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SOVEREIGN DEBT AND CONSUMPTION  
SMOOTHING

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**ABSTRACT**

This paper shows that whether or not a sovereign can borrow to smooth consumption depends both on how consumption smoothing is achieved, whether by contingent debt issuance or by contingent debt servicing, and on the exact nature of the penalty for debt repudiation. If a sovereign that repudiated its debt could not borrow again, but could continue to save and to dissave, then contingent debt issuance, without contingent debt servicing, cannot support a positive amount of uncollateralized sovereign debt. But, under this same specification of the penalty for repudiation, contingent debt servicing supports a positive amount of uncollateralized sovereign debt.

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The existing literature widely recognizes that sovereign debt is often used to smooth consumption intertemporally in the face of a stochastic income stream. The literature, however, reveals no consensus on whether or not this motive for debt issue can support a positive amount of uncollateralized sovereign debt, without a binding commitment to service debt. In the analysis that follows, we show that whether or not a sovereign can borrow to smooth consumption depends both on how consumption smoothing is achieved and on the exact nature of the penalty for debt repudiation.

In some models — see, for example, Jonathan Eaton and Mark Gersovitz (1981) and V. V. Chari and Patrick Kehoe (1993) — consumption smoothing is achieved by making debt issuance contingent on the realization of income. Specifically, the sovereign issues additional debt whenever it has a low realization of income and the sovereign retires debt whenever it has a high realization of income. In these models, debt servicing depends on the amount of accumulated debt and the interest rate, but debt servicing is not contingent on the realization of income. In equilibrium, the sovereign always services its debt in full.

In other models — see, for example, Herschel Grossman and John Van Huyck (1988) — consumption smoothing is achieved by making debt servicing contingent on the realization of income. Specifically, the sovereign issues an amount of debt that depends, *inter alia*, on the probability distribution of income and on the interest rate, but does not depend on the realization of income. To smooth consumption, the sovereign services its debt in full only when it has a high realization of income. In the event of a low realization of income, the sovereign defaults either partially or fully.

In any case, if the sovereign is unable to make a binding commitment to service its debt and also does not collateralize its debt, then in the event of a high realization of income the sovereign will be tempted to repudiate its debt.<sup>1</sup> Models of sovereign debt typically

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<sup>1</sup>The appropriate definition of repudiation depends on whether debt issuance or debt servicing is contingent on income. Without contingent debt servicing, repudiation is equivalent to failure to service the accumulated debt in full. In contrast, with contingent debt servicing, repudiation occurs only if the sovereign fails to service its debt according to the understood debt-servicing schedule. In their model of contingent

assume that lenders use a two-part strategy to deter the sovereign from repudiating. First, lenders impose a ceiling on the amount of debt that the sovereign can issue. Second, lenders would punish repudiation by denying the sovereign access to further loans.

The ambiguity in the literature involves the exact specification of the options that would remain open to a sovereign that suffered this punishment. Various authors have considered at least three different possibilities, each of which has radically different implications.

At one extreme, Jeremy Bulow and Kenneth Rogoff (1989) assume that, even if a sovereign that repudiated its debt could not borrow again, it could continue to smooth consumption by buying a standard insurance policy against low realizations of income. Under this assumption, regardless of whether debt issuance or debt servicing is contingent on income, the ability to borrow to smooth consumption would have no value for the sovereign, and the penalty of no further borrowing would not deter repudiation. Consequently, a sovereign could not issue any uncollateralized debt. This model, however, has limited interest both because standard insurance policies against low realizations of income do not seem to exist and because sovereigns actually seem to issue large amounts of uncollateralized debt.

At another extreme, many discussions abstract from saving by the sovereign. Examples include Eaton and Gersovitz (1981), section 2.1, Grossman and Van Huyck (1988), Tim Worrall (1990), and Eaton (1993). With no saving, the penalty of no further borrowing would cause the sovereign's future consumption stream to match exactly its realized future income stream. Under this assumption, regardless of whether debt issuance or debt servicing is contingent on income, the penalty of no further borrowing generally would be a sufficient deterrent to repudiation that lenders would allow a positive debt ceiling. This model, however, also has limited interest because it would always seem possible, even in autarky, for a sovereign to save.

In contrast to both of these uninteresting models, other discussions — see, for example,

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debt servicing Grossman and Van Huyck distinguish repudiation from excusable default, which occurs when the sovereign understandably fails to service its debt in full because of a low realization of income.

