Introduction to Spatial Econometrics

James LeSage
Texas State University-San Marcos
San Marcos, Texas, U.S.A.

R. Kelley Pace
Louisiana State University
Baton Rouge, Louisiana, U.S.A.

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

© 2009 by Taylor & Francis Group, LLC

Contents

1 Introduction 1
- 1.1 Spatial dependence . . . 1
- 1.2 The spatial autoregressive process . . . 8
  - 1.2.1 Spatial autoregressive data generating process . . . 12
- 1.3 An illustration of spatial spillovers . . . 16
- 1.4 The role of spatial econometric models . . . 20
- 1.5 The plan of the text . . . 22

2 Motivating and Interpreting Spatial Econometric Models 25
- 2.1 A time-dependence motivation . . . 25
- 2.2 An omitted variables motivation . . . 27
- 2.3 A spatial heterogeneity motivation . . . 29
- 2.4 An externalities-based motivation . . . 30
- 2.5 A model uncertainty motivation . . . 30
- 2.6 Spatial autoregressive regression models . . . 32
- 2.7 Interpreting parameter estimates . . . 33
  - 2.7.1 Direct and indirect impacts in theory . . . 34
  - 2.7.2 Calculating summary measures of impacts . . . 39
  - 2.7.3 Measures of dispersion for the impact estimates . . . 39
  - 2.7.4 Partitioning the impacts by order of neighbors . . . 40
  - 2.7.5 Simplified alternatives to the impact calculations . . . 41
- 2.8 Chapter summary . . . 42

3 Maximum Likelihood Estimation 45
- 3.1 Model estimation . . . 46
  - 3.1.1 SAR and SDM model estimation . . . 46
  - 3.1.2 SEM model estimation . . . 50
  - 3.1.3 Estimates for models with two weight matrices . . . 52
- 3.2 Estimates of dispersion for the parameters . . . 54
  - 3.2.1 A mixed analytical-numerical Hessian calculation . . . 56
  - 3.2.2 A comparison of Hessian calculations . . . 59
- 3.3 Omitted variables with spatial dependence . . . 60
  - 3.3.1 A Hausman test for OLS and SEM estimates . . . 61
  - 3.3.2 Omitted variables bias of least-squares . . . 63
  - 3.3.3 Omitted variables bias for spatial regressions . . . 67
- 3.4 An applied example . . . 68
  - 3.4.1 Coefficient estimates . . . 69

© 2009 by Taylor & Francis Group, LLC

3.4.2 Cumulative effects estimates 70
3.4.3 Spatial partitioning of the impact estimates 72
3.4.4 A comparison of impacts from different models 73
3.5 Chapter summary 75

4 Log-determinants and Spatial Weights **77**
4.1 Determinants and transformations 77
4.2 Basic determinant computation 81
4.3 Determinants of spatial systems 84
4.3.1 Scalings and similarity transformations 87
4.3.2 Determinant domain 88
4.3.3 Special cases 89
4.4 Monte Carlo approximation of the log-determinant 96
4.4.1 Sensitivity of ρ estimates to approximation 100
4.5 Chebyshev approximation 105
4.6 Extrapolation 108
4.7 Determinant bounds 108
4.8 Inverses and other functions 110
4.9 Expressions for interpretation of spatial models 114
4.10 Closed-form solutions for single parameter spatial models . . 116
4.11 Forming spatial weights 118
4.12 Chapter summary 120

5 Bayesian Spatial Econometric Models **123**
5.1 Bayesian methodology 124
5.2 Conventional Bayesian treatment of the SAR model 127
5.2.1 Analytical approaches to the Bayesian method 127
5.2.2 Analytical solution of the Bayesian spatial model . . . 130
5.3 MCMC estimation of Bayesian spatial models 133
5.3.1 Sampling conditional distributions 133
5.3.2 Sampling for the parameter ρ 136
5.4 The MCMC algorithm 139
5.5 An applied illustration 142
5.6 Uses for Bayesian spatial models 145
5.6.1 Robust heteroscedastic spatial regression 146
5.6.2 Spatial effects estimates 149
5.6.3 Models with multiple weight matrices 150
5.7 Chapter summary 152

