0$ ,  $\sigma \neq 1$ ,  $\gamma \neq 1$ , and  $h \in [0,1)$ . Preferences over consumption thus incorporate habit formation, with the functional form used in (4.1) being a special case of that in Carroll, Weil and Overland (1995), the special case being suggested by Fuhrer's (1998) estimates. In (4.1),  $v_i$  is a preference shock whose behavior we specify in Section 7.

Households consume many goods — all of them domestically produced. The  $C_t$  variable appearing in (4.1) is the quantity consumed in t of an aggregate of these goods, with the index constructed in the Dixit-Stiglitz (1977) manner, with  $C_t = \left[\int_0^1 C_t(j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}$ , where  $C_t(j)$  denotes the household's period t consumption of good j, and  $\theta > 1$ . All goods are differentiated from each other.

While a typical household consumes an aggregate of all goods, it specializes in production, using the CES technology

$$Y_{t} = \left[a_{1}(A_{t}N_{t}^{d})^{\nu_{1}} + (1 - a_{1})(IM_{t}^{d})^{\nu_{1}}\right]^{1/\nu_{1}}$$

$$(4.2)$$

<sup>&</sup>lt;sup>10</sup> Other open-economy generalizations of this specification appear in Batini and Haldane (1998) and Svensson (1998).

<sup>&</sup>lt;sup>11</sup> Note that our  $(\sigma-1)/\sigma$  is Fuhrer's  $(1-\sigma)$ .

where  $a_1 \in (0,1]$ , and  $v_1 \in (-\infty,\infty)$ . In (4.2),  $A_t$  is an exogenous technology shock entering all households' production functions;  $N_t^d$  is the amount of labor hired by the household in period t; and  $IM_t^d$  is the quantity purchased by the household of the foreign-produced good, which is an input in production. There is only one type of foreign good explicitly recognized, but it too could be viewed as a Dixit-Stiglitz aggregate with different goods being used by different domestic producers.<sup>12</sup>

Due to its monopoly power over the good it produces, household i treats the good's price, which is  $P_t$  in domestic-currency units, as a choice variable, while taking the domestic aggregate price level  $P_t^A$ , the nominal exchange rate  $S_t$ , and the foreign price level  $P_t^*$  as given. Having chosen  $P_t$ , the household produces whatever quantity of its output is demanded. The household has two types of buyer for its good: domestic residents (i.e., other households in the same country) and the rest of the world (to which it may export its good). The household may not price-discriminate, so the price of good i to foreign purchasers is simply  $(P_t/S_t)$ . Let  $DY_t^d$  be the total quantity demanded by all domestic households of the representative household's output, and let  $EX_t^d$  be the quantity of the good demanded by foreigners. (Thus,  $Y_t^d = DY_t^d + EX_t^d$ ). It may be shown (e.g., Obstfeld and Rogoff, [1996, ch. 10]) that domestic households' demand function for good i is given by

$$DY_{t}^{d} = (P_{t} / P_{t}^{A})^{-\theta} DY_{t}^{A}$$
(4.3)

where  $P_t^A = [\int_0^1 P_t(j)^{(1-\theta)} dj]^{1/(1-\theta)}$ , and  $DY_t^A$  is an aggregate of the  $DY_t^A$ . We assume that foreigners' demand function for the households' exports is similarly given by:

$$EX_{t}^{d} = (P_{t} / P_{t}^{A})^{-\theta} EX_{t}^{A}$$
(4.4)

where  $EX^A_i$  is the domestic economy's aggregate exports. (The exchange rate does not appear in (4.4) because it cancels from the relative price term on the right-hand side.) Furthermore, we assume that the total demand for exports is given by:

$$EX^{A}_{t} = (S_{t}P_{t}^{*}/P^{A}_{t})^{\eta}Y_{t}^{*b}$$
(4.5)

where  $\eta > 0$ , b > 0. Thus, aggregate export demand is positively related to the real exchange rate,  $Q_t \equiv S_t P_t^* / P_t^A$ . We assume that the domestic economy's exports form an insignificant fraction of foreigners' consumption, and thus their weight in the foreign economy's aggregate price index is negligible. This is one way in which the domestic economy is small.

<sup>&</sup>lt;sup>12</sup> Our model abstracts from growth in output and other real quantities.

<sup>&</sup>lt;sup>13</sup> Similarly, foreign producers may not price-discriminate when selling their output.

<sup>&</sup>lt;sup>14</sup> Thus,  $P_t^*$  is simply the foreign-currency price of the single foreign good.

Labor is immobile across countries and, in addition to being an employer of labor, each household is endowed with one unit of potential work-time each period, which it supplies inelastically to the domestic labor market. Governments, both domestic and foreign, are assumed not to issue debt, but each country has a private security denominated in units of its own output.<sup>15</sup> Domestic households may purchase the domestic security for  $(1+r_t)^{-1}$  per unit in period t, and it is redeemed for one unit of domestic output in period t+1. Foreigners purchase only the bond denominated in their own output, which they may purchase for  $(1+r_t^*)^{-1}$  units of foreign output and which is redeemed for one unit of foreign output one period later. Domestic households may also purchase this bond, but the price they must pay (expressed in foreign output units) is  $(1+\kappa_t)^{-1}(1+r_t^*)^{-1}$ . Let  $B_{t+1}$  and  $B_{t+1}^*$  denotes the quantity of domestic and foreign bonds, respectively, purchased in t by the representative household.

The household also receives  $TR_t$  in lump-sum real transfers from the home government. The budget constraint for a typical household, expressed in real terms, is therefore

$$(P_{t}/P^{A}_{t})DY_{t}^{d} + (P_{t}/P^{A}_{t})EX_{t}^{d} - C_{t} + (W_{t}/P^{A}_{t})N_{t}^{S} - (W_{t}/P^{A}_{t})N_{t}^{d} + TR_{t} - (M_{t}/P^{A}_{t}) + (M_{t-1}/P^{A}_{t}) - B_{t-1}(1+r_{t})^{-1} + B_{t} - Q_{t}IM_{t}^{d} - Q_{t}B_{t+1}^{*}(1+\kappa_{t})^{-1}(1+r_{t}^{*})^{-1} + Q_{t}B_{t}^{*} = 0,$$

$$(4.6)$$

where  $W_t$  is the nominal wage, and  $N_t^S$  denotes household labor supply in period t.

Let  $\xi_t$  denote the Lagrange multiplier on constraint (4.2) and  $\lambda_t$  the multiplier on (4.6). Then the household's first order conditions with respect to  $C_t$ ,  $(M_t/P^4_t)$ ,  $B_{t+1}$ , and  $B_{t+1}^*$  are:<sup>17</sup>

$$\exp(v_{t})(1/C_{t-1}^{h})^{(\sigma-1)/\sigma}C_{t}^{-1/\sigma} -\beta h E_{t} \exp(v_{t-1})C_{t}^{(h-\sigma h-\sigma)/\sigma}C_{t+1}^{(\sigma-1)/\sigma} = \lambda_{t}$$
(4.7)

$$(M_t/P_t^A)^{-\gamma} + \lambda_t E_t [(1+r_t)^{-1} (P_t^A/P_{t-1}^A) - 1] = 0$$
(4.8)

$$\lambda_t = \beta E_t \lambda_{t+1} (1+r_t) \tag{4.9}$$

$$Q_{t} \lambda_{t} = \beta E_{t} Q_{t+1} \lambda_{t+1} (1 + \kappa_{t}) (1 + r_{t}^{*}). \tag{4.10}$$

<sup>&</sup>lt;sup>15</sup> The model could be expressed in a manner that has governments issuing debt that is a perfect substitute for private bonds of the same country.

<sup>&</sup>lt;sup>16</sup> This is one way of introducing a random "risk-premium" term that reflects temporary (but persistent) departures from uncovered interest parity.

<sup>&</sup>lt;sup>17</sup> Transversality conditions regarding household accumulation of money and bonds are assumed to hold. Other equations that must hold in equilibrium are the government's budget constraint, linking its creation of nominal money to the transfers it pays to the household, and the market-clearing condition  $B_{t+1} = 0$ .

As labor supply is inelastic, another optimality condition is  $N_t^S = 1$  for all t. However, with the price adjustment specification that we employ (detailed in the next section), the level of output will in general differ from its natural — i.e., flexible-price — value; output will be demand-determined and producers will hire the required level of labor input. Thus, there will not be labor market clearing: the realized value of labor supplied will vary from period to period depending on demand conditions, usually departing from the desired level  $N_t^S = 1$ .

