

Land Use and Density in Cities with Congestion*

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It was well documented that monocentric spatial models with congestion require driving tolls to generate market efficiency. Because driving and location are equivalent, tolling congestion is the same as regulating density. This paper shows that internalizing the congestion externality always requires upward adjustments to market density—which are greatest at the urban center. This holds whether or not transportation capacity is optimally provided. Simulations suggest optimal cities should have central densities that are orders of magnitude greater than market cities. © 1998 Academic Press

I. INTRODUCTION

The dense development of urban land, such as occurs in cities throughout the world, frequently is viewed as a social cost. The notion that density is a negative *public* good also was theoretically formulated, and received some empirical support in the economics literature (Richardson [17], Diamond and Tolley [6]). In many countries, land use policies re-inforce this view by encouraging lower densities through minimum lot size zoning, height limits, and yard setbacks. In contrast, spatial equilibrium models of the urban land market suggest that density actually generates a *positive* externality. Greater density in these models shortens worker commuting distances and hence reduces the most studied of urban externalities—traffic congestion. With a fixed number of commute trips, pricing congestion and regulating density are identical policies. The central question of this paper is what would city density look like if the congestion externality was correctly internalized? Should urban densities be higher than market densities? If so, by how much, and is the difference uniform across locations?

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The literature on urban density and congestion was quite well formulated, but the results of the current paper were only hinted at or implied in numerous previous papers. This is probably because much of the focus was on developing results about optimal transportation policy rather than about land development policy. The earliest papers are by Mills and de Ferranti [12] and Solow-Vickery [19]. These assume fixed density, and either spatial commuting or freight shipment between locations. This travel generates congestion, whose cost gives rise to land rent in a private market. If roadways are built so as to equate marginal benefits with costs, market land rents turn out not to be the correct cost with which to obtain land for roadways—because of the congestion externality. Solow [18] greatly improves on these efforts by making density fully endogenous, but obtains the same result: a divergence between private land rents and the “shadow rent” for land to be used in transportation.

Over the intervening years, a series of papers continued to probe the more general question of what (land) resources should be devoted to transport capacity in an urban spatial model: Kanemoto [10, 11], Arnott [2], Pines and Sadka [15], Fujita [7]. The results hinge crucially on whether the congestion externality is internalized. If the appropriate road tolls are used to correct the congestion externality then private market land rents were shown to provide the right price signal for allocating land to transportation capacity. When tolls are not in effect, then the transportation investment decision becomes what in public finance is called a “Second-Best” decision. In this case, the shadow rent for land (road capacity) will exceed private market rents at the center, but be less at the urban border.

In all of these papers, when the shadow rent for land differs from market rent because congestion was not internalized, the implication is that land density will also be inefficient. The literature, however, was not very explicit about these differences. More importantly, when congestion tolls *are* in effect, how does the market rent for land compare to that when congestion is not internalized? The closest result to the one here is in Kanemoto [11]. Kanemoto compares the market rent for land when congestion tolls are in place and when optimal transport capacity is provided, with the market rent for land when there are no tolls and when transport capacity is not Second-Best. Social rents are higher (and so presumably density), but it is not clear how much of this difference is due to the widely different transportation capacities in effect. Fujita [7] carefully develops the optimal congestion toll under a variety of transportation investment levels (land allocation rules), but never compares density gradients. Against this background, the objectives of the current paper are two:

1. First, the paper shows that the full internalization of congestion *always* requires *upward*, but *differential*, adjustments to market density.

The adjustments are highest at the center and zero at the urban border, and can be achieved either by direct land regulation or by congestion tolls. Further, these upward density adjustments are necessary for *any* pattern of transport capacity, including the optimal one. Transport capacity policy is not a complete substitute for such land use corrections.

2. Second, the current paper develops these results with an approach that differs from that in the literature. Previous research either allocated land to maximize utility (e.g., Solow, Kanemoto), or to maximize *private* rent (Fujita). In contrast, this paper directly allocates resources to maximize aggregate rent. The distinction between the two is subtle, but important. Private rent is maximized when land at each location is individually developed to its highest use. With externalities, this is not equivalent to maximizing aggregate rent. A land use pattern that maximizes aggregate rent is shown to fully internalize all externalities, while a land use pattern that maximizes the private rent at each site still requires corrective taxes or subsidies.