6 Model Comparison **155**
6.1 Comparison of spatial and non-spatial models 155
6.2 An applied example of model comparison 159
6.2.1 The data sample used 161
6.2.2 Comparing models with different weight matrices . . . 161
6.2.3 A test for dependence in technical knowledge 163

© 2009 by Taylor & Francis Group, LLC

6.2.4 A test of the common factor restriction 164
6.2.5 Spatial effects estimates 165
6.3 Bayesian model comparison 168
6.3.1 Comparing models based on different weights 169
6.3.2 Comparing models based on different variables 173
6.3.3 An applied illustration of model comparison 175
6.3.4 An illustration of MC^3 and model averaging 178
6.4 Chapter summary 184
6.5 Chapter appendix 185

7 Spatiotemporal and Spatial Models **189**
7.1 Spatiotemporal partial adjustment model 190
7.2 Relation between spatiotemporal and SAR models 191
7.3 Relation between spatiotemporal and SEM models 196
7.4 Covariance matrices 197
7.4.1 Monte Carlo experiment 200
7.5 Spatial econometric and statistical models 201
7.6 Patterns of temporal and spatial dependence 203
7.7 Chapter summary 207

8 Spatial Econometric Interaction Models **211**
8.1 Interregional flows in a spatial regression context 212
8.2 Maximum likelihood and Bayesian estimation 218
8.3 Application of the spatial econometric interaction model . . 223
8.4 Extending the spatial econometric interaction model 228
8.4.1 Adjusting spatial weights using prior knowledge 229
8.4.2 Adjustments to address the zero flow problem 230
8.4.3 Spatially structured multilateral resistance effects . . . 232
8.4.4 Flows as a rare event 234
8.5 Chapter summary 236

9 Matrix Exponential Spatial Models **237**
9.1 The MESS model 237
9.1.1 The matrix exponential 238
9.1.2 Maximum likelihood estimation 239
9.1.3 A closed form solution for the parameters 240
9.1.4 An applied illustration 241
9.2 Spatial error models using MESS 243
9.2.1 Spatial model Monte Carlo experiments 246
9.2.2 An applied illustration 247
9.3 A Bayesian version of the model 250
9.3.1 The posterior for α 250
9.3.2 The posterior for β 252
9.3.3 Applied illustrations 253
9.4 Extensions of the model 255

© 2009 by Taylor & Francis Group, LLC

9.4.1 More flexible weights 255
9.4.2 MCMC estimation 256
9.4.3 MCMC estimation of the model 257
9.4.4 The conditional distributions for β, σ and V 258
9.4.5 Computational considerations 259
9.4.6 An illustration of the extended model 260
9.5 Fractional differencing 265
9.5.1 Empirical illustrations 270
9.5.2 Computational considerations 275
9.6 Chapter summary 277

10 Limited Dependent Variable Spatial Models **279**
10.1 Bayesian latent variable treatment 281
10.1.1 The SAR probit model 283
10.1.2 An MCMC sampler for the SAR probit model 284
10.1.3 Gibbs sampling the conditional distribution for y^* . . 285
10.1.4 Some observations regarding implementation 287
10.1.5 Applied illustrations of the spatial probit model 289
10.1.6 Marginal effects for the spatial probit model 293
10.2 The ordered spatial probit model 297
10.3 Spatial Tobit models 299
10.3.1 An example of the spatial Tobit model 302
10.4 The multinomial spatial probit model 306
10.4.1 The MCMC sampler for the SAR MNP model 307
10.4.2 Sampling for β and ρ 308
10.4.3 Sampling for Σ 308
10.4.4 Sampling for $\tilde{y}^*$ 310
10.5 An applied illustration of spatial MNP 312
10.5.1 Effects estimates for the spatial MNP model 314
10.6 Spatially structured effects probit models 316
10.7 Chapter summary 320

References **323**

© 2009 by Taylor & Francis Group, LLC