As a producer, each household chooses the optimal values of its inputs,  $N_t^d$  and  $IM_t^d$ . This leads to the following pair of conditions for the typical household:

$$[(\lambda_t/\xi_t)\cdot(W_t/P^A_t)]^{1/(1-\nu_1)} = a_1^{1/(1-\nu_1)} A_t^{\nu_1/(1-\nu_1)} (Y_t/N_t^d).$$
 (4.11)

$$[(\lambda_t/\xi_t)\cdot Q_t]^{1/(1-\nu_1)} = (1-a_1)^{1/(1-\nu_1)} (Y_t/IM_t^d).$$
(4.12)

Equations (4.11) and (4.12) indicate that, in a symmetric equilibrium, the aggregate markup (i.e., the ratio of the price level to aggregate marginal cost) is given by  $(\lambda_t/\xi_t)$ . The household has one more decision to make, namely its choice of  $P_t$ . We defer discussion of this until the next section, other than to note that, in common with other general equilibrium models that use Dixit-Stiglitz aggregation (such as Ireland [1997]), our model has the property that under price flexibility, the markup  $(\lambda_t/\xi_t)$  is constant, and equal to  $\theta/(\theta-1)$ .

Define the domestic and foreign nominal interest rates as  $R_t = r_t + E_t \Delta p_{t+1}$  and  $R_t^* = r_t^* + E_t \Delta p_{t+1}^*$ , where  $p_t = \log P_t^A$ ,  $p_t^* = \log P_t^*$  and  $\Delta$  denotes the first difference operator. Then (4.9) and (4.10) imply that, as a first-order approximation, the following uncovered interest parity condition holds:

$$R_t = R_t^* + E_t \Delta s_{t+1} + \kappa_t, \qquad (4.13)$$

where  $s_t = \log S_t$ . Our analysis treats  $R_t^*$ ,  $\Delta p_t^*$  and  $y_t^*$  as exogenous variables. To complete the model, we need to specify laws of motion for these variables as well as the other exogenous processes  $A_t$ ,  $\kappa_t$ , and  $v_t$ . We specify these processes in Section 6. We also need a domestic policy rule for  $R_t$  or  $M_t$ ; in this paper we consider a variety of alternative rules for  $R_t$ , as will be discussed in Section 8. Finally, we need to specify price adjustment behavior, a task to which we now turn.

## 5. Price Adjustment

## 5.1 The P-Bar Model

The typical household has one more choice other than those we have already

analyzed, namely its choice of  $P_t$ , the price that it charges for its output. This section analyzes this decision and thereby introduces our specification of price adjustment. Since the adjustments are gradual, our model belongs to the general category discussed recently by Goodfriend and King (1997).

Taking logs of equations (4.3) and (4.4), we have

$$dy_t^d = dy_t^A - \theta(p_t - p_t^A)$$
 (5.1)

$$ex_t^d = ex_t^A - \theta(p_t - p_t^A)$$
 (5.2)

where lower case letters denote logged variables. Since all output produced is sold to one type of buyer or the other, we have (making log-linear approximations)  $y_t = (1 - EX^{ss}/Y^{ss}) dy_t^d + (EX^{ss}/Y^{ss}) ex_t^d$  and  $y_t^A = (1 - EX^{ss}/Y^{ss}) dy_t^A + (EX^{ss}/Y^{ss}) ex_t^A$ , under the assumption of symmetry across households in their steady-state ratios of exports to output,  $(EX^{ss}/Y^{ss})$ . The following relationship between "relative output",  $y_t - y_t^A$ , and "relative price",  $p_t - p_t^A$ , is implied by (5.1)-(5.2):

$$y_t - y_t^A = -\theta(p_t - p_t^A). (5.3)$$

Let  $\bar{y}^A_t$  denote the log level of total domestic output that would be produced under price flexibility, and  $\bar{p}^A_t$  the log price level that supports  $\bar{y}_t$ . (Section 5.2 discusses the definition of  $\bar{v}^A_t$  under our specification of technology). Then from (5.3),

$$v_t - \overline{v}^A_t = -\theta(p_t - \overline{p}^A_t). \tag{5.4}$$

If there were no costs of adjusting prices or output for the household, then its optimal choice of  $p_t$  would be  $\overline{p}_t$  (which, in symmetric equilibrium, would simply be equal to  $\overline{p}^A_t$ ). We suppose, however, that households do *not* set actual prices to this level because, as in McCallum and Nelson (1998), they must set prices one period in advance, and, furthermore, they face costs of adjusting output (relative to capacity) from period to period. Specifically, the household faces the problem:

Minimize

$$E_{t-1} \sum_{j=0}^{\infty} \beta^{j} \{ (p_{t+j} - \overline{p}_{t+j})^{2} + \gamma_{1} (y_{t+j} - \overline{y}_{t+j} - [y_{t+j-1} - \overline{y}_{t+j-1}])^{2} \}$$
 (5.5)

where  $\gamma_1 > 0$ . Output is thus costly to adjust, with the adjustment cost measured in terms of  $(y_{t+j} - \overline{y}_{t+j})$ , the log of output relative to capacity. Expressing costs in terms of this variable, instead of  $y_{t+j}$ , reflects the notion that output adjustment costs principally arise from the implied changes the producer must make in the level of employment of labor input. Changes in  $y_t$  that are matched by a corresponding change in  $\overline{y}_t$  are unlikely to be associated with such costly changes in labor input,

since fluctuations in  $\overline{y}$ , tend to be driven mainly by changes in technology and non-labor inputs.

Define  $\tilde{p}_t = p_t - \bar{p}_t$ ,  $\tilde{y}_t = y_t - \bar{y}_t$ . Also note that from (5.4),  $(\tilde{y}_t)^2 = \theta^2 \tilde{p}_t^2$ . Thus if we define  $c = \gamma_1 \theta^2$ , (5.5) becomes:

minimize 
$$E_{t-1} \sum_{j=0}^{\infty} \beta^{j} \{ (\tilde{p}_{t+j})^{2} + c(\tilde{p}_{t+j} - \tilde{p}_{t+j-1})^{2} \}$$
 (5.6)

The first order condition for this problem is:

$$E_{t-1}[\widetilde{p}_t + c(\widetilde{p}_t - \widetilde{p}_{t-1}) - \beta c(\widetilde{p}_{t+1} - \widetilde{p}_t)] = 0.$$
(5.7)

McCallum and Nelson (1998) show that rearrangement of equation (5.7) establishes that the aggregate price behavior implied by problem (5.5) is the same as that associated with the "P-bar" model of price adjustment (e.g., McCallum [1994]). They additionally show that solving the Euler equation (5.7) produces the following decision rule for  $p_t$ :

$$E_{t-1} \widetilde{p}_t = \phi \widetilde{p}_{t-1}, \tag{5.8}$$

where  $\phi = [1 - (1 - 4\alpha^2\beta)^{1/2}] / 2\alpha\beta$ ,  $\alpha = c / (1 + c + \beta c)$ . McCallum and Nelson (1998) prove that the parameter  $\phi$  lies in (0,1). Furthermore, (5.8) implies via (5.4) that the expectation of the "output gap",  $\tilde{y}_b$ , satisfies the condition

$$\mathbf{E}_{t-1} \widetilde{\mathbf{y}}_t = \phi \widetilde{\mathbf{y}}_{t-1}. \tag{5.9}$$

While this analysis pertains to the individual household's pricing decision, in a symmetric equilibrium equation (5.9) will also apply to the economy-wide aggregate output gap. Therefore, from now on we use the notation  $y_t$ ,  $\bar{y}_t$ ,  $\bar{y}_t$ ,  $ex_t$ , and  $p_t$  to refer to the aggregates of the corresponding individual-household variables. It should be mentioned that equation (5.9) permits a "one-line proof" that the strict version of the natural rate hypothesis, due to Lucas (1972), is valid in the P-bar model: simply apply the unconditional expectations operator to both sides of (5.9) and note the resulting implication that  $E[\bar{y}_t] \equiv 0$ , regardless of the monetary policy in place. This is important because most models with gradual price adjustment (i.e., sticky prices) do not satisfy the strict natural rate hypothesis.