Eaton and Gersovitz (1981), section 2.2, Eaton, Gersovitz, and Joseph Stiglitz (1986), and Chari and Kehoe (1993) — have suggested a more realistic specification of the penalty for repudiation, according to which a sovereign that repudiated its debt could not borrow again but could continue to save and to dissave. We now show that under this specification of the penalty for repudiation contingent debt issuance, without contingent debt servicing, cannot support a positive amount of sovereign debt.

### 1. Contingent Debt Issuance

Assume that debt issuance is contingent on the realization of income. Moreover, as long as the sovereign services its debt and as long as it has not reached a ceiling on accumulated debt, it can continue to issue new debt at a constant interest rate. Thus, if the sovereign services its debt in period  $t$  and plans to continue to service its debt in all future periods, then consumption in every period  $t+i$ , denoted by  $c_{t+i}$ , is given by

$$(1) \quad c_{t+i} = y_{t+i} - rb_{t+i-1} + b_{t+i} - b_{t+i-1}, \quad i = 0, 1, 2, \dots,$$

where  $y_{t+i}$  is the stochastic realization of income in period  $t+i$ ,  $r$  is the interest rate,  $b_{t+i-1}$  is the amount of debt accumulated through period  $t+i-1$ , and  $b_{t+i} - b_{t+i-1}$  is either the amount of additional debt issued or the amount of debt retired in period  $t+i$ .

If the realization of income in any period is large enough that consumption smoothing would call either for debt to be retired or for debt servicing to exceed new debt issued, then consumption in that period would be larger if the sovereign were to repudiate its accumulated debt than if the sovereign services its debt. In this event, the sovereign would be tempted to repudiate its debt and this temptation would be greater the larger the amount of accumulated debt. Without either a binding commitment to service debt or collateralization, the only deterrent to repudiation is that repudiation would preclude future borrowing. But, given that the value of the ability to smooth consumption is finite, the value of the ability to borrow in the future is also finite. Accordingly, there is a maximum amount of accumulated debt such that the temptation to repudiate would not outweigh the deterrent to repudiation. Let

$\bar{b}$  denote this amount of accumulated debt. Lenders will not knowingly allow the sovereign's accumulated debt to exceed  $\bar{b}$ .

Suppose that  $b_{t-1} = \bar{b}$  — that is, suppose that the sovereign already has reached its debt ceiling. If  $b_{t-1} = \bar{b}$ , and if the sovereign services its debt in period  $t$ , then equation (1) implies that current and future consumption are subject to the following constraints:

$$(2) \quad E_t \sum_{\tau=t}^{\infty} (1+r)^{t-\tau} (y_{\tau} - c_{\tau}) = \bar{b}$$

and

$$(3) \quad E_t \sum_{\tau=t}^n (1+r)^{t-\tau} (c_{\tau} - y_{\tau}) \leq 0 \text{ for all values of } n = t, t+1, \dots,$$

where  $E_t$  denotes an expectation conditional on information available in period  $t$ .

Condition (2) is a solvency constraint. It says that, if the sovereign services its accumulated debt, then the expected present value of current and future consumption is less than the expected present value of current and future income by the amount of current accumulated debt.

Condition (3) represents the assumption that the sovereign has reached its debt ceiling and cannot accumulate any more debt. Condition (3) says that over any horizon the expected present value of current and future consumption cannot exceed the expected present value of current and future income. Condition (3) allows the sovereign to borrow and to consume in excess of its income in some future period if and only if it has first repaid some debt.

Assume further that, if the sovereign were ever to fail to service its debt, then it could never issue any new debt, although it could continue to save and to dissave. Thus, if the sovereign were not to service its debt in period  $t$ , then current and future consumption would be subject to the following constraints:

$$(4) \quad E_t \sum_{\tau=t}^{\infty} (1+r)^{t-\tau} (y_{\tau} - c_{\tau}) = 0$$

and

$$(5) \quad E_t \sum_{\tau=t}^n (1+r)^{t-\tau} (c_{\tau} - y_{\tau}) \leq 0 \text{ for all values of } n = t, t+1, \dots$$

Condition (4) is an alternative solvency constraint. It says that, if the sovereign were to repudiate its accumulated debt, then the expected present value of current and future consumption would equal the expected present value of current and future income. The difference between condition (4) and condition (2) represents the temptation to repudiate the accumulated debt. If  $\bar{b}$  were positive, then repudiation would permit the average level of consumption to be unambiguously higher.