The next section of the paper lays out the approach of directly maximizing aggregate rent paying ability to illustrate the symmetry between this and the traditional consumer optimization problem without externalities. The third section adds transportation congestion to this model, and derives the fundamental result, when transportation capacity is exogenous. In the fourth section, the allocation of land between residential use and transportation capacity is incorporated, to demonstrate the density adjustments that are required when transportation capacity is provided optimally. Finally, in the fifth section I present some numerical simulations to illustrate the magnitude of the density adjustments that are necessary in the case of a simple transportation technology. Even with very modest congestion, optimal central densities are shown to be orders of magnitude greater than market densities.

II. RENT MAXIMIZING LAND USE WITHOUT CONGESTION

It is important to emphasize at the outset that the problem raised in this paper presumes that identical consumers are mobile across locations. Hence with locational arbitrage consumers derive equal welfare at all sites within a city. Accordingly, as Mirrlees [13] has shown, in some contexts, this itself need not be optimal, but here attention is restricted to a class of planning problems in which a government must treat equals equally. Hence any maximization of aggregate rent paying ability is subject to a constraint on utility.

Land use that maximizes aggregate urban rent can be different from that which results when land is allocated to the maximum rent paying use. The latter notion was first introduced by Alonso [1] and Herbert-Stevens

[8]. Wheaton [20] and most recently Fujita [7] have shown that this principle yields the equivalent resource allocation to utility maximization. *When land is allocated to the highest rent paying use, aggregate land rent will be maximized only in the absence of externalities.* When there are externalities, it may be optimal to develop land at some locations to less than maximum rent—in order to achieve rent gains at other locations. Directly allocating resources to maximize aggregate rent achieves this, while allocating land to the highest rent paying use does not—unless corrective taxes or subsidies are in place. Developing land to its highest use is what multiple individual property owners will achieve through trade. Developing land to yield the highest aggregate rent is what a single monopolistic land owner seeks to achieve.¹

It is instructive to first see the equivalence between maximizing aggregate rent and the private market allocation of resources in a model without congestion externalities. Consider the traditional monocentric city where N consumers commute to a central employment zone from a distance t , that ranges from zero to as far out as the city border b . Consumer utility depends on “other” consumption x , as well as land q , and transportation costs (including time) are exogenous. The well known market solution to this problem has the consumption of q increasing with commute distance along a compensated demand curve. This requires that market land rents decline with distance at a rate equal to the savings in travel expenditure—per acre of residential development. The particular level of compensated utility must be determined so that all N households are situated, while the urban border occurs at that distance where urban rents equal some alternative opportunity cost (R_a).

The market solution previously described, easily can be achieved by determining density directly so as to maximize aggregate rent paying ability (income minus all non-land expenditure). This is done with the constrained optimization problem shown in the expression.

$$\text{Max}_{x(t), q(t), b} : \int_0^b \left[\frac{y - T(t) - x}{q} \right] 2\pi t dt + [A - \pi b^2] R_a + \beta_1 [U(x, q) - U_0], \quad (1)$$

Where: N : urban household population

$x(t)$: consumption of other goods

$q(t)$: consumption of land

y : household income (uniform)

¹Brueckner [5] demonstrates that a single shopping center owner will maximize aggregate rent, as opposed to allocating space to the highest bidder. The latter is what will happen in a commercial district with individual parcel ownership. With inter-store externalities, the two resource allocations are quite different.

$T(t)$: exogenous cost of travel to distance t

b : border of urban land use

R_a : opportunity rent of urban land

A : total land area

$U(x, q)$: household utility

Expression (1) is a particularly simple problem of variational control, in which there are only the two control variables, $x(t)$, $q(t)$, and the scalar b . There are no state variables, or equations of motion. Applying, for example, the calculus of variations, the condition (2) must hold at all t , while (3) holds at the border. Then

$$\left[\frac{y - t(t) - x}{q} \right] = \frac{U_q}{U_x}; \quad U(x, q) = U_0, \quad (2)$$

$$\left[\frac{y - T(b) - x_b}{q_b} \right] = R_a. \quad (3)$$

Equations (2) and (3) are exactly those conditions that characterize a market equilibrium in an "open" monocentric city, when market urban land rent $R(t)$ equals rent paying ability, $(y - T - x)/q$, or the left-hand side of either equation. In the open version of the model, utility levels are exogenous, while the city population is endogenous. The "closed" city version is obtained by simply determining U_0 so that (4) holds at the rent maximizing solutions to b and $q(t)$.