## 5.2 Calculation of Y-bar

In this subsection we describe our definition of flexible price output,  $\overline{Y}$ . The actual value of output is given by (4.2), which we rewrite for convenience:

$$Y_{t} = \left[a_{1}(A_{t}N_{t}^{d})^{\nu_{1}} + (1 - a_{1})(IM_{t}^{d})^{\nu_{1}}\right]^{1/\nu_{1}}$$
(5.10)

Recalling that under price flexibility, labor input equals  $N_t = N_t^S = 1$  for all t, the natural, or flexible-price, level of output is given by

$$\overline{Y}_{t} = \left[ a_{1} (A_{t})^{\nu_{1}} + (1 - a_{1}) \left( I \overline{M}_{t} \right)^{\nu_{1}} \right]^{1/\nu_{1}}$$
(5.11)

where  $\overline{IM}_t$  = the level of imports in period t under price flexibility. To a log-linear approximation, then,

$$\overline{y}_t = (1 - \delta_1) a_t + \delta_1 i \overline{m}_t \tag{5.12}$$

where  $\delta_1 = (1 - a_1)(I\overline{M}^{ss}/\overline{Y}^{ss})^{\nu_1} = (\theta/(\theta - 1))(Q^{ss}IM^{ss}/Y^{ss})$ , ss again denoting steady-state value. 18

Letting  $q_i$  denote the logarithm of  $Q_i$ , we have from (4.12) that

$$im_{t} = y_{t} - (1/(1-v_{1})) \log(\lambda_{t}/\xi_{t}) - (1/(1-v_{1})) q_{t} + (1/(1-v_{1})) \log(1-a_{1}).$$
(5.13)

Under price flexibility,  $(\lambda_t/\xi_t)$  is a constant,  $\theta/(\theta-1)$ . Thus (5.13) implies that, neglecting the intercept term, the value of  $i\overline{m}_t$ , conditional on the value of the real exchange rate, is given by:

$$i\overline{m}_{t} = \overline{y}_{t} - (1/(1-v_{1})) q_{t}.$$
 (5.14)

Then (5.12) and (5.14) together imply

$$\overline{y}_t = a_t - \omega \, q_t, \tag{5.15}$$

where  $\omega \equiv \delta_1 / \{(1 - v_1)(1 - \delta_1)\}$ . Equation (5.15) indicates that the flexible price level of log output,  $\bar{y}_t$ , is a function of both the technology shock and the real exchange rate. This relation displays the route by which exchange rate changes affect the price of domestic goods. Since the P-bar model implies that  $p_t$  is set in response to  $E_{t-1}\bar{p}_t$ , changes in  $s_t$  that affect  $q_t$  lead to rapid changes in  $p_t$ .

# 6. Log-Linearization and Solution of the Model

Let lower case letters denote logarithms of the corresponding upper-case variables. Loglinearizing (4.7), we then have (neglecting constants)

<sup>&</sup>lt;sup>18</sup> The second equality in the definition of  $\delta_1$  follows from the marginal product condition (4.12) as well as the fact that our aggregate supply specification satisfies the natural rate hypothesis.

$$\log \lambda_{t} = \{ (\beta h^{2} \sigma + \beta h \sigma - \beta h^{2} - 1) / (\sigma [1 - \beta h]) \} c_{t}$$

$$-h \{ (\sigma - 1) / (\sigma [1 - \beta h]) \} c_{t-1}$$

$$-\beta h \{ (\sigma - 1) / (\sigma [1 - \beta h]) \} E_{t} c_{t+1} + (1 - \beta h)^{-1} (1 - \beta h \rho_{v}) v_{t}.$$
(6.1)

Also, loglinearizing (4.9) we have

$$\log \lambda_t = \mathcal{E}_t \log \lambda_{t+1} + R_t - \mathcal{E}_t \Delta p_{t+1}. \tag{6.2}$$

Note that (6.1) and (6.2) imply the following expectational difference equation for the change in consumption:

$$\beta g_1 E_t \Delta c_{t+2} + g_2 E_t \Delta c_{t+1} + g_3 E_t \Delta p_{t+1} = g_1 \Delta c_t + g_3 R_t + g_4 v_t. \tag{6.3}$$

Here 
$$g_1 = (h - \sigma h)$$
,  $g_2 = (1 + \beta h^2 - \sigma \beta h^2 - \sigma \beta h)$ ,  $g_3 = \sigma (1 - \beta h)$ , and  $g_4 = -\sigma (1 + \rho_v - \beta h \rho_v^2 + \beta h \rho_v)$ .

In the case of no habit persistence (h = 0), (6.3) collapses to the standard consumption Euler equation relating  $E_t \Delta c_{t+1}$  to the real interest rate. Furthermore, in a closed economy, the h = 0 case would imply a version of the optimizing IS equation that is used in Kerr and King (1996), Woodford (1996), and McCallum and Nelson (1997).

The other equations in our model include:

$$ex_t = \eta q_t + by_t^* \tag{6.4}$$

$$q_t = s_t - p_t + p_t^* \tag{6.5}$$

$$\bar{y}_t = a_t - \omega q_t \tag{6.6}$$

$$R_{t} = R_{t}^{*} + E_{t} s_{t+1} - s_{t} + \kappa_{t}$$
(6.7)

$$x_t = p_t + y_t \tag{6.8}$$

$$\widetilde{y}_t = y_t - \overline{y}_t \tag{6.9}$$

$$y_{t} = (C^{ss}/Y^{ss})c_{t} + (EX^{ss}/Y^{ss})ex_{t}$$
(6.10)

$$E_{t}\widetilde{y}_{t+1} = \phi \widetilde{y}_{t} \tag{6.11}$$

$$im_t = y_t + (1 / \{\theta(1 - v_1)\}) \tilde{y}_t - (1 / (1 - v_1)) q_t.$$
 (6.12)

Here (6.10) shows that output is consumed or exported; imports do not appear because  $Y_t$  is gross output, not value added. Equation (6.11) is our aggregate supply equation (5.9), shifted forward one period. Equation (6.12) is obtained from (5.13) by first using the relationship  $[(\lambda_t/\xi_t)]/[\theta/(\theta-1)] = ([(\lambda_t/\xi_t)\cdot MC_t]/[\theta/(\theta-1)] + MC_t] = (P_t/\overline{P}_t)$ , where  $MC_t$  is aggregate marginal cost, then substituting in (5.4).

We assume that  $a_t$ ,  $\kappa_t$ , and  $v_t$  are univariate exogenous processes with normally distributed innovations (again, omitting constants):

$$a_t = \rho_a a_{t-1} + e_{at},$$
  $e_{at} \sim N(0, \sigma_{ea}^2),$  (6.13)

$$\kappa_{t} = \rho_{\kappa} \kappa_{t-1} + e_{\kappa t}, \qquad e_{\kappa t} \sim N(0, \sigma_{e\kappa}^{2}), \qquad (6.14)$$

$$v_t = \rho_v v_{t-1} + e_{vt},$$
  $e_{vt} \sim N(0, \sigma_{ev}^2).$  (6.15)

There are three foreign exogenous variables,  $R_t^*$ , and  $\Delta p_t^*$ , and  $y_t^*$ . We assume that the first two of these are constant for all t and that log foreign output  $y_t^*$  is an exogenous AR(1) process:

$$y_t^* = \rho_{Y^*} y_{t-1}^* + e_{Y_t^*}, \qquad e_{Y_t^*} \sim N(0, \sigma_{eY^*}^2).$$
 (6.16)

It is straightforward to show that equations (6.1)-(6.2) and (6.4)-(6.12), together with definitional identities and a Taylor-style policy rule for  $R_t$ , may be written in matrix form as

$$A \mathbf{E}_t \mathbf{y}_{t+1} = B \mathbf{y}_t + C \mathbf{z}_t \tag{6.17}$$

Where A is 17 x 17, B is 17 x 17, C is 17 x 6, with  $\mathbf{y}_t = [y_t, ex_t, R_t, q_t, s_t, im_t, \overline{y}_t, \overline{y}_t, x_t, c_t, \log \lambda_t, \Delta x_t, p_t, \Delta p_t, c_{t-1}, x_{t-1}, R_{t-1}]'$ , and  $\mathbf{z}_t = [a_t, \kappa_t, y_t^*, v_t, R_t^*, p_t^*]'$ .

The law of motion for  $z_t$  may be expressed in vector form as  $z_t = \phi z_{t-1} + \varepsilon_t$ , where the elements of  $\phi$  are determined by (6.13)-(6.16) plus the assumed constancy of  $R_t^*$  and  $p_t^*$ . A rational expectations solution to (6.17) is then given by

$$\mathbf{y}_t = \pi_1 \mathbf{k}_t + \pi_2 \mathbf{z}_t \tag{6.18}$$

and

$$\begin{bmatrix} \mathbf{k}_{t+1} \end{bmatrix} = G \begin{bmatrix} \mathbf{k}_{t} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{z}_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{\varepsilon}_{t+1} \end{bmatrix}$$
(6.19)

where  $\mathbf{k}_t = [p_t, \Delta p_t, c_{t-1}, x_{t-1}, R_{t-1}]$ '. The solution thus expresses the endogenous variables in terms of predetermined endogenous variables  $\mathbf{k}_t$  and the exogenous processes  $\mathbf{z}_t$ .