Condition (5) represents the assumption that repudiation would preclude future borrowing. Condition (5) allows the sovereign to consume in excess of its income in some future period if and only if it has first accumulated some savings. The critical observation is that condition (5) is identical to condition (3). Specifically, condition (5) would constrain the sovereign's ability to smooth future consumption exactly as does condition (3).

Taken together, conditions (2) – (5) imply that, if the debt ceiling  $\bar{b}$  were positive, then once its accumulated debt had reached  $\bar{b}$  the sovereign by repudiating its debt would increase its average future consumption without reducing its ability to smooth future consumption. Put another way, once it has reached its debt ceiling the sovereign gains nothing in the way of additional consumption smoothing by incurring the cost of continuing to service its debt. Accordingly, without a binding commitment to service debt and without collateralization, lenders cannot allow the sovereign a positive debt ceiling.

Chari and Kehoe (1993) derived this same result, albeit with more complicated mathematics, but Chari and Kehoe abstracted from the possibility of contingent debt servicing. We argue next that under the same assumption that a sovereign that repudiated its debt could not borrow again, but could continue to save and to dissave, contingent debt servicing can support a positive amount of sovereign debt.

## 2. Contingent Debt Servicing

Assume that the realization of income during any period  $\tau$ ,  $y_\tau$ , can be either  $y_H$ , which is a high probability event, or  $y_L$ , which is a low probability event, or  $y_{LL}$ , which

is a very low probability event, where  $y_H > \bar{y} > y_L > y_{LL}$ , and where  $\bar{y}$  denotes the average of realizations of income. In other words, the realization of income is larger than average with high probability, smaller than average with low probability, and much smaller than average with very low probability. Let  $\rho$  denote the interest rate on risk-free assets. Assume further that prior to each period the sovereign issues an amount of debt  $\hat{b}$ , where  $\hat{b} = (\bar{y} - y_{LL})/(1 + \rho)$  and that the sovereign invests the proceeds from this debt issue in risk-free assets. Thus, at the end of period  $\tau$ , when the sovereign has realized income  $y_\tau$ , the sovereign also has assets worth  $(1 + \rho)\hat{b}$ .

Let  $s_\tau$  denote the amount that the sovereign spends on servicing its debt at the end of period  $\tau$ . Assume that the sovereign services its debt according to the state-contingent schedule

$$(6) \quad s_\tau = y_\tau - \bar{y} + (1 + \rho)\hat{b}.$$

We can interpret equation (6) as follows: If the realization of income is high, a high probability event, then the sovereign services its debt in full. If the realization of income is low, a low probability event, then the sovereign defaults partially on its debts. If the realization of income is very low, a very low probability event, then the sovereign defaults fully on its debts.

Full debt-servicing implies paying an interest rate,  $r$ , in excess of the risk-free interest rate. Specifically, if  $y_\tau$  equals  $y_H$ , then

$$1 + r = \frac{s_\tau}{\hat{b}} = \frac{y_H - y_{LL}}{\bar{y} - y_{LL}}(1 + \rho).$$

Equation (6) also implies that the expected return to the lenders equals the risk-free interest rate. In addition, equation (6) implies that  $s_\tau$  is non-negative in all states, including the worst state, in which  $y_\tau$  equals  $y_{LL}$ .

Given this debt-servicing schedule, consumption in each period  $\tau$  is

$$(7) \quad c_\tau = y_\tau + (1 + \rho)\hat{b} - s_\tau = \bar{y} \text{ for any realization of } y_\tau.$$



In this simple example contingent debt servicing makes  $c_\tau$  independent of  $y_\tau$  and, hence, results in complete consumption smoothing. The amount of debt  $\tilde{b}$  is just large enough so that even in the worst state, in which  $y_\tau$  equals  $y_{LL}$ , the sovereign, by defaulting on its debts, would be able to avoid a decrease in consumption. Moreover, because in this example the lenders do not require a return in excess of the risk-free interest rate, complete consumption smoothing is costless, and consumption always equals  $\bar{y}$ .