$$\int_0^b \frac{2\pi t}{q(t)} dt = N. \quad (4)$$

III. RENT MAXIMIZING LAND USE WITH CONGESTION, BUT EXOGENOUS TRANSPORT CAPACITY

When congestion is introduced into the model just described, the optimization problem in (1) is expanded in two ways. First, an intermediate variable is created for convenience, equal to the number of households residing beyond a distance t , $n(t)$. Following Solow [18], this variable represents the number of commuters passing distance t on their way into work (i.e., travel demand). Second, the marginal cost of travel at t , $T'(t)$, now becomes a positive function of this travel demand, and a negative function of the transportation capacity provided at distance t . Transportation capacity comes from devoting a fraction of the land at t , $v(t)$, to highways (as opposed to residential use). Following the transportation

literature, the marginal travel cost function $c(-)$ depends on the ratio of demand to capacity.²

With these conditions, the optimization problem now becomes a more complete variational control model, shown in.

$$\begin{aligned} \text{Max}_{x(t), q(t), b} : & \int_0^b \left[\frac{y - T(t) - x}{q} \right] 2\pi t [1 - v] dt + [A - \pi b^2] R_a \\ & + \beta_1 [U(x, q) - U_0], \end{aligned} \quad (5)$$

subject to:

$$\begin{aligned} T'(t) &= c \left(\frac{n(t)}{2\pi t v} \right), & \text{costate variable: } Z_1, \\ n'(t) &= \frac{-2\pi t [1 - v]}{q(t)}, & \text{costate variable: } Z_2, \end{aligned}$$

with boundary conditions:

$$T(0) = 0, \quad n(b) = 0,$$

where: N : urban household population

$x(t)$: consumption of other goods

$q(t)$: consumption of land

y : household income (uniform)

$T(t)$: cost of travel to distance t

b : border of urban land use

R_a : opportunity rent of urban land

A : total land area

$U(x, q)$: household utility

$n(t)$: number of households living beyond t

$v(t)$: Percent of land at t devoted to transportation

$c(-)$: marginal travel cost function.

For a closed city model, the utility level U_0 must again be selected so that every household gets situated. This requires choosing U_0 so that $n(0) = N$, using the $n(-)$ function that emerges from the optimization of (5). In problem (5), $x(t)$ and $q(t)$ again are control variables, while T and n are now state variables. The Hamiltonian function for the optimization

²The results of the paper do not require the assumption that the congestion function $c(-)$ is homogenous (depends on the ratio: $n(t)/2\pi t v$). It is made only for ease of exposition, and because the literature suggests that empirically this is the case. It also is assumed that both c' and c'' are positive (Keeler-Small [9]).

problem is in (6), where the "dynamic" lagrangian or costate variable for T 's equation of motion is Z_1 , while that for the n equation is Z_2 . Then

$$H = \left[\frac{y - T(t) - x}{q} \right] 2\pi t[1 - v] + Z_1 c \left(\frac{n(t)}{2\pi tv} \right) - Z_2 \left[\frac{2\pi t(1 - v)}{q} \right] + \beta_1 [U(x, q) - U_0]. \quad (6)$$

Following optimal control theory, the first necessary condition for maximizing (5) is that the Hamiltonian (6) must be optimized at each location with respect to q , x , and b . Thus at all locations, $t < b$, Eq. (7) must hold while (3) continues to hold at the urban border. So

$$\frac{U_q}{U_x} = \left[\frac{y - (T(t) + Z_2) - x}{q} \right] = \left[\frac{y - T(t) - x}{q} \right] - \frac{Z_2}{q}. \quad (7)$$

In terms of the costate variables, we know that each must equal zero at the urban border, since the Hamiltonian (H) has no terminal condition that depends upon $T(b)$ or $n(b)$. Again following control theory, the second necessary condition for optimization is that the change in each costate variable across locations equals the derivative of the Hamiltonian with respect to the corresponding control variable. Thus (8) describes the full properties of Z_1 and Z_2 . Then

$$\begin{aligned} Z'_1 &= -\frac{\partial H}{\partial T} = \frac{2\pi t(1 - v)}{q} > 0; & Z_1(b) &= 0, \\ Z'_2 &= -\frac{\partial H}{\partial n} = -\frac{Z_1 c'}{2\pi tv} > 0; & Z_2(b) &= 0. \end{aligned} \quad (8)$$