## 7. Model Calibration

We base our calibration of our model's preference parameters on Fuhrer's (1998) estimates. Accordingly, we set the intertemporal elasticity of substitution,  $\sigma$ , to 1/6 (a value which is also close to our estimate in McCallum and Nelson [1998]) and h to 0.8. The IS shocks  $\{v_t\}$  are assumed to be white noise  $(\rho_v = 0)$  and we set their variance  $\sigma_{ev}^2$  to Fuhrer's estimate of  $(0.011)^2$ . We assign  $\beta$  the conventional value of 0.99.

We report results in the next section for both closed and open-economy versions of our model. The closed-economy version sets  $(C^{ss}/Y^{ss}) = 1$ ,  $(EX^{ss}/Y^{ss}) = 0$  in (6.10), and  $\omega = 0$  in (6.6). The open-economy version sets  $(C^{ss}/Y^{ss}) = 0.89$ ,  $(EX^{ss}/Y^{ss}) = 0.11$ ,  $(Q^{ss}IM^{ss}/Y^{ss}) = 0.12$ , and  $v_1 = -2$ . Our choice of  $v_1$  was motivated primarily by the need for an elasticity of substitution in production,  $1/(1 - v_1)$ , that does not generate excessive variability, via the real exchange rate, in  $\overline{y}_t$ . The coefficient  $\delta_1$  in (5.12) is equal to the steady-state markup,  $\theta/(\theta-1)$ , multiplied by the steady state share of imports (in domestic output units) in GDP,  $Q^{ss}IM^{ss}/Y^{ss}$ . We set  $\theta = 6$ , following Ireland (1997). These choices imply  $\delta_1 = 0.144$  and  $\omega = 0.048$ . As in our previous paper (McCallum and Nelson [1998]) we set the aggregate supply parameter  $\phi$  in (6.11) to 0.89.

We assume that the logs of both foreign output and the technology shock are random walks ( $\rho_{I^*} = \rho_a = 1$ ). The innovation variance for foreign output is  $\sigma_{eI^*}^2 = (0.02)^2$ . Our calibration of the technology innovation variance necessarily differs across the closed and open-economy versions of the model. We set  $\sigma_{ea}^2$  so that it generates approximately the same amount of variability of  $\Delta \bar{y}_I$  in both closed and open economy models, where this variability is roughly consistent with the standard deviation of  $\Delta \bar{y}_I$  of 0.028 (annualized) that is estimated in McCallum and Nelson (1998). To this end, we set  $\sigma_{ea}^2 = (0.007)^2$  in the closed-economy version of our model and  $(0.0035)^2$  in the open-economy version. The export demand function (6.4) is assumed to have income and exchange rate elasticities of b = 1 and  $\eta = 0.333$ . Finally, we set the risk premium shock parameters  $\rho_{\kappa}$  and  $\sigma_{e\kappa}^2$  to 0.50 and  $(0.04)^2$ , using values suggested by Taylor (1993b, pp. 84 and 114).

To examine the properties of this model, we have calculated impulse response functions for the main endogenous variables in response to the system's five shocks. For this exercise, we have used the following policy rule, which features

policy responses quite similar to those estimated in Section 3 as actually prevailing in the United States over 1979-1997:

$$R_t = 0.25 \ E_{t-1} \Delta x_t + 0.05 \ E_{t-1} \ \widetilde{y}_t + 0.8 \ R_{t-1} + \varepsilon_{Rt}. \tag{7.1}$$

Impulse response functions, depicting the reaction to unit shocks to  $\varepsilon_{Rt}$ ,  $(1 - \beta h)^{-1} v_t$ ,  $e_{Y^*t}$ ,  $e_{Kt}$ , and  $e_{at}$  are plotted in Figures 1-5, respectively.

The main interest, probably, resides in Figure 1, which gives responses to a unit shock in the monetary policy rule (7.1). In the upper left panel of Figure 1 we see that output drops by 0.4 units in response to a one unit unexpected increase in  $R_t$ . The largest effect is in the first period after the shock and there is considerable persistence, so that about half of the effect still remains 10 quarters after the shock. This response pattern for output is fairly similar to that depicted by Rotemberg and Woodford (1997, p. 306) — and used as one of the three impulse response functions that their estimation procedure seeks to match. The other two functions considered by Rotemberg and Woodford (1997, pp. 321-323) are the responses of inflation and  $R_t$  to an  $R_t$  policy shock. For the  $R_t$  response, our pattern matches the Rotemberg-Woodford VAR pattern rather nicely, although theirs returns to approximately zero after two periods while some effect remains in ours (see the bottom right panel). As for the inflation variable, the maximum response in ours (bottom left panel of Figure 1) is much larger than Rotemberg and Woodford's, but only for a very few periods. There is, evidently, less inflation persistence in our model than is the case in reality, but there is some present in our Figure 1 plot, nevertheless.

The remaining panels in Figure 1 show responses of the price level, the nominal exchange rate, and the real exchange rate. The price level response begins only after one quarter, of course, and bottoms out after four quarters, which is perhaps a bit sooner than in reality. But the contrast with  $s_t$ , the (log of the) nominal exchange rate, is qualitatively as one would expect. Thus the exchange rate response (an appreciation) occurs in the first period and is almost four times as large as the maximum price level response. Then  $s_t$  moves back, as in "overshooting" models, to reflect a much less strong long-term effect. The real exchange rate  $q_t$  moves with  $s_t$  in the first quarter, but returns more quickly to its original value, which reflects long-run monetary neutrality.

In Figure 2, we see that a unit aggregate demand shock leads to an upward jump in output that tails away as time passes. This blip in  $y_t$  leads to a real depreciation (i.e.  $q_t$  rises), because of the increased import demand; via (5.15), the depreciation brings about a drop in  $\overline{y}_t$ . The fall in inflation results from complicated dynamic expectational effects involving both price adjustment and consumption behavior; inflation would rise instead of fall if there was positive serial correlation in the  $v_t$  process. The fall in inflation brings about a small temporary decline in  $R_t$ .

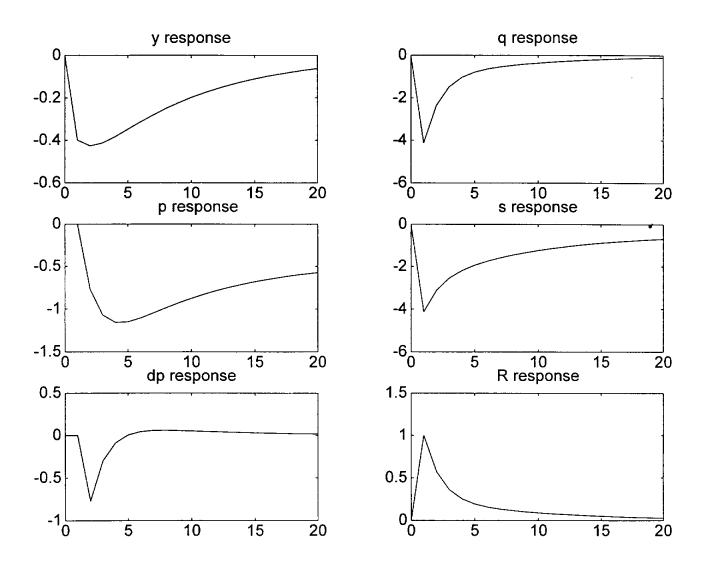
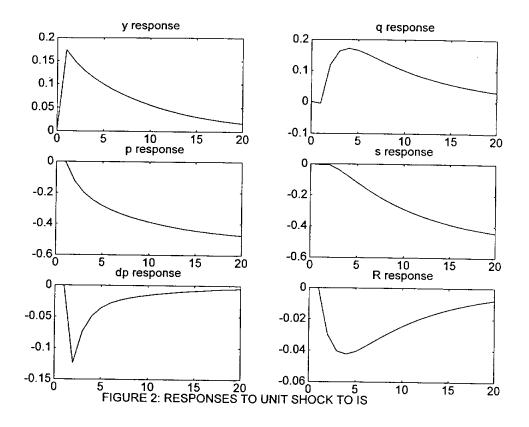