Let  $U_t$  denote the present value of current and expected future utility from consumption, given the realization of  $y_\tau$  in period  $\tau = t$ . Assume that

$$(8) \quad U_t = u(c_t) + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u(c_\tau),$$

where the utility function,  $u(c_\tau)$ , is increasing and concave, and the discount factor,  $\beta$ , is a positive fraction. If in period  $\tau = t$  the sovereign services its debt according to equation (6) and expects to continue to service its debt according to equation (6) in future periods, then  $U_t$  equals  $U_t^*$ , where

$$(9) \quad U_t^* = u(\bar{y}) + \frac{\beta}{1-\beta} u(\bar{y}).$$

Assume further that, if in period  $t$  the sovereign were to provide less debt servicing than the amount required by equation (6), then it would be unable to issue debt in future periods. In that event, as we have seen, it would be able to consume in excess of its income in any future period if and only if it has first accumulated some savings. Accordingly, if in period  $t$  the sovereign were to succumb to the temptation to repudiate its debt, then, setting  $s_t$  equal to zero,  $U_t$  would equal  $U_t^o$ , where

$$(10) \quad U_t^o = u[y_t + (1 + \rho)\tilde{b}] + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u(c_\tau^o),$$

and where  $c_\tau^o$  is the value of  $c_\tau$  that in each future period would be consistent with maximizing  $E_\tau \sum_{x=\tau}^{\infty} \beta^{x-\tau} u(c_x^o)$  subject to condition (5). Note that the expected value of  $c_\tau^o$  equals  $\bar{y}$ , but, because condition (5) precludes complete consumption smoothing,  $c_\tau^o$  is stochastic.

Comparing  $U_t^*$  with  $U_t^o$  we see that, if  $y_t$  equals  $y_{LL}$ , so that that  $s_t$  as given by equation (6) equals zero, then  $y_t + (1 + \rho)\tilde{b}$  equals  $\bar{y}$ . But, if the realization of  $y_t$  is larger than  $y_{LL}$ , so that  $s_t$  as given by equation (6) is positive, then  $y_t + (1 + \rho)\tilde{b}$  is larger than  $\bar{y}$ . Also, the difference between  $y_t + (1 + \rho)\tilde{b}$  and  $\bar{y}$  is an increasing function of  $y_t$ . Thus, the better is the realized state, the greater is the temptation to repudiate the debt.

But, given diminishing marginal utility, because  $c_\tau^o$  is stochastic,  $\frac{\beta}{1-\beta}u(\bar{y})$  is larger than  $E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} u(c_\tau^o)$ . Thus, the attraction of being able to smooth consumption in the future counters the temptation to repudiate the debt. If the utility function is sufficiently concave, then  $U_t^*$  is as large as  $U_t^o$  for all possible realizations of  $y_t$ . In this case, contingent debt servicing supports complete consumption smoothing, with the sovereign able to issue an amount of debt equal to  $\tilde{b}$ .

More generally, even if  $U_t^*$  is not as large as  $U_t^o$  for all possible realizations of  $y_t$ , contingent debt servicing would support the issuance of a smaller, but positive amount of debt. In this event, the sovereign would use both saving and dissaving and contingent debt servicing to smooth consumption. But, the sovereign would not be able to smooth consumption completely. For example,  $c_\tau$  could be the same for  $y_\tau$  equal to  $y_H$  and for  $y_\tau$  equal to  $y_L$ , but smaller for  $y_\tau$  equal to  $y_{LL}$ . Nevertheless, as long as the sovereign can issue a positive amount of debt, contingent debt servicing would permit more consumption smoothing than would saving and dissaving alone.

The essential observation is that, because there is a finite limit to accumulated savings, as there is a finite limit to accumulated debt, the possibility of contingent saving, like the possibility of contingent debt issuance, would not permit complete smoothing of consumption. Thus, even if the sovereign could save and dissave, it would still be valuable to be able to issue debt with contingent servicing in order to achieve more complete consumption smoothing. Accordingly, the sovereign will resist the temptation to repudiate its debt as long as the amount of debt outstanding is not too large.

### 3. Summary

What is the critical difference between consumption smoothing by means of contingent debt servicing and consumption smoothing by means of contingent debt issuance that makes a positive amount of sovereign debt possible with contingent debt servicing but not with contingent debt issuance? The answer is that with contingent debt issuance once the sovereign reaches any positive debt ceiling repudiation would not reduce the possibilities for future consumption smoothing, whereas with contingent debt servicing repudiation always would reduce the possibilities for future consumption smoothing.

Grossman and Van Huyck (1988) motivated their model of contingent debt servicing by arguing that viewing sovereign debt as a contingent claim introduced an important element of realism into the analysis of sovereign debt. They argued that their contingent-claim model was useful in explaining why actual defaults are associated with identifiably bad states of the world, why defaults are usually partial rather than complete, and why sovereigns often are able to borrow again soon after default. The present note suggests further that state-contingent debt servicing also is a critical element in understanding the very existence of sovereign debt. Specifically, we have shown that without contingent debt servicing, a sovereign could not use uncollateralized debt to smooth consumption.

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