Comparing the condition for Z'_1 in (8) with the equation of motion and boundary condition for n in (5), it is clear that $Z_1(t) = -n(t)$. Incorporating this equality into the second equation of (8), Z'_2 equals $c'[n(t)/2\pi tv]$, and it is easy to interpret the costate variable $Z_2(t)$. As $n(t)$ travelers pass through distance t on their inward commute, Z'_2 represents the marginal increase in their travel time that results when an extra resident is placed at t (and hence joins them on the road). An extra resident locating at t , however, will also have an impact on the time per mile of these commuters at all locations inward of t . Integrating these marginal impacts, $-Z_2$ is interpreted as the social cost of locating an additional resident at distance t . When divided by land consumption, $-Z_2/q$ becomes the social cost of developing or consuming land at that distance. Thus in the marginal

condition (7), the right-hand side represents the full shadow rent for land: composed of the normal private willingness-to-pay, $(y - T(t) - x)/q$, plus the annual social cost of land use, $-Z_2/q$. The pattern of Z_2 across different locations is easy to infer.

Because the costate variable Z_2 is zero at the urban border, there is no social cost of land consumption there. Given the congestion function, and with no residents living beyond b , this makes obvious sense.³ Moving inward from the border, $-Z_2$ rises, and $-Z_2/q$ does so even more rapidly (because q is falling). Thus the wedge between the private willingness to pay for land and its full shadow rent will be zero at the border and significantly greater at the urban center. In Eq. (7), this clearly implies that private decisions about land consumption are efficient at the urban border, but there will be over consumption of land in the urban center. Market density gradients are not steep enough.

To get a better idea of how the private willingness to pay and the true shadow rent for land differ, the slopes of these two rent gradients can be compared. Denote the private willingness to pay, $[y - T(t) - x]/q$, as WTP and the full shadow rent, $WTP - Z_2/q$, as R . The ratio of the slopes of the two rent gradients is⁴

$$\frac{R'}{WTP'} = 1 + \frac{c'}{c} \frac{n}{2\pi tv}. \quad (9)$$

The right-hand side of (8) is one plus the ratio of marginal-to-average congestion costs (per mile). Most congestion functions exhibit increasing marginal costs, as the ratio of demand/capacity rises. In most cases this means that the ratio on the right-hand side of (9) will be greater than one and thus the full right-hand side will exceed two: the social rent gradient could be at least twice as steep as the private willingness to pay.⁵

It is instructive to consider policies that might achieve the optimal density gradient within a private land market with atomistic ownership. The traditional solution, a congestion toll, could be implemented with a tax per mile of *driving* equal to Z'_2 . Because residential location and

³In a more complicated inter-temporal growth model, it might well be the case that the consumption of land at the current border would impose (discounted) social costs on future residents. Planning transport capacity for future growth was recently researched by Braid [4].

⁴To determine the slope of each rent gradient, the envelope theorem can be applied: WTP is maximized with respect to q in a private market, while R is so maximized in the aggregate rent maximizing solution.

⁵If $c = (n/2\pi tv)^\beta$, then the right-hand side of (9) equals $1 + \beta$, where $\beta > 1$ to meet the condition of increasing marginal congestion costs.

Comparing the ratios in (9) is only a partial equilibrium analysis. The slope of each gradient depends on the solution values for $q(t)$ and $n(t)$, which of course would be different in a general equilibrium solution with and without social pricing.

driving mileage are identical, it might easier to simply create a tax or subsidy of Z_2 that would be attached to *living* at each location.

It is tempting to suggest that an ad valorem land "tax" on private market rent might achieve efficiency, with a rate that varied upward from zero at the border. However, this will not work. Any tax on land consumption purely within the urban border merely reduces private market rents, rather than raising the total rent paid by urban users (Polinsky–Rubinfeld [16]). Still within a privately operating land market, governments might attempt to regulate development decisions with some form of (MLS) *maximum* lot size zoning!

It is possible to imagine institutional changes or ownership patterns that could implement the efficient solution. With public land ownership, a revenue maximizing city government or planning agency would implement or decree the appropriate density gradient. Single private ownership of a city, such as sometimes occurs with resorts or "new towns" would also create the correct incentive to develop land efficiently.