FIGURE 1: RESPONSES TO UNIT SHOCK TO R



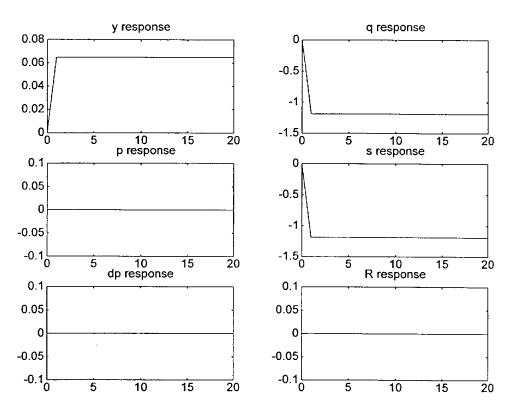


FIGURE 3: RESPONSES TO UNIT SHOCK TO YSTAR

More interesting, perhaps, are the responses to an unexpected unit increase in real income abroad. The dominant effect, according to Figure 3, is a unit appreciation in the exchange rate, both real and nominal. This increases  $\bar{y}_i$  and  $y_t$  hand-in-hand, and leaves  $p_t$ ,  $\Delta p_t$ , and  $R_t$  almost entirely unaffected. Figure 4 shows that a unit increase in  $\bar{y}_t$  — a favorable technology shock — also has no appreciable effect on  $p_t$ ,  $\Delta p_t$ , and  $R_t$  but drives  $q_t$  and  $s_t$  in the opposite direction from a  $y_t^*$  shock. Here the reason is that the increase in  $\bar{y}_t$  leads to a sizable (though smaller) increase in income (output), which involves an upward jump in import demand that can only be satisfied by a depreciation in the real (and nominal) exchange rate. The lack of gradual adjustment in these two figures is a consequence of the exact random-walk nature of the shocks. If stationary AR(1) processes were posited instead, all panels would look quite different.

Finally, we have in Figure 5 the responses to a shock to the "risk premium" term in the UIP relation. From (4.13) it is clear that an upward blip in  $\kappa_t$  will lead to a jump in the same direction in  $s_t$  (and, with sticky prices, in  $q_t$ ). The increase in  $q_t$  leads to an expansion of export demand and therefore to an upward jump in output that wears off as time passes. But what is the explanation for the fall in  $p_t$  shown in the middle left panel, in Figure 5? In the period of the  $q_t$  jump, there is no effect on  $p_t$  since the latter is predetermined. Then there are in succeeding periods expected real exchange rate appreciations, which, by reducing import costs, lead to price level decreases. These wear off as time passes, as the value of  $q_t$  returns to its initial level.

It will be clear from the foregoing discussion that many — perhaps most — of the responses of interest are intimately linked to the open-economy features of our model. If it were closed down, by assigning a value of zero to the parameters  $(EX^{ss}/Y^{ss})$  and  $(Q^{ss}IM^{ss}/Y^{ss})$ , the model would behave quite differently in several respects.

The characteristics of the model's impulse response functions are dependent, in our forward-looking rational expectations setup, on the policy rule. It may be of interest to examine the effects — for the  $R_t$  policy shock only — of two separate alterations of that rule. First, let us set  $\mu_2 = 0$ , thereby eliminating the rule's response to  $\tilde{y}_t$  (which is of questionable significance, according to Section 3). The result of this change is shown in Figure 6. Here the responses are qualitatively similar to those in Figure 1, except that  $p_t$  falls substantially more and does not rise between periods 5 and 20 subsequent to the shock. Thus, in Figure 6 the shock to  $R_t$  has a permanent effect on the price level. Our understanding of this is that in this case the policy rule treats nominal income growth as the target variable, thus permitting

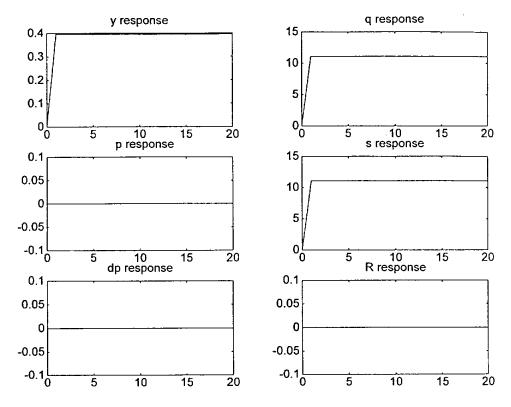


FIGURE 4: RESPONSES TO UNIT SHOCK TO YBAR

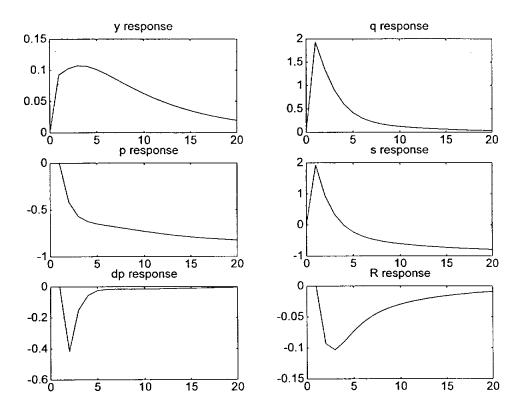


FIGURE 5: RESPONSES TO UNIT SHOCK TO UIP

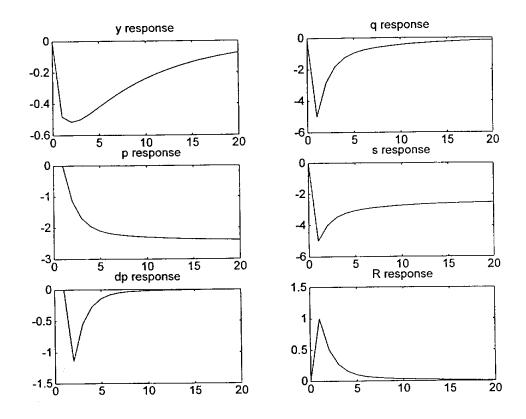


FIGURE 6: RESPONSES TO UNIT SHOCK TO R

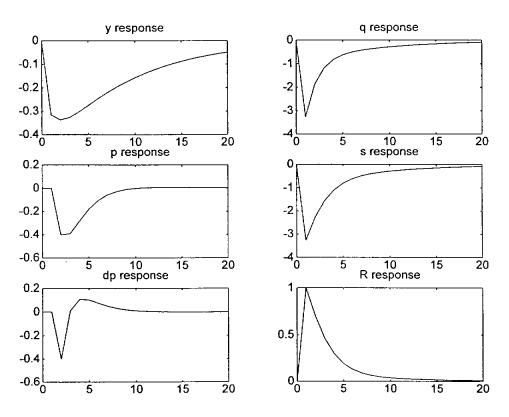


FIGURE 7: RESPONSES TO UNIT SHOCK TO R

drift in the nominal price level  $p_t$ . The nominal exchange rate  $s_t$  in turn inherits unit root behavior from  $p_t$ .

To check on this last conjecture, we have examined two cases in which, with  $\mu_3 = 0.8$ , a coefficient of  $\mu_1 = 0.25$  in the policy rule is attached to  $p_t$  and (alternatively) to  $E_{t-1}x_t$ . The impulse responses for the former cases are shown in Figure 7 (for an  $R_t$  shock). Here it is evident that  $p_t$  and  $s_t$  do indeed ultimately return to their initial values. When we consider the second case, with  $E_{t-1}x_t$  replacing  $p_t$  in the policy rule, the plots are quite similar, except that  $p_t$  climbs back above its initial value for a number of periods beginning 6-7 quarters after the  $R_t$  shock.

## 8. Simulation Results

In this section we report results of stochastic simulations conducted with open and closed-economy versions of the model just described. In all cases we report standard deviations of quarterly values of the inflation rate  $\Delta p_t$ , the output gap  $\tilde{y}_t$ , and the nominal interest rate  $R_t$ . The magnitudes reported are expressed in percentage terms and are annualized in the case of  $\Delta p_t$  and  $R_t$ . In these simulations, constant terms are omitted so the standard deviation of  $\Delta p_t$  is interpretable as the root-mean-square (RMS) deviation from the inflation target value and the standard deviation of  $\tilde{y}_t$  is similarly interpretable as the RMS deviation of  $y_t$  from  $\bar{y}_t$ . In all cases, the reported magnitudes are mean values averaged over 100 replications, with each simulation pertaining to a sample period of 200 quarters (after 53 startup periods are discarded). In calculating the rational expectations solutions, we relied upon Klein's (1997) algorithm.

Before turning to the topic of nominal income targeting, let us consider the background results reported in Table 2 for cases involving several variants of the Taylor rule (Taylor [1993a]) with both the open and closed economy versions of our model. Here the policy rule is of the form

$$R_{t} = \mu_{0} + \mu_{1} \Delta p_{t} + 0.25 \ \mu_{2} \ \widetilde{y}_{t} + \mu_{3} R_{t-1}, \tag{8.1}$$

where  $\mu_0$  is in principle set to deliver the desired average inflation rate and  $\mu_3$  reflects the extent of interest rate smoothing. In the top half of the table a value of  $\mu_3 = 0$  is used, as in the original Taylor rule, whereas  $\mu_3$  equals the more realistic value of 0.8 in the bottom half of the table. The left half of the table pertains to the closed economy version of our model and the right half to the open economy. Within each of the quadrants, we see that larger values of  $\mu_1$  lead to reduced values of the standard deviation of  $\Delta p_i$ , i.e. to lower RMS target-error values for the inflation rate, while increasing the variability of  $\tilde{y}_i$ . In most (but not all) cases, an analogous effect is obtained from increasing  $\mu_2$ , i.e., variability of  $\tilde{y}_i$  is reduced

and  $\Delta p_t$  increased. The quantitative extent of the reduction in  $\tilde{y}_t$  variability is small, however, in comparison to the increased variability of  $\Delta p_t$ . Thus these results are evidently favorable to the idea that (pure) inflation targeting, with  $\mu_1 > 0$  and  $\mu_2 = 0$ , is an attractive policy option.

It is clear from Table 2 that, for almost all policy parameter values, the variability of  $\Delta p_t$ ,  $\tilde{y}_t$ , and  $R_t$  is considerably greater in the open-economy version of our model. It might also be noted that the closed-economy simulations generate much smaller standard deviations of  $\Delta p_t$ ,  $\tilde{y}_t$ , and  $R_t$  than in analogous cases — i.e., with the same policy-rule parameters — reported for a similar model in McCallum and Nelson (1998). Since nearly all other features are the same as in our previous study, it is evident that this reduction in variability stems from use of the habit-formation consumption formulation, which introduces some inertia to aggregate demand (and in the process reduces the variability of  $\overline{y}_t$  and therefore  $\overline{p}_t$ ).

It is also clear from Table 2 that the macroeconomic results obtained with the smoothing parameter  $\mu_3$  set at 0.8 are substantially more desirable than those generated with the original Taylor rule value of 0.0. Since 0.8 is also much more realistic, according to our Section 3 and to the more extensive investigation of Clarida, Gali, and Gertler (1997), we shall henceforth concentrate our attention on policy-rule specifications with  $\mu_3 = 0.8$ .

In Table 3 we turn to the topic of nominal income targeting. In particular, we now compare the macroeconomic consequences of changing rule (8.1) to one of the form

$$R_t = \mu_0 + \mu_1 \Delta x_t + 0.25 \ \mu_2 \ \widetilde{y}_t + \mu_3 R_{t-1}, \tag{8.2}$$

Here  $\Delta x_t$  enters where  $\Delta p_t$  appears in rule (8.1). With  $\mu_2 = 0$ , the rule implies NIT, leavened with realistic interest rate-smoothing behavior. A comparison of the first two columns of Table 3 indicates that with values of  $\mu_1 < 1$ , the NIT policy rule would perform somewhat better than inflation targeting: it would keep  $\Delta p_t$  closer to its target value (and entail less  $R_t$  variability) while keeping variability in  $\tilde{y}_t$  down almost as effectively as the inflation targeting rule. For stronger adjustments of  $R_t$  in response to target misses, however — i.e., with  $\mu_1 > 1$  — substantially more  $\Delta p_t$  and  $R_t$  variability would be generated by the NIT-type rule (8.2).

<sup>&</sup>lt;sup>19</sup> For example, with  $\mu_1 = 1.5$ ,  $\mu_2 = 0.5$ , and  $\mu_3 = 0$ , the standard deviations in our previous paper are 9.67, 2.39, and 13.70

<sup>2.39,</sup> and 13.70.

The two other major differences between our previous paper's model and the closed-economy model in Table 2 are that in our earlier work (i) current output depended on  $E_{t-1}y_{t+1}$ , not  $E_ty_{t+1}$ , a choice which tended to hold down the standard deviations of all three variables in our previous paper, and (ii) only 80% of domestic demand was interest-sensitive, with exogenous variability for the remaining portion.

Table 2
Taylor Rule,  $R_t = \mu_1 \Delta p_t + 0.25 \mu_2 \tilde{y}_t + \mu_3 R_{t-1}$ Reported figures are standard deviations of  $\Delta p_t$ ,  $\tilde{y}_t$ ,  $R_t$  respectively (percent per annum)

	Closed Economy Version				Open Economy Version			
Values of μ <sub>1</sub> , μ <sub>3</sub>	$\mu_2 = 0$	$\mu_2 = 0.25$	$\mu_2 = 0.50$	$\mu_2 = 1.0$	$\mu_2 = 0$	$\mu_2 = 0.25$	$\mu_2 = 0.50$	$\mu_2 = 1.0$
1.0*, 0.0	7.04	9.32	11.39	15.82	14.03	16.27	17.95	20.30
	1.27	1.26	1.23	1.22	1.81	1.69	1.57	1.33
	7.11	9.14	10.97	14.91	14.03	15.91	17.28	19.17
1.5, 0.0	1.49	2.00	2.44	3.47	5.01	5.63	6.03	7.15
	1.35	1.37	1.33	1.36	2.51	2.43	2.29	2.18
	2.24	2.71	3.10	4.07	7.51	7.93	8.09	8.92
3.0, 0.0	0.44	0.58	0.73	1.01	1.85	2.02	2.28	2.62
,	1.39	1.38	1.39	1.36	2.78	2.69	2.65	2.56
	1.33	1.46	1.62	1.95	5.54	5.50	2.60	2.78
0.5, 0.8	1.07	2.31	3.59	5.85	3.38	4.62	5.67	7.56
	1.33	1.29	1.32	1.25	2.36	2.13	1.91	1.65
	1.09	1.69	2.38	3.51	3.27	3.85	4.28	5.15
1.0, 0.8	0.49	0.97	1.45	2.41	1.95	2.48	3.13	4.11
	1.36	1.35	1.32	1.33	2.53	2.38	2.40	2.09
	1.01	1.30	1.60	2.25	3.63	3.83	4.25	4.56
3.0, 0.8	0.16	0.29	0.42	0.69	0.75	0.93	1.11	1.48
	1.41	1.39	1.37	1.35	2.83	2.74	2.64	2.50
	0.98	1.08	1.20	1.47	4.18	4.13	4.14	4.23
* 1.01 in closed-economy cases.								

We have argued previously that a rule specification that requires knowledge of current-quarter values of real GDP is not operational, for such values are very imperfectly known (at least in the United States) even at the end of a quarter. Accordingly, in the third column of Table 3 we have used  $E_{t-1}\Delta x_t$  as the target variable. Somewhat surprisingly, perhaps, the performance is actually better than with hypothetical responses to  $\Delta x_t$ . Qualitatively, however, the comparisons with  $\Delta p_t$  targeting remain as stated in the previous paragraph.

In the two final columns of Table 3 we have combined inflation and (operational) NI targeting with some response to  $E_{t-1}\tilde{y}_t$ . The simulation results indicate an improvement in NI relative to inflation targets for values of  $\mu_1 > 1$ , but diminishes the absolute and relative attractiveness of NIT for values of  $\mu_1 \le 1$ . Overall, inclusion of the  $E_{t-1}\tilde{y}_t$  terms does not seem attractive.

In Table 4 we report results for analogous cases using the open-economy version of our model. In principle, these should be of much greater interest unless the open-economy aspects of our model are especially poorly constructed. Here we find that again the results based on feedback from  $E_{t-1}\Delta x_t$  are somewhat better than those that (unrealistically) assume feedback from  $\Delta x_t$ . In comparison with inflation targeting, there is little basis for preference except in cases with extremely strong feedback — i.e., with  $\mu_1 > 0.5$  — which are more favorable with the  $\Delta p_t$  target. Again, with moderate or small values of  $\mu_1$ , the addition of a response to  $\tilde{y}_t$  (with  $\mu_2 > 0$ ) is arguably counterproductive as it gives rise to much more  $\Delta p_t$  (and  $R_t$ ) variability while reducing variability of  $\tilde{y}_t$  only slightly.