IV. RENT MAXIMIZING LAND USE WITH CONGESTION AND OPTIMAL TRANSPORT CAPACITY

When transportation capacity is provided optimally, the distortion between private and socially desirable land consumption still remains—at least qualitatively. Optimal capacity investment does not alleviate the need for corrective land pricing and land density adjustments, although it will surely change their magnitude. To see this, the optimization problem in (5) is expanded to make the fraction of land devoted to roads at each distance $v(t)$ a control variable along with $x(t)$ and $q(t)$. In all other respects the problem is identical to that in (5). Along the optimal trajectory, the following additional marginal condition must hold at all locations t . Then

$$-\frac{Z_1 c' n}{v 2 \pi t} = c' \left[\frac{n}{v 2 \pi t} \right]^2 = \left[\frac{y - T(t) - x}{q} \right] - \frac{Z_2}{q}. \quad (10)$$

Interpreting (10) is straight forward. The marginal benefit of expanding capacity and reducing congestion (the left-hand side) must equal the full shadow rent of land (the right-hand side). Thus in the "first best" city, the use of land for either highway capacity or residences must be based on the full and optimal shadow rent. There are no mitigating factors to consider, or counter balancing distortions to impose, as occur in the "second best" investment rule, where residential land use is based only on market rents (Solow [18], Arnott [2], Kanemoto [10]).

Following the investment rule (10), as the shadow rent for land falls with increased commuting distance, the ratio of demand to capacity, $n/v 2 \pi t$, likewise will fall. This follows since both c' and c'' are positive. Thus, with

first best land use and infrastructure investment, the time/mile spent commuting will fall or travel speeds will rise as one moves out further.

V. NUMERICAL EXAMPLES

A major objective of this paper is to place the qualitative conclusions previously described into a more concrete context. Just how distorted is market density? How much more dense should central areas be? Solving the optimization problem in (5) is quite simple as long as explicit equations can be developed for the marginal conditions in (7), (8), and (10). To answer the distortion question most directly, numerical solutions to the optimization problems will be compared to market equilibrium solutions under identical parameters. All of these simulated cities are based on the following functional and parameter specifications. So

$$\text{Utility function: } U^0 = xq^\alpha, \quad \alpha = 0.1,$$

$$\text{Travel cost function: } c(-) = 80 \left[\frac{n(t)}{2\pi tv(t)40,000} \right]^\beta, \quad \beta = 1.0,$$

$$y = \$40,000,$$

$$R_a = \$4,000,000 \text{ (annually, per sq mile),}$$

$$N = 3,000,000.$$

The parameter values in the preceding text reflect approximate values for large modern cities. Roughly one third of consumer expenditure is housing related, and roughly one third of that goes to land, hence $\alpha = 0.1$. The parameters of the congestion function are chosen to provide realistic congestion conditions, at capacity levels typically found in U.S. cities. Setting the parameter $\beta = 1.0$ is very conservative, but makes the equations particularly easy to solve. The numerical simulations also begin the residential portion of the city at a radius of 1 mile, and assume that commuting within this ring is costless. The land within a ring of this size is sufficient to contain the city's workers at typical central business district office density levels. Finally, in all simulations, $1/N$ of total land rental payments are added to consumer income when determining equilibrium indifference levels U^0 . Thus utility levels will capture the benefits to both land owners and users.

Four simulations are presented in Table 1. The first two, labelled Mkt1 and Opt1, are based on exogenous road capacity levels, in which 30% of the land at each location is set aside for highways. In most U.S. cities, this fraction actually ranges from about 35% in urban centers to 15% in lower density residential suburbs (Keeler-Small [9], Solow [18]). The Mkt1 simulation is purely a market outcome, while the Opt1 simulation presents the solution to (5). Thus $R(t)$ represents the shadow rent for land (in the Opt1 solution) but the private willingness to pay (in the Mkt1 solution).