In Table 5 we consider additional rule specifications designed to shed light on various issues concerning nominal income, inflation, and exchange rate targeting. First we consider three cases relating to the issue of growth-rate versus growing-levels targets for nominal variables. In the present case the implicit rate of increase is zero, but that magnitude is irrelevant for the issue at hand. In column 1, the target variable is  $p_i$  so the comparison between this column and the first column of Table 4 constitutes an example of price-level versus inflation targeting. It is interesting to note that for the same values of  $\mu_1$  and  $\mu_3$ , price-level targeting results in less variability (in the model at hand) of the inflation rate than does inflation

<sup>&</sup>lt;sup>21</sup> In McCallum and Nelson (1998, Section 2) we cite evidence suggesting that an end-of-quarter 95% confidence interval for that quarter's growth rate has a width of over 5% (annualized).

We do not make an analogous argument regarding  $\Delta p_t$  because it is, in our model, a predetermined variable. Also,  $\Delta p_t$  has a much smaller one-period-ahead forecast variance.

Table 3
Inflation versus Nominal Income Targeting: Closed Economy
Reported figures are standard deviations of  $\Delta p_t$ ,  $\tilde{y}_t$ ,  $\Delta x_t$ , and  $R_t$  respectively (percent per annum)

		Interest Rate	Rule with Coe	fficient $\mu_1$ on:	. <u>.</u>	
	$\Delta p_t$	$\Delta x_t$	$\mathbf{E}_{t-1}\Delta x_t$	$\Delta p_t$	$\mathbf{E}_{t-1}\Delta x_t$	
Values of				With coefficient 0.25/4		
$\mu_1, \mu_3$				on $E_{t-1}\widetilde{y}_t$		
0.25, 0.8	2.70	1.21	0.93	6.25	4.56	
	1.33	1.37	1.38	1.24	1.29	
	3.26	1.71	1.56	6.73	5.07	
	1.40	0.66	0.61	2.83	2.16	
0.5, 0.8	1.09	0.93	0.48	2.13	1.10	
	1.38	1.40	1.37	1.36	1.35	
	1.85	1.32	1.13	2.83	1.84	
	1.14	0.68	0.47	1.77	1.12	
1.0, 0.8	0.50	0.92	0.52	0.85	0.29	
	1.35	1.38	1.41	1.35	1.39	
	1.42	1.25	1.05	1.71	1.18	
	1.00	1.05	0.43	1.29	0.73	
2.0, 0.8	0.24	0.92	0.56	0.39	0.39	
	1.40	1.39	1.40	1.38	1.40	
	1.29	1.17	1.02	1.40	1.05	
	0.99	1.86	0.41	1.12	0.56	
5.0, 0.8	0.13	0.87	0.58	0.14	0.52	
	1.40	1.34	1.38	1.36	1.38	
	1.23	1.05	1.02	1.25	1.03	
	1.00	4.09	0.39	0.99	0.45	

inflation rate targeting.<sup>23</sup> It entails more variability of  $\tilde{y}_i$ , as conjectured by McCallum (1997a, p. 235; 1997b) and many others, but not vastly more.

Next, column 2 of Table 5 treats  $x_t$  as the target variable, as in nominal income levels targeting, while column 3 features a more operational version with  $E_{t-1}x_t$  as the feedback variable. Here we see that in comparison with the NI growth rate rules in Table 4, the results are slightly better with levels targeting when the response coefficient  $\mu_1$  is small, but are distinctly poorer for large values of  $\mu_1$ . The Table 4 figures also permit a comparison of  $p_t$  vs.  $x_t$  or  $E_{t-1}x_t$  targets as well. In that regard it is clear that  $p_t$  targeting delivers much less variability of  $\Delta p_t$  than does either variant of NI targeting, but entails somewhat greater variability of  $\tilde{y}_t$ .

In columns 4 and 5 we consider the use of *future* inflation,  $\Delta p_{t+1}$ , as the variable that is responded to. Results for  $E_t \Delta p_{t+1} = \Delta p_{t+1}$  and  $E_{t-1} \Delta p_{t+1}$  are reported. It can be seen that with  $\mu_1$  values of 0.25, 0.50, and 1.0, performance is much poorer with respect to inflation variability as compared with a rule that responds to  $\Delta p_t = E_{t-1} \Delta p_t$ . With small  $\mu_1$  values and the  $E_{t-1} \Delta p_{t+1}$  variable, it is the case that  $\tilde{y}_t$  variability is reduced relative to that in column 1 of Table 4. But all in all, these results are not supportive of the belief that it is appropriate to focus on future inflation.<sup>24</sup> We do not want to make too much of this result, however, since it is rather obviously a property of our particular price-adjustment specification (which implies little inflation inertia in most cases).

Finally, Table 5 includes columns in which the feedback response is to the exchange rate variables  $s_t$  and  $\Delta s_t$ . The former, in column 6, implies a fixed exchange rate target, whereas column 7 is for a targeted constant rate of depreciation (here, zero). Clearly, inflation variability is very large when the exchange rate is targeted, but  $\tilde{y}_t$  variability is held down quite effectively. In our model this occurs because the exchange rate target entails policy responses to reduce the variability of  $s_t$ . This holds down variability of  $s_t$  and that in turn prevents  $s_t$  from fluctuations as large as those that are typical in Table 4.

<sup>&</sup>lt;sup>23</sup> Svensson (1997b) proves that this will be the case with optimal discretionary policy in a particular model, but the present result is for a rule that is not optimal or discretionary, and is obtained in a very different model. Our finding is nevertheless at least partially supportive of the spirit of Svensson's argument.

<sup>&</sup>lt;sup>24</sup> With values of  $\mu_1 \ge 1.8$ , Klein's algorithm gives a solution that is not the minimum state variable solution. We have determined this by examining the generalized eigenvalues for a number of values of  $\mu_1$ . See McCallum (1998) for a very brief discussion of the minimum state variable criterion and its relation to Klein's algorithm.

Table 4
Inflation versus Nominal Income Targeting: Open Economy
Reported figures are standard deviations of  $\Delta p_t$ ,  $\tilde{\gamma}_t$ ,  $\Delta x_t$ , and  $R_t$  respectively (percent per annum)

	Interest Rate Rule with Coefficient µ <sub>1</sub> on:						
	$\Delta p_t$	$\Delta x_t$	$E_{t-1}\Delta x_t$	$\Delta p_t$	$E_{t-1}\Delta x_t$		
Values of				With coeff	icient 0.25/4		
$\mu_1, \mu_3$				on $E_{t-1}\widetilde{y}_t$			
0.25, 0.8	5.53	5.56	4.92	8.11	7.71		
	1.95	1.88	2.07	1.53	1.65		
	5.78	5.70	5.22	8.25	7.81		
	2.85	2.01	2.16	3.97	3.59		
0.5, 0.8	3.36	3.92	3.31	4.48	4.26		
	2.37	2.08	2.33	2.18	2.14		
	4.11	4.32	3.96	5.01	4.75		
	3.27	2.47	2.63	3.97	3.32		
1.0, 0.8	1.96	2.75	2.18	2.46	2.50		
	2.59	2.07	2.57	2.57	2.45		
	3.38	3.33	3.31	3.67	3.53		
	3.68	3.15	3.13	4.13	3.44		
2.0, 0.8	1.08	1.90	1.48	1.29	1.56		
	2.72	1.91	2.69	2.69	2.72		
	3.17	2.56	3.12	3.25	3.16		
	3.99	4.46	3.48	4.17	3.72		
5.0, 0.8	0.46	1.12	1.01	0.55	0.99		
	2.81	1.56	2.84	2.83	2.80		
	3.21	1.69	3.08	3.23	3.09		
	4.22	7.40	3.74	4.31	3.81		

Table 5 Miscellaneous Open Economy Results Reported figures are standard deviations of  $\Delta p_t$ ,  $\widetilde{y}_t$ ,  $\Delta x_t$ , and  $R_t$  respectively (percent per annum)