TABLE 1
City Simulations *

| Simulation | <i>Mkt1</i> | <i>Opt1</i> | <i>Opt2</i> | <i>Mkt2</i> |
|------------|-------------|-------------|-------------|-------------|
| U_0 | 24246 | 25301 | 27872 | 27230 |
| b | 28.9 | 25.9 | 27.6 | 28.4 |
| $R(1)$ | 46.3m | 198.5m | 39.6m | 16.5m |
| $R(b)$ | 4.0m | 4.0m | 4.0m | 4.0m |
| $q(1)$ | 0.000084 | 0.000023 | 0.000110 | 0.00024 |
| $q(b)$ | 0.00078 | 0.00081 | 0.00088 | 0.00087 |
| $T(1)$ | 0 | 0 | 0 | 0 |
| $T(b)$ | 8588 | 7600 | 4510 | 5240 |
| $Z_2(1)$ | 0 | -7600 | -4510 | 0 |
| $Z_2(b)$ | 0 | 0 | 0 | 0 |
| $v(1)$ | 0.3 | 0.3 | 0.99 | 0.99 |
| $v(b)$ | 0.3 | 0.3 | 0.001 | 0.001 |

* Distances in miles, area units in square miles, land rents in dollars per square mile, m = million.

In the first two simulations, the optimal city has a shadow rent at its center that is *more than 4 times* that in the private market city (\$198 million versus \$46 million). Given the utility function, this generates central land consumption that is roughly one fourth as high as in the market solution (0.000023 versus 0.000084). Optimal central density levels are thus 4 times as great as market density. At the border, shadow and market rents are equal, although land consumption in the optimal city is slightly higher. This results because the border in the optimal city is closer (by 3 miles), and this in turn gives the resident at the urban edge somewhat more money after transportation expenses.

After redistributing land (or shadow) rental income, residents in the optimal city wind up with a level of utility that is higher by roughly \$1000.⁶ This is close to the difference in commuting expenses that edge residents must pay in each of the two cities. This makes sense, because the edge resident in each city has similar land expenditures, and so travel savings go directly into other expenditures. While total welfare improvements of this magnitude (4%) might seem small, the gain is much higher when judged against the consumption category being reallocated: land expenditure and travel are only 10% of income.

In Fig. 1, the difference between market and optimal density is traced out across locations.⁷ It is interesting that virtually all of the difference in

⁶Because the marginal utility of income is one with this utility function, the change in utility (measured in utility units) is a first order approximation to compensated variation (measured in dollars).

⁷Given a utility function with constant expenditure shares, the corresponding rent shadow-rent gradients look identical to the density gradients in Fig. 1.

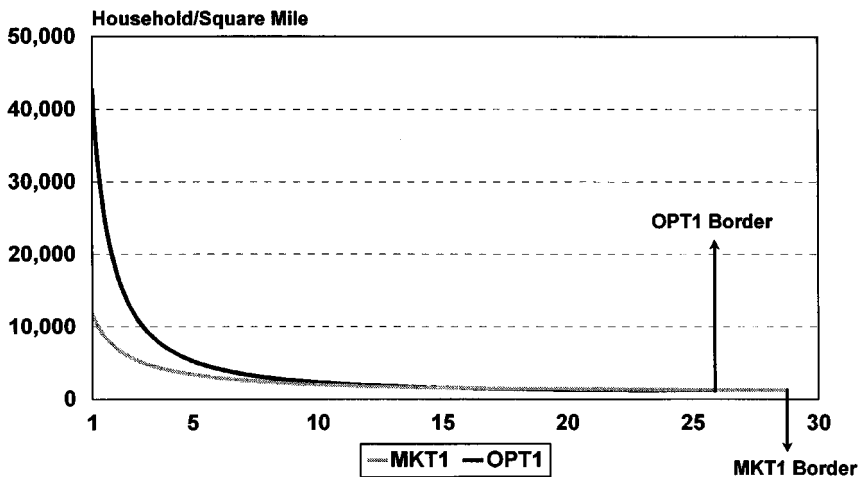


FIG. 1. Density vs. distance.

density occurs within the first 5 miles of the urban center. The reason for this pattern has to do with the uniform (exogenous) fraction of land devoted to roads. Close to the center, the 30% allocation results in quite high congestion levels, while near to the urban edge, it creates much excess capacity and insures completely free flow travel. The shadow rent for land reflects the social cost of land consumption, which given the particular provision of capacity, is greatly skewed toward the city center.⁸