	Interest Rate Rule with Coefficient μ <sub>1</sub> on:							
Values of	$p_t$	$x_t$	$E_{t-1} x_t$	$\Delta p_{t+1}$	$E_{t-1}\Delta p_{t+1}$	$S_t$	$\Delta s_t$	
$\mu_1, \mu_3$			_					
0.25, 0.8	2.74	4.27	4.17	7.02	10.39	13.61	9.83	
	2.48	2.12	2.23	1.96	1.09	1.55	2.15	
	3.69	4.59	4.59	7.17	10.37	13.72	10.05	
	2.23	1.44	1.54	3.55	1.74	6.76	8.40	
0.5, 0.8	2.05	3.87	3.72	4.69	7.60	13.32	9.64	
	2.50	2.06	2.32	2.50	1.63	1.52	1.64	
	3.37	4.22	4.24	5.26	7.66	13.49	9.85	
	2.69	1.92	1.93	4.41	2.35	9.05	11.60	
1.0, 0.8	1.43	3.38	3.38	3.02	5.06	12.90	9.66	
	2.68	2.01	2.28	2.92	2.04	1.52	1.48	
	3.23	3.76	4.03	4.25	5.38	13.12	9.89	
	3.22	2.70	2.18	5.54	2.97	11.43	14.23	
2.0, 0.8	0.94	2.89	3.15	2.01*	3.36*	12.07	9.86	
	2.75	1.79	2.40	3.18*	2.28*	1.54	1.49	
ļ	3.19	3.27	3.82	3.99*	4.08*	12.36	10.13	
	3.62	4.13	2.49	6.29*	3.38*	13.65	15.94	
5.0, 0.8	0.47	2.06	2.99	**	**	11.07	10.00	
1	2.81	1.44	2.39	**	**	1.63	1.61	
	3.23	2.33	3.77	**	**	11.42	10.35	
	4.01	7.05	2.67	**	**	15.73	17.24	

<sup>\*</sup> Calculated with  $\mu_1 = 1.75$ . \*\* Minimum state variable solution not obtainable with existing software.

#### 9. Conclusion

In this paper we have developed stochastic simulation results pertaining to the performance of nominal income targeting, represented by a policy rule that sets quarterly values of an interest rate instrument in response to departures of nominal income from its specified target path. Performance is evaluated principally in terms of root-mean-square deviations of inflation and real output from desired paths, but some attention is also paid to the implied variability of the interest rate instrument. Thus our analysis views nominal income as a potential intermediate target variable, comparable to an intermediate target based on expected future inflation rates, as in inflation targeting as currently practiced by several major central banks.

Other studies meeting the foregoing description have been conducted previously, of course. Our intention here is to improve upon existing studies by conducting the analysis in the context of a structural macroeconomic model that is carefully designed to respect both neoclassical economic theory and actual empirical regularities. Accordingly, the basic theoretical framework is one in which individual economic agents are depicted as solving dynamic optimization problems with rational expectations, as in the real business cycle literature. The model presumes, however, that prices do not adjust freely within each period but instead respond gradually — so it belongs to the general category of models recently surveyed by Goodfriend and King (1997). The specific price adjustment mechanism utilized here is a variant of the P-bar model, which differs from most alternative sticky-price formulations by conforming to the strict version (Lucas [1972]) of the natural-rate hypothesis.

Relative to our previous work (McCallum and Nelson [1998]), the present model features two major improvements. First, the agents' intertemporal utility function is not time-separable but instead reflects "habit formation," in a manner suggested by the recent estimates of Fuhrer (1998). This modification lends some inertia to agents' consumption choices and results in econometric estimates in which much less explanatory power stems from unobserved residuals, according to Fuhrer (1998). Second, the model economy is one that is open to international flows of goods and securities. In our setup, imports are intermediate goods used in production of the finished goods which are either consumed by the economy's households or exported, while uncovered interest parity holds but with a timevarying "risk premium" that is first-order autoregressive. Because real exchange rate changes affect capacity (i.e., natural-rate) output, they have rapid affects via the P-bar relation on the prices of produced goods. Both of these changes have major effects on the model's properties. Quantitatively, the model is calibrated so as to match central features of the post-Bretton Woods quarterly data for the United States economy.

Substantively, our results suggest that nominal income targeting deserves serious consideration as a monetary policy strategy. For most policy-parameter configurations, NIT gives rise to root-mean-square values for inflation and  $y_t - \overline{y}_t$  that are approximately the same magnitude as those provided by inflation targeting. Somewhat surprisingly, inclusion of the expected output gap (i.e.,  $E_{t-1}[y_t - \overline{y}_t]$ ) as an additional variable to which the instrument responds, is rather unproductive according to our results: such an inclusion sharply increases inflation variability while reducing output gap variability only very slightly. Also, our results indicate that the use of growing-level as contrasted with growth-rate target paths is quite attractive whereas rules that respond to expected future inflation rates perform less well than those that respond to current (predetermined) inflation. Finally, use of the nominal exchange rate as the target variable leads to greatly increased variability of inflation.

All of these results are, unfortunately but inevitably, model dependent. We hope in future work to improve upon our price adjustment specification, to consider rules with a monetary base instrument variable, and to examine the robustness of our findings to various aspects of the model's specification.

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## Appendix A

It is our impression that when most analysts speak of monetary policy targeting they do so in the way that we have in this paper, e.g., "X-targeting" is a regime in which the central bank sets its instrument according to a rule involving responses to deviations of X from its desired path. In a number of recent papers, Svensson (1997a, 1998; Rudebusch and Svensson [1998]) has argued for a different terminology, one that identifies X-targeting as a regime in which the central bank (i) has deviations of X from its desired path as one argument of its objective function, and (ii) behaves optimally in light of its model of the economy. While we applaud Svensson's desire for terminological precision, we are not persuaded that adoption of his proposed terminology is warranted.

Svensson's basic criticism of traditional terminology is as follows. A rule that responds to deviations of X does not constitute targeting because "to target X" means "using all relevant information to bring [X] in line with the target path" (Svensson [1998, p. 2]; Rudebusch and Svensson [1998, p. 8]). And in typical cases, optimal instrument rules will entail responses to other variables in addition to X. But here "optimal" actually means optimal with respect to one particular objective function and one particular model of the economy. But the point of a simple rule such as  $R_t = \mu_0 + \mu_1(\Delta x_t - \Delta x^*) + \mu_3 R_{t-1}$  is that with  $\mu_1 > 1 - \mu_3$  it will call for  $R_t$  adjustments that will keep  $\Delta x_t$  close to its target value  $\Delta x^*$ , without being dependent upon any particular objective function or model. For the merits of this "robustness" approach to rule design, see McCallum (1997b, Section 3).

Furthermore, the stated basic criticism of traditional terminology is evidently applicable only when there is only a single variable appearing in the central bank's objective function. For if both X and Z appear in the objective function, then optimal behavior does not involve bringing either variable fully into line with its target path, but rather in achieving a specified compromise between doing so for either of the two variables. That multiple objective function arguments create difficulties for the proposed terminology is suggested by two aspects in Svensson's papers. First, the arguments regarding this issue are typically expressed in terms of cases in which only one variable is relevant — see Svensson (1998, p. 2) and Rudebusch and Svensson (1998, pp. 8-9). Second, multiple variable cases have led in practice to questionable terminology. For example, optimal central bank behavior when both  $\Delta p_i$  and  $\tilde{y}_i$  appear in its objective function is referred to by Svensson (1998) and Rudebusch-Svensson (1998) as "flexible inflation targeting." But there seems to be no logical basis for that choice over "flexible output-gap targeting."

A second reason for retaining the traditional language is that it corresponds more closely, in our judgment, to actual practice of "inflation targeting" as represented by the central banks of Canada, New Zealand, and the United Kingdom. That is, these central banks adjust their interest rate instruments in response to (expected future) inflation rates only, except under exceptional circumstances. Furthermore, these central banks have also evidently not possessed well-formulated utility functions (relating to their goals, which involve the output gap as well as inflation) or explicit quantitative models considered appropriate for use in optimization exercises. In expressing this judgment — this interpretation of actual practice — we realize that it is not shared by Svensson.

The foregoing discussion should not be interpreted as lacking sympathy with Svensson's desire for improved terminology, or as expressing disagreement with particular substantive aspects of his highly productive policy rule research. It should be understood, rather, as a defense of our reluctance to condemn the widespread practice of referring to a rule that responds to X alone as "X-targeting." Our extension of this practice to cases in which instrument smoothing is undertaken constitutes sheer terminological expediency: we should actually refer to such cases as "X-targeting with smoothing."

We do not dispute that existing terminology is somewhat unsatisfactory, but at least it has no pretensions to precision. The proposed replacement terminology is also less than fully satisfactory, as explained above, so we believe that the case for change is not compelling. Terminological conventions are just that, conventions; if altered too frequently they become useless and may distract attention from important substantive matters.