The second set of simulations address the issue of how the provision of road capacity alters the optimal density gradient. Clearly, it is better to “target” the share of land devoted to roads—as opposed to providing a uniform fraction. In the Opt2 simulation, the combined density–capacity problem posed with the addition of Eq. (10) is solved. The results are interesting. Given the parameters and functions of this simulation, the share of land devoted to roads must be constrained (to be less than or equal to 100%) within the first few miles of the employment district. This allocation is perfectly reasonable, and in fact, tends to mirror the dense highways networks that surround more modern central business districts (e.g., Los Angeles, Dallas). One might even argue that the constraint should be relaxed and land allocations allowed to exceed 100%—as in the case of double-decked highways. In this case, however, it clearly would be

⁸If the (constant) fraction of land devoted to roads is reduced (e.g., to 20%), then the simulated difference between market and optimal density increase at the urban center, while looking little different at farther distances (where 20% still yields considerable excess capacity).

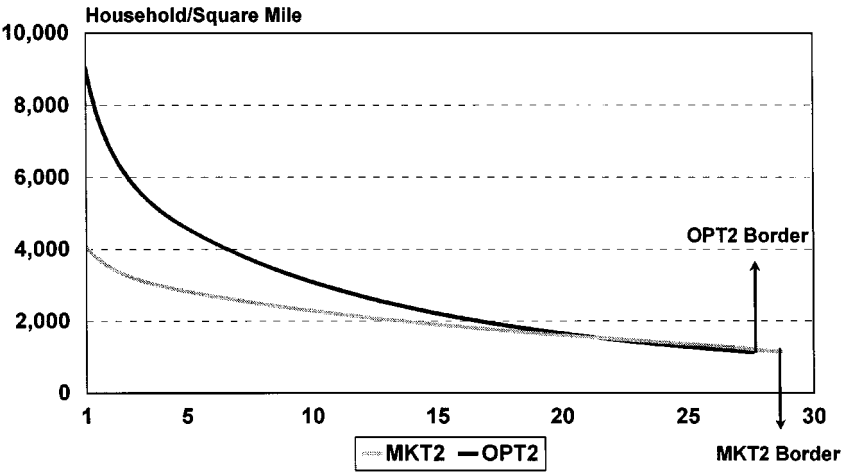


FIG. 2. Density vs. distance.

necessary to introduce capital costs into the model. In any event, the optimal allocation of land to roads (in the simulation Opt2) is clearly highly focused to match both the pattern of demand and the shadow price of land. At the urban border, for example, virtually no land is devoted to roads.

The market comparison to the simulation Opt2, involves taking the optimal provision of road capacity, and then letting the market develop land at levels which ignore the congestion externality.⁹ The results are shown in the Mkt2 column of Table 1, and are also displayed in Fig. 2.

With a pattern of targeted road capacity, congestion becomes somewhat more dispersed throughout the city and less concentrated right at the center. This of course means that the shadow rent for land will exceed the market rent at a broader range of locations and not just at the center. Thus at the center, the distortion drops from 4-fold to only somewhat more than 2-fold. In Fig. 2, optimal density—in the presence of more focused transport capacity—is now only 2.5 times market at the center. At a distance of 10 miles, however, it now is 50% greater, where previously it was virtually identical (Fig. 1).

⁹ It is important to point out that the first best capacity levels from the Opt2 simulation are not second best optimal—given market density behavior. This is the issue raised previously in the literature by authors such as Solow [18], Arnott [2], and Kanemoto [10].

VI. CONCLUSIONS

Commuting and congestion are inherent features of an urban landscape in which workplaces are distinct from residences. Without regulations or other forms of intervention, a private land market will ignore the social consequences of site development and will disperse residences *more* than should be the case. Significant improvements in welfare can be had by increasing central densities. Simulations suggest that such increases in density are likely to be many orders of magnitude. These simulations, however, also highlight the issue of whether workplaces should be so concentrated—particularly in a single center. This pattern creates much congestion, which even the complete devotion of land to infrastructure cannot fully overcome.

The results of this paper can be fully generalized to a city with multiple centers. Around each center, there will continue to exist a failure in the market to develop residential land at sufficient density. The result is excessive congestion and a waste of resources spent in traveling. The question that the current paper cannot address, is whether in a polycentric city, there will exist the optimal level of employment dispersal. It seems reasonable to presume that the individual locational decisions of firms will not necessarily lead to the optimal level of employment dispersal. This, however, is the subject of future research